Abstract. The work is basically a review article and partially a research paper. Problems for evaluation and impact of published scientific works and their authors are discussed at theoretical level. The role of citations in this process is pointed out. Different bibliometric indicators are reviewed in this connection and ways for generation of new bibliometric indices are given. The influence of different circumstances, like self-citations, number of authors, time dependence and publication types, on the evaluation and impact of scientific papers are considered. The repercussion of works citations and their content is investigated in this respect. Attention is paid also on implicit citations which are not covered by the modern bibliometrics but often are reflected in the peer reviews. Some aspects of the Web analogues of citations and new possibilities of the Internet resources in evaluating authors achievements are presented.

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1. Introduction

Can the scientific output of a scientists be measured quantitatively? We often said that someone has better achievements than other person but explain this with non-strict words and opinions of experts in the corresponding field of research which certainly can not be measured quantitatively (except counting some kind of votes via a qualitative procedure of rating). To such qualifications often are added strict number measures like the number of published papers and their (known) number of total citations. The former is a measure of author productivity while the latter one is considered as his/her impact on (other) authors. Just here comes into
action the bibliometrics\(^1\) which has as input data the raw information about an author published (and publicly available) works and their recorded influence on other published works and as an output gives quantitative conclusions concerning the author.\(^2\) This process is well described in [49]. A deep investigation in this area is performed in the ACUMEN project [23].

The bibliometrics provides a number of already established numerical characteristics of authors publications and their citations [20,53], known as bibliometric indicators, such as number of publications (total and for some period of time), number of publications in top journals, number of citations (total and for some period of time), citations per publication, top 5% citations, etc. Starting from 2005 Hirsch paper [29] there were introduced a number of new bibliometric indicators [4] like the Hirsch index \(h\) and different (Hirsch-like) indices that modify it in ways that compensate some its disadvantages. Regardless of these rigorous measures, the peer judgements remain leading in taking decisions about the achievements of papers and their authors. On statistical level is observed a correlation between assessment by different bibliometric indicators and quality judgment of peers [11,51,63]. This naturally suggest the both methods to be used as complimentary to each other.

This paper is an extended and revised version of [31]. It has aspects of a review article and a research paper simultaneously. Section 2 points to some peculiarities of citing in different types of publications and concerns the problem of self-citations. Section 3 is devoted to citations lists and ways for preparing them. Different forms of citation lists are presented in Section 4. Special attention is paid to citations of works with more than one author and to citing papers with multiple authors. Section 5 deals with some bibliometric indices. The Hirsch index, certain its modifications and complimentary to it indices are recalled. Ways for generation of new bibliometric indices are provided. Section 6 concerns problems like self-citation, number of authors, highly/low cited papers, and time dependence of the citations. Connections between citations and scientific achievements are discussed in Section 7. In Section 8 are presented some aspects of the problem on how the content of a paper may influence its evaluation and impact. The implicit citations are discussed in Section 9. The role of the peers is mentioned in Section 10. The paper ends with a final discussion in Section 11.

As the author of this paper works mainly in the field of (mathematical) physics and mathematics, the problems investigated in it concern physics literature but it is likely that they apply also to other scientific fields.

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\(^1\)Sometimes the bibliometrics is called scientometrics but these are different overlapping things [27,30].

\(^2\)Here and below we talk about author(s) but in the most cases the text is true for group of authors, journal, university, county etc.
2. When a Work is Cited?

In physics any scientist builds his/her work on the base of earlier existing works and for this reason new works/publications cite the works they are build upon. A deep analysis of this process is contained in [44] and more particular reasons for citing are presented in [26, Section 4.1]. In this way is made a link with already existing knowledge and is paid tribute to the work of the scientist that have contributed to it. In this sense the citation is part of the process of linking of a work with the knowledge preceding it. It is known that the more citations a published paper has, the more impact it has on the other authors [48] but the problem is to evaluate this impact quantitatively. As a consequence of this the (number of) citations of a paper is a measure for its impact on other works and scientists. Respectively, the (number of) citations of the papers of an author is a measure for his/her impact.

The reasons for citing a particular work and putting it in a reference/bibliography list of papers are numerous and depend on the type of the publication in which it is cited, its content and the authors. Below we shall try to analyze this problem for some kinds of works and to make conclusions which may be useful for finding criterions for evaluation of publishing (and, possibly scientific) activity of a person. A comprehensive analysis of the reasons for citing can be found in [44]. An analysis of the citation process can be found in [41, 46, 64]. In [52, pages 11–13] is presented a list of factors that affect the number of citations.

