EXACT INTEGRATION OF A NONLINEAR MODEL OF STEADY HEAT CONDUCTION/RADIATION IN A WIRE WITH INTERNAL POWER

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Abstract. The paper treats in one dimensional mixed heat transfer problem of steady conduction and radiation in a wire with internal source. We are led to a Cauchy problem consisting of a second order nonlinear ordinary differential equation. A special integrable case with two non independent left boundary conditions requires a hyperelliptic integral, for which a representation theorem has been established through the Gauss hypergeometric function $\,_{2}F_{1}$. The relevant steady solution is then found to grow monotonically with the distance from boundary, up to a certain limiting position where it suddenly jumps unbounded.

1. Introduction

Conduction, namely the flow of thermal energy through solid bodies, was modelled by Jean B. Fourier (1768-1830) who first inquired into the general principles of it. Throughout his Théorie analytique de la chaleur (1822), he established a partial differential equation (PDE) for analyzing the temperature distribution within a conducting body. His analytical conduction theory disregards the molecular structure of a body and thinks of it as a continuum, but after Fourier it has been understood that-on the contrary- conduction is actually caused by particle collisions. His linear PDE, in one dimensional geometry, is

$$\rho c_{p} \frac{\partial T}{\partial t}(t, x) = \chi \frac{\partial^{2} T}{\partial x^{2}}(t, x)$$

where the material data are: thermal conductivity $\chi$, specific heat capacity $c_{p}$ and volumetric density $\rho$. As far as it concerns the spatial effects, the PDE has to be solved with suitable boundary conditions (BC).

Transient problems ($\partial T/\partial t \neq 0$) will also need initial conditions (IC) on $T$ for every position in the system: the PDE is parabolic and heat propagates at infinite