MODULAR FORMS ON BALL QUOTIENTS OF NON-POSITIVE KODAIRA DIMENSION

AZNIV KASPARIAN

Communicated by Vasil V. Tsanov

Abstract. The Baily-Borel compactification \( \mathbb{B}/\Gamma \) of an arithmetic ball quotient admits projective embeddings by \( \Gamma \)-modular forms of sufficiently large weight. We are interested in the target and the rank of the projective map \( \Phi \), determined by \( \Gamma \)-modular forms of weight one. This paper concentrates on the finite \( H \)-Galois quotients \( \mathbb{B}/\Gamma_H \) of a specific \( \mathbb{B}/\Gamma_{(6,8)} \), birational to an abelian surface \( A_{-1} \). Any compactification of \( \mathbb{B}/\Gamma_H \) has non-positive Kodaira dimension. The rational maps \( \Phi^H \) of \( \mathbb{B}/\Gamma_H \) are studied by means of the \( H \)-invariant abelian functions on \( A_{-1} \).

The modular forms of sufficiently large weight are known to provide projective embeddings of the arithmetic quotients of the two-ball

\[
\mathbb{B} = \{ z = (z_1, z_2) \in \mathbb{C}^2 ; |z_1|^2 + |z_2|^2 < 1 \} \cong \text{SU}(2,1)/\text{S}(U_2 \times U_1).
\]

The present work studies the projective maps, given by the modular forms of weight one on certain Baily-Borel compactifications \( \overline{\mathbb{B}}/\Gamma_H \) of Kodaira dimension \( \kappa(\mathbb{B}/\Gamma_H) \leq 0 \). More precisely, we start with a fixed smooth Picard modular surface \( A'_{-1} = \left( \mathbb{B}/\Gamma_{(6,8)} \right)' \) with abelian minimal model \( A_{-1} = E_{-1} \times E_{-1} \), \( E_{-1} = \mathbb{C}/\mathbb{Z} + \mathbb{Z}i \). Any automorphism group of \( A'_{-1} \), preserving the toroidal compactifying divisor \( T' = \left( \mathbb{B}/\Gamma_{(6,8)} \right)' \setminus \left( \mathbb{B}/\Gamma_{(6,8)} \right) \) acts on \( A_{-1} \) and lifts to a ball lattice \( \Gamma_H \), normalizing \( \Gamma_{(6,8)} \). The ball quotient compactification \( A'_{-1}/H = \overline{\mathbb{B}}/\Gamma_H \) is birational to \( A_{-1}/H \). We study the \( \Gamma_H \)-modular forms \( \left[ \Gamma_H, 1 \right] \) of weight one by realizing them as \( H \)-invariant of \( \left[ \Gamma_{(6,8)}, 1 \right] \). That allows to transfer \( \left[ \Gamma_H, 1 \right] \) to the \( H \)-invariant abelian functions, in order to determine \( \dim_{\mathbb{C}}[\Gamma_H, 1] \) and the transcendence dimension of the graded \( \mathbb{C} \)-algebra, generated by \( \left[ \Gamma_H, 1 \right] \). The last one is exactly the rank of the projective map \( \Phi : \overline{\mathbb{B}}/\Gamma_H \longrightarrow \mathbb{P}([\Gamma_H, 1]) \).

69