Besides the types of works considered below, there may be distinguished many other types of published works. Moreover, there exist works that are of mixed type, e.g. a handbook or review paper containing new results and thus having elements of a research work. Here we are not going to present a “complete” list and analysis of publication types and note that an honest citation of a paper is intended to point readers attention to it and may mean that the author(s) has (have) used some information from the cited work.

2.1. Citation in a Research Paper

The research papers are regular discovery accounts and are usually in the form of journal articles, preprints, electronic preprints and others. As their short versions can be regarded the meeting communications (abstracts, full or part text articles), short communications, letters (possibly to editors), notes, corrections/additions of/to earlier works, etc. At present a typical (full text) research paper has an introduction, main body and concluding part.

Some of the roles of the introduction are: i) to present the main problems that will be investigated further in the work; ii) to pay attention to (some of) the existing
results on them; iii) to fix certain notations, concepts and results that will be used in the work; iv) to point to the history and possible future developments of the items of the work. So, citing a paper in the introductory part of a research work may mean different things like

1. It belongs to a general list of references on some item considered in the work.

2. It contains essential results that will be used, developed, commented, etc. in the work.

3. Contains problem(s) that will be investigated in the citing work.

4. It is of pure historical interest; e.g. representing a wrong theory.

The main body of a research work contains rigorous statement of some problems, their analysis and, possibly, their solutions. Respectively, normally a paper is cited here when it is directly connected with these problems and its content is (partially) used in the work. This means that a paper cited in the main body has, generally, more impact on the work than a paper cited in the introduction (if something else is not stated explicitly).

At last, the purpose of the conclusion may be: i) summarizing the outcome of the main body of the work, ii) comments/analysis of the results obtained, iii) making connections with other works containing results of interests, iv) pointing to non-solved problems and further developments. Correspondingly, here typically a paper is cited when it poses similar problems but its results do not influence directly the main developments of the work.

A deeper analysis of the citation process can be found in [41, 46, 64].

2.2. Citation in a Review Work

The main aim of a review work is to bring together results obtained in research papers for some period of time. However, the particular realization of such a work may be done in quite different ways, for example

1. A simple list of literature with possible comments.

2. An independent presentation of the material, e.g. in a book or book-like paper.

3. An unified presentation of groups of papers in different sections forming the main body of the work.
In any case, a review paper is generally not suppose to contain new results. Its main purpose is to put in a single place results that can be found in different sources which form the main part of the citation list of such kind of a work. In this sense, most of the papers cited in a review work are essentially used. Besides, a citation of a particular part of a review work may be considered, in some sense, as citation of the original papers on which the cited part rest.

2.3. Citation in a Handbook, Encyclopedia and Similar Works

The handbooks and encyclopedias may be regarded as review works but they have more specific structure, presentation and usually cover larger areas of materials. A typical work of this kind consists of series of (alphabetically ordered) separate papers (articles) with possible cross-references between them. They contain normally only presentation of facts (results, theorems, methods) with little comments and their reference lists are restricted to represent (details on) these facts. So, any paper cited in a handbook or encyclopedia is essentially used in it. Besides, citing an article of a such a work may be regarded as an indirect citation (of some) of the papers in its reference list.

2.4. Citation in Textbook

The purpose of a textbook is learning the material presented in it. This usually limits the citations in it, if any, to publications that are: i) other textbooks on the same or similar material, ii) containing original (e.g. historical) material on the covered items, iii) further developments on the subject(s) covered, iv) used by the author(s) to write it.

2.5. Self-Citations

There are many reasons when an author cites his/her own paper(s). Normally this is done when the author has previous publications on the subject(s) of the work where self-citations appear and he/she finds them essential in the context where they are cited. In this sense, the self-citations reveal the self-impact of an author and should be treated on the same footing as any other citations.

It should be said that there are authors that intentional cite their own papers for, let us say, “non-scientific” reasons; e.g. popularizing own works, extending the list of citations of their works etc. The author of these lines would like to think that these are exceptional cases, at least in the case of research papers and may be neglected in the general case. However, if there are facts that a particular author belongs
to this category of authors, then he/she may be blamed as non-honest with respect to his/her citation list and the self-citations in it should be considered critically or neglected at all.

We shall return on the problem for self-citations in Subsection 6.1.

