ON THE GENERALIZED $f$-BIHARMONIC MAPS AND STRESS $f$-BIENERGY TENSOR

DJAA MUSTAPHA AND AHMED CHERIF

Abstract. In this paper, we investigate some properties for generalized $f$-harmonic and $f$-biharmonic maps between two Riemannian manifolds. In particular we present some new properties for the generalized stress $f$-energy tensor and the divergence of the generalized stress $f$-bienergy.

1. Introduction

Consider a smooth map $\varphi : M \rightarrow N$ between Riemannian manifolds $M = (M^m, g)$ and $N = (N^n, h)$ and $f : M \times N \rightarrow (0, +\infty)$ is a smooth positive function, then the $f$-energy functional of $\varphi$ is defined by

$$E_f(\varphi) = \frac{1}{2} \int_M f(x, \varphi(x)) |d_x \varphi|^2 v_g$$

(or over any compact subset $K \subset M$).

A map is called $f$-harmonic if it is a critical point of the $E_f(\varphi)$. In terms of Euler-Lagrange equation, $\varphi$ is harmonic if the $f$-tension field of $\varphi$ is:

$$\tau_f(\varphi) = f \varphi \nabla \varphi + d \varphi(\nabla f \varphi) - e(\varphi)(\nabla f) \circ \varphi.$$

The $f$-bienergy functional of $\varphi$ is defined as

$$E_{2,f}(\varphi) = \frac{1}{2} \int_M |\tau_f(\varphi)|^2 v_g.$$ 

A map is called $f$-biharmonic if it is a critical point of the $f$-bienergy functional.

The $f$-harmonic and $f$-biharmonic concept is a natural generalization of harmonic maps (Eells and Sampson [8]), and biharmonic maps (Jiang [9]).

In mathematical physics, $f$-harmonic maps, are related to the equations of the motion of a continuous system of spins (see [6]) and the gradient Ricci-soliton structure (see [12]).