LINEAR CONNECTIONS AND EXTENDED ELECTRODYNAMICS

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Abstract. In this paper we give a presentation of the basic vacuum relations of Extended Electrodynamics in terms of linear connections.

1. Linear Connections

Linear connections are first-order differential operators in vector bundles. If such a connection \( \nabla \) is given and \( \sigma \) is a section of the bundle, then \( \nabla \sigma \) is one-form on the base space valued in the space of sections of the vector bundle, so if \( X \) is a vector field on the base space then \( i(X) \nabla \sigma = \nabla_X \sigma \) is a new section of the same bundle [2]. If \( f \) is a smooth function on the base space then \( \nabla(f \sigma) = df \otimes \sigma + f \nabla \sigma \), which justifies the differential operator nature of \( \nabla \): the components of \( \sigma \) are differentiated and the basis vectors in the bundle space are linearly transformed.

Let \( e_a \) and \( \varepsilon^b, a, b = 1, 2, \ldots, r \) be two dual local bases of the corresponding spaces of sections \( \langle \varepsilon^b, e_a \rangle = \delta_a^b \), then we can write

\[
\sigma = \sigma^a e_a, \quad \nabla = d \otimes id + \Gamma^b_{\mu a} dx^\mu \otimes (\varepsilon^a \otimes e_b), \quad \nabla(e_a) = \Gamma^b_{\mu a} dx^\mu \otimes e_b
\]

and therefore

\[
\nabla(\sigma^m e_m) = d\sigma^m \otimes e_m + \sigma^m \Gamma^b_{\mu a} dx^\mu \langle \varepsilon^a, e_m \rangle \otimes e_b = \left[ d\sigma^b + \sigma^a \Gamma^b_{\mu a} dx^\mu \right] \otimes e_b
\]

where \( \Gamma^b_{\mu a} \) are the components of \( \nabla \) with respect to the coordinates \( \{x^\mu\} \) on the base space and with respect to the bases \( \{e_a\} \) and \( \{\varepsilon^b\} \).

Since the elements \( (\varepsilon^a \otimes e_b) \) define a basis of the space of (local) linear maps of the local sections, it becomes clear that in order to define locally a linear connection it is sufficient to specify some one-form \( \theta \) on the base space and a