2.6. Inferences

Without considering other types of publications and treating self-citations as ordinary citations, we may point to some of the main reasons for citations

1. Using particular information, like results, methods and formulae, form the cited works.

2. Pointing to texts from the cited work without using them.

3. Pointing the readers attention to works connected to the subject(s) considered in the citing work.

4. Presenting list(s) of publications on some item(s).

The impact of a cited paper on the citing one depends on the category to which it belongs. It seems that most weight should be given to citing paper from the first of the above category. However, it is unlikely that particular numerical weights can be assigned to some or all categories of the citations and, as a result of this, the arrangement of these categories by weights is qualitative. Of course, the impact of a paper depends on its content and the contents of the works citing it.

3. Lists of Citations

Nowadays there is an understanding that the more citations an author has, the greater is his/her impact in Science.\textsuperscript{3} A list of author citations may have different purposes like

- To show other scientists how his/her works are used by other authors.

- It is needed for some official (possibly internal) account/report.

- It may be a part of the reasons for obtaining scientific degree or a promotion.

\textsuperscript{3}Here is excluded the problem of the content of the papers cited as well as the context in which the citations are made. For instance, an evident counter example of this understanding is a citation in which is pointed plagiarism in the cited work.
• It may be a reason for author proud or simply a way to tell other scientist which authors have used his/her works.

A preparation of author citation list is not an easy task in times when there are literally tens of thousands of scientific journals, institutional/university annual reports, books etc. published e.g. monthly or annually. The easiest way to make such a list is via the Internet based databases like (see [8] for some instructions on that item):

5. CiteSeerX (free) with URL http://citeseerx.ist.psu.edu/ . It has replaced the database CiteSeer.
7. The program Publish or Perish (free), http://www.harzing.com/pop.htm

The above databases cover differently different scientific fields and types of publications [20, pages 349–350] like journal articles, electronic preprints, books/monographs, conference reports, theses, etc. A concise and good analysis of them is given in [35]. In general, they give overlapping but not identical results [4,7,38,61].

A less efficient way for finding citations is to search the Web for some combinations of key-words including the name(s) of the author whose citations are looked for and possibly the names of the authors who may cite him/her.

For preparation of citation lists in the field of physics and/or mathematics one can use also the sites

2. IOP eprint web with URL http://eprintweb.org which is based on the arXiv.

\(^4\)The Web of Science (WoS) is an electronic version of the Science Citation Index (SCI) [22].
3. SAO/NASA Astrophysics Data System (ADS), http://adsabs.harvard.edu

Of course, for making citation lists one may use more “conventional” resources like

1. (accidental) reading of scientific papers.
2. personal acquaintance with scientists.
3. consultations with the Science Citation Index (SCI) of the Thomson Reuters Institute for Scientific Information which is a paper version of The Web of Science (WoS).

It is important to note that the data in a citation list should be publicly available as otherwise it is (almost) impossible to check/verify independently its trueness.

The completeness of a citation list depends on the sources used, i.e., the data sets from which it is prepared. In this sense, a particular citation list gives also a lower limit on the number of works with non-zero citations as well the number of their citations.

For the purposes of this paper we assume below that an author citation list includes all his/her published papers; in particular, these with zero number of citations.

4. Analysis and Forms of Citation Lists

For our purposes it is convenient to arrange the author’s papers in a citation list in order of descending number of their citations.\(^5\) Besides, if some works have equal number of citations, then we consider their relative order as insignificant and, consequently, they can be arrange in such a list in an arbitrary way relative to each other, e.g. alphabetically by their titles.\(^6\) The consecutive number of a paper in such a list is called its rank (in this list). So, at this stage, a citation list of an author with \(n \geq 1\) published papers can be represented as like the Table 1 on the following page.

A little information can be obtained from Table 1 on the next page without a comparison with similar tables for other authors. The main inference is that the more

\(^5\)Of course, the publication can be arrange by other criteria like number of author, date/time of publishing, impact (by some measure/metric) etc. Any such criterion has it pros and con and by its application can be drown different conclusions.

\(^6\)If one want to make finer analysis, this arrangement may become important. For instance, one may arrange them by date/time of publishing, publication type, field of research, number of authors, and so on and make conclusions on this base.
Table 1. Initial example form of a citation list. Here \( c_i, \ i = 1, \ldots, n, \) is the number of citation of the paper with rank \( i \) and description \( p_i \). By definition \( c_i \geq c_{i+1} \) for \( i = 1, \ldots, n - 1 \) and it is possible that \( c_i = 0 \) for \( i \geq n_0 \) for some \( n_0 \in \{1, \ldots, n\} \).

