SOLUTIONS FOR THE CONSTANT QUANTUM YANG-BAXTER EQUATION FROM LIE (SUPER)ALGEBRAS

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Abstract. We present a systematic procedure to obtain singular solutions of the constant quantum Yang-Baxter equation in arbitrary dimension. This approach, inspired by the Lie (super)algebra structure, is explicitly applied to the particular case of (graded) contractions of the orthogonal real algebra $\mathfrak{so}(N+1)$. In this way we show that "classical" contraction parameters which appear in the commutation relations of the contracted Lie algebras, become quantum deformation parameters, arising as entries of the resulting quantum $R$-matrices.

1. Introduction

Quantum $R$-matrices are solutions of the constant quantum Yang-Baxter equation (cQYBE)

$$R_{12}R_{13}R_{23} = R_{23}R_{13}R_{12}$$

(1)

where $R = \sum_i a_i \otimes b_i$ is a linear operator acting on a $D^2$-dimensional space and

$$R_{12} \equiv \sum_i a_i \otimes b_i \otimes 1, \ R_{13} \equiv \sum_i a_i \otimes 1 \otimes b_i, \ R_{23} \equiv \sum_i 1 \otimes a_i \otimes b_i.$$  

(2)

The cQYBE can be considered as a limiting case of the QYBE with spectral parameters, which constitutes the algebraic keystone for the integrability properties of $(1+1)$ solvable models [1, 2]. Constant quantum $R$-matrices have been shown to be relevant in quantum group theory and non-commutative geometry [3], since constant quantum $R$-matrices can be used to get the defining relations for non-commutative spaces such as the ones obtained under different generalizations/deformations of the special relativity theory (see [4] and references therein). Several classifications for the solutions of the cQYBE, mainly concerning low dimensions, can be found in [5–9]. However, few constructive procedures for solutions in arbitrary dimensions $D$ are available. The aim of this contribution is to present a systematic construction of multiparametric solutions of the cQYBE