Abstract. We analyze several types of soliton solutions to a family of Tzitzeica equations. To this end we use two methods for deriving the soliton solutions: the dressing method and Hirota method. The dressing method allows us to derive two types of soliton solutions. The first type corresponds to a set of six symmetrically situated discrete eigenvalues of the Lax operator $L$; to each soliton of the second type one relates a set of twelve discrete eigenvalues of $L$. We also outline how one can construct general $N$ soliton solution containing $N_1$ solitons of first type and $N_2$ solitons of second type, $N = N_1 + N_2$. The possible singularities of the solitons and the effects of change of variables that relate the different members of Tzitzeica family equations are briefly discussed. All equations allow quasi-regular as well as singular soliton solutions.

MSC: 35Q51, 35Q53, 37K40

Keywords: Tzitzeica equations, singular soliton solutions, Zakharov-Shabat dressing method, Hirota method

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