<table>
<thead>
<tr>
<th>Number of citations</th>
<th>Rank</th>
<th>Paper description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( c_1 )</td>
<td>1</td>
<td>( p_1 )</td>
</tr>
<tr>
<td>( c_2 )</td>
<td>2</td>
<td>( p_2 )</td>
</tr>
<tr>
<td>\vdots</td>
<td>\vdots</td>
<td>\vdots</td>
</tr>
<tr>
<td>( c_n )</td>
<td>( n )</td>
<td>( p_n )</td>
</tr>
</tbody>
</table>

A paper is closer to the top of the list, the more it has been used by the authors and vice versa, the closer a paper is to the table end, the less it has been used. At this stage, the paper rank is a measure of its importance for the authors: the less the rank, the more important a paper is and vice versa. As a quantitative measure for this opinion may serve the numbers

\[
c^r_i := \frac{c_i}{\sum_{i=1}^{n} c_i} \tag{1}
\]

which are the citation numbers normalized by their sum, so that \( 0 \leq c^r_i \leq 1 \) and \( \sum_{i=1}^{n} c^r_i = 1 \). Of course, here we suppose that the author has at least one published work with at least one citation.

Usually, there is a number \( n_0 \in \{1, \ldots, n\} \) such that \( c_i = 0 \) for \( i \geq n_0 \), i.e., the papers with rank greater then or equal to \( n_0 \) have no citations and the first \( n_0 - 1 \) papers in the table (with rank less than \( n_0 \)) have at least one citation. If such a number \( n_0 \) exists, the ratio

\[
E := \frac{n_0 - 1}{n} \tag{2}
\]

can be called author effectiveness (or coefficient of performance (COP) or coefficient of efficiency) as it measures how much of his/her published works have been used by (other) authors. If there is no a number \( n_0 \) with the properties required, we set \( n_0 = n + 1 \) and \( E = 1 \). So that \( 0 \leq E \leq 1 \).

Obviously, the greater the author efficiency, the more of his/her published works have been used by authors and possibly influenced their papers. However, this measure is individual and is inadequate for comparing authors; e.g. two authors with equal efficiencies may have essentially different number of citations.
4.1. Cited Papers with Multiple Authors

Till this point we have not mentioned problems concerning the number of authors of any particular work in which the author has contributed (as a coauthor). Since we aim to make conclusions concerning a particular person, the above written is valid in a case when all papers in Table 1 on the facing page are written by a single person, i.e., there are not other co-authors. However, in the general case, when the paper $p_i$ has $a_i \geq 1$ authors, the needed for our purposes modification of Table 1 on the preceding page may look like the next Table 2.

Table 2. Citation list including the number of authors. Here $a_i$, $i = 1, \ldots, n$ is the number of authors of the paper $p_i$.

<table>
<thead>
<tr>
<th>Number of citations</th>
<th>Rank</th>
<th>Number of authors</th>
<th>Paper description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_1$</td>
<td>1</td>
<td>$a_1$</td>
<td>$p_1$</td>
</tr>
<tr>
<td>$c_2$</td>
<td>2</td>
<td>$a_2$</td>
<td>$p_2$</td>
</tr>
<tr>
<td>\vdots</td>
<td>\vdots</td>
<td>\vdots</td>
<td>\vdots</td>
</tr>
<tr>
<td>$c_n$</td>
<td>$n$</td>
<td>$a_n$</td>
<td>$p_n$</td>
</tr>
</tbody>
</table>

How we should proceed if there is at least one paper with at least two authors? It is intuitively clear that in such a case the personal impact ("fame") of a particular author should be connected somehow with his/her contributions in a multiple author paper (see the discussion on this item in [49, page 4, Case 3]). Generally we can distinguish the following main cases.

1. The authors do not supply any information about their personal contributions in their joint paper or they write that these contributions cannot be distinguished.

2. It is explicitly said which parts of the work by who of the coauthors are personally written.

3. The authors present concrete information about their contributions in a form of numbers.

Evidently, there may be many other cases, e.g. different parts of a work realize some/all of the above three possibilities. As we do not want to overload the presentation with too much details, we shall restrict our consideration to the above cases.
The most clear is Case 3. Suppose we talk about paper \( p_i \) of Table 2 on the previous page for some fixed \( i \). Then to the \( j \)-th, \( j = 1, \ldots, a_i \), coauthor corresponds a number (weight) \( w_{aj}^{ai} \) such that \( 0 < w_{aj}^{ai} < 1 \), \( \sum_{j=1}^{a_i} w_{aj}^{ai} = 1 \) and the contribution of the \( j \)-th author is exactly \( w_{aj}^{ai} \).

