ANALYSIS OVER $C^\ast$-ALGEBRAS AND THE OSCILLATORY REPRESENTATION

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Abstract. Since the last two decades, several differential operators appeared in connection with the so-called oscillatory geometry. These operators act on sections of infinite rank vector bundles. Definitions of the oscillatory representation, metaplectic structure, oscillatory Dirac operator, as well as some necessary fundamental results in the analysis in $C^\ast$-Hilbert bundles are recalled here. These results are used for a description of the kernel of a certain second order differential operator arising from oscillatory geometry and the cohomology groups of the de Rham complex of exterior forms with values in the oscillatory representation.

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