A STRONG VERSION OF THE BIRKHOFF–JAMES ORTHOGONALITY IN HILBERT $C^*$-MODULES

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This paper is dedicated to Professor T. Ando

Communicated by M. S. Moslehian

Abstract. In this paper we introduce a strong version of the Birkhoff–James orthogonality in Hilbert $C^*$-modules. More precisely, we consider elements $x$ and $y$ of a Hilbert $C^*$-module $V$ over a $C^*$-algebra $\mathcal{A}$ which satisfy $\|x\| \leq \|x + ya\|$ for all $a \in \mathcal{A}$. We show that this relation can be described as the Birkhoff–James orthogonality of appropriate elements of $V$, and characterized in terms of states acting on the underlying $C^*$-algebra $\mathcal{A}$. Some analogous relations of this type are considered as well.

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Date: Received: 27 August 2013; Revised: 16 September 2013; Accepted: 21 September 2013.

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2010 Mathematics Subject Classification. Primary 46L08; Secondary 46L05, 46B20.
Key words and phrases. Birkhoff–James orthogonality, state, Hilbert $C^*$-module, $C^*$-algebra.