CLASSICAL-QUANTUM CORRESPONDENCE AND
WAVE PACKET SOLUTIONS OF THE DIRAC
EQUATION IN A CURVED SPACE-TIME

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Abstract. The idea of wave mechanics leads naturally to assume the
well-known relation $E = h\omega$ in the specific form $H = hW$, where $H$ is
the classical Hamiltonian of a particle and $W$ is the dispersion relation of
the sought-for wave equation. We derive the expression of $H$ in a curved
space-time with an electromagnetic field. Then we derive the Dirac equation
from factorizing the polynomial dispersion equation corresponding with $H$.
Conversely, summarizing a recent work, we implement the geometrical op-
tics approximation into a canonical form of the Dirac Lagrangian. Euler-
Lagrange equations are thus obtained for the amplitude and phase of the
wave function. From them, one is led to define a four-velocity field which
obeys exactly the classical equation of motion. The complete de Broglie
relations are then derived as exact equations.

1. Introduction

1.1. Context of This Work

The long-standing problem of quantum gravity may mean, of course, that we
should try to better understand gravity and the quantum. More specifically, it may
mean that we should try to better understand the transition between the classical
and the quantum, especially in a curved space-time. Quantum effects in the clas-
sical gravitational field are indeed being observed on neutral particles such as neu-
trons [11, 15, 19] or atoms [13, 18], with the neutrons being spin $\frac{1}{2}$ particles. This
together motivates investigating the “classical-quantum correspondence”—the cor-
respondence between a classical Hamiltonian and a quantum wave equation—for
the Dirac equation in a curved space-time.