The complete lack of information about personal authors contributions in Case 1 leads to only one hypothesis for rigorous analysis, namely that all coauthors have equal contribution in the work. This hypothesis, which we assume, reduces Case 1 to Case 3 with \( w_{aj}^{ai} = 1/a_i \).

Case 2 does not supply sufficient information for a rigorous analysis. For example, a judgement of an author’s contribution by the number of pages he/she has written is not serious. Our intention is to reduce this case to Case 3 but there is not enough information to do this. So again, we shall assume that \( w_{aj}^{ai} = 1/a_i \). However, regardless of the equalization of authors contributions, the information given in Case 2 may lead to some consequences for our next considerations.

We shall call the numbers \( w_{aj}^{ai} \) *personal authors weights*. We assume that \( w_{aj}^{ai} = 1 \) for \( a_i = 1 \) to cover also the single-author case.

The general approach to the fractionalizing and weighting the number of publications and of the citations is outlined in [52, pages 22–23].

Let us now return to a citation list form from the viewpoint of the contributions of the author to whom it belongs. Taking into account the above discussion, we should add to the citation list a new column containing in its \( i \)-row the personal author weight \( w_i^a \) for the paper \( p_i \). At this point it becomes evident that not all of the fame for the paper \( p_i \) having \( c_i \) citations belongs to the considered author if \( a_i > 1 \), i.e., for \( w_{ai}^i < 1 \). Since the number \( w_i^a \) is the only measure for the author’s particular contribution, we shall assume that from all \( c_i \) citations of the paper \( p_i \) only the part \( c_i^a := w_i^a c_i \) belong to that author. We shall call the numbers \( c_i^a := w_i^a c_i \) *author-reduced number of citations* of the paper \( p_i \). Its inclusion in a citation list leads to the Table 3 on the facing page as a new form of citations lists. Now the reduced citation numbers \( c_i^a \) play the role of the citation numbers \( c_i \) at the beginning of this section, so we shall rearrange Table 3 on the next page by their descending order and will introduce the reduced rank that numbers the rows of the rearranged table. In this way we obtain Table 4 on the facing page as a new form of a citation list.

From Table 4 on the next page can be drown conclusions similar to the ones at the beginning of this section, but now covering the multiple author case.

If \( c_i = 0 \) for \( i \geq n_0 \) for some \( n_0 \in \{1, \ldots, n\} \), then \( c_i^a = w_i^a \cdot 0 = 0 \). For this reason the works with zero citations sit at the bottom of Table 4 on the facing page and their relative order from Table 3 on the next page can be preserved.
Table 3. Citation list including data for author personal contributions. Here \( w^a_i, \ i = 1, \ldots, n \), is the personal author weight for the paper \( p_i \).

<table>
<thead>
<tr>
<th>Number of citations</th>
<th>Rank</th>
<th>Number of authors</th>
<th>Author weight</th>
<th>Reduced number of citations</th>
<th>Paper description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( c_1 )</td>
<td>1</td>
<td>( a_1 )</td>
<td>( w^a_1 )</td>
<td>( c^1_1 = w^a_1 c_1 )</td>
<td>( p_1 )</td>
</tr>
<tr>
<td>( c_2 )</td>
<td>2</td>
<td>( a_2 )</td>
<td>( w^a_2 )</td>
<td>( c^2_2 = w^a_2 c_2 )</td>
<td>( p_2 )</td>
</tr>
<tr>
<td>( \vdots )</td>
<td>( \vdots )</td>
<td>( \vdots )</td>
<td>( \vdots )</td>
<td>( \vdots )</td>
<td>( \vdots )</td>
</tr>
<tr>
<td>( c_n )</td>
<td>( n )</td>
<td>( a_n )</td>
<td>( w^a_n )</td>
<td>( c^n_n = w^a_n c_n )</td>
<td>( p_n )</td>
</tr>
</tbody>
</table>

Table 4. Citation list arranged by descending order of the reduced number of citations. Here \((r_1, \ldots, r_n)\) and \((k_1, \ldots, k_n)\) are permutations of \((1, \ldots, n)\) and \( c^a_k, \ k = 1, \ldots, n \), \( c^a_{k+1} \), \( i = 1, \ldots, n - 1 \).

<table>
<thead>
<tr>
<th>Reduced number of citations</th>
<th>Reduced rank</th>
<th>Number of authors</th>
<th>Rank</th>
<th>Number of authors</th>
<th>Author weight</th>
<th>Paper description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( c^a_{k_1} = w^a_{k_1} c_{k_1} )</td>
<td>1</td>
<td>( c_{k_1} )</td>
<td>( r_{k_1} )</td>
<td>( a_{k_1} )</td>
<td>( w^a_{k_1} )</td>
<td>( p_{k_1} )</td>
</tr>
<tr>
<td>( c^a_{k_2} = w^a_{k_2} c_{k_2} )</td>
<td>2</td>
<td>( c_{k_2} )</td>
<td>( r_{k_2} )</td>
<td>( a_{k_2} )</td>
<td>( w^a_{k_2} )</td>
<td>( p_{k_2} )</td>
</tr>
<tr>
<td>( \vdots )</td>
<td>( \vdots )</td>
<td>( \vdots )</td>
<td>( \vdots )</td>
<td>( \vdots )</td>
<td>( \vdots )</td>
<td>( \vdots )</td>
</tr>
<tr>
<td>( c^a_{k_n} = w^a_{k_n} c_{k_n} )</td>
<td>( n )</td>
<td>( c_{k_n} )</td>
<td>( r_{k_n} )</td>
<td>( a_{k_n} )</td>
<td>( w^a_{k_n} )</td>
<td>( p_{k_n} )</td>
</tr>
</tbody>
</table>

In multi-author paper may be important the order of the authors as it may (implicitly) point to different role of the authors in writing the work. A good analysis on this topic is presented in [17] in which is concluded that “There is a strong trend for signatures of younger researchers and those in the lower professional ranks to appear in the first position (junior signing pattern), while more veteran or highly-ranked ones, who tend to play supervisory functions in research, are proportionally more likely to sign in the last position (senior signing pattern)”.

4.2. Citations in Papers with Multiple Authors

The consideration of the number of authors of the citing papers leads to other form of citation lists that reveals in a finer way the impact of the author of the cited papers on (other) authors. The simple number of citations of a work shows only how many times it has been used in other works. However, it is not one and the same when a citing paper has one or more than one authors. It is reasonable to suppose...
that all authors of a citing paper have equal acquaintance with all references contained in it if it is not stated explicitly something else in the paper. Assuming this hypothesis, we see that the impact of a paper on a work citing it can be measured not only the number one (representing only the fact of citation) but more precisely by the number of authors of the citing work each of which we suppose to know the cited paper and have some benefit of it. Similarly, the number of authors of all papers citing a given work can be taken as a measure of the influence of the cited work.\footnote{Some of the citing authors may coincide.}

**Remark 1.** There are works whose number of authors may be classified as “quite large”. Examples of such papers can be found in the region of experimental physics of elementary particles, where can be found papers with, say, 100–150 and more authors; for instance, in the work [6] we see more than 2500 authors. Usually as authors of such works are pointed whole experimental collaborations. We do not want to speculate on how such works are written and what is the particular contribution of their authors and so on. However, it seems that the hypothesis of acquaintance of all authors with all references breaks down for works with “quite large” number of authors.

Let us note that a situation with too much authors is described as “hyperauthorship” and “within the biomedical world it has been proposed that authors be replaced by lists of contributors (the radical model), whose specific inputs to a given study would be recorded unambiguously” [19].

**Remark 2.** It seems that as a “normal” ("reasonable" and statistical) upper limit on the number of authors of a research paper or a book/monograph can be taken 7 or 4 respectively. With some reserve we may replace these numbers by 9 and 6 respectively. These numbers seem to be relevant for the physics and certainly depend on the particular scientific field. Our opinion is that the hypothesis of equal acquaintance of all authors with all references is not true for research articles or books whose number of authors is greater than the pointed numbers. Similar (statistical) limits may be pointed and for other types of publications such as review article or articles in encyclopedias. In any way, if the number of authors of a work is greater then some “reasonable” number, which should depend on works types, then the mentioned hypothesis seems not to be valid.

**Remark 3.** When the hypothesis of equal acquaintance of all authors of a work with all references in it is not true and there is not other information concerning the acquaintance of the authors with the references, we cannot make any conclusions
on the impact of a cited work (and its authors) on the authors of the citing work based on the fact of citation. In such cases we shall consider the citing work as written by only one author for the purposes of our analysis.

So, to any citing paper we assign a number, citing paper impact, which is equal to the number of its authors that are acquainted with the cited paper or to one, if such an information is missing in the citing paper or cannot be found by means of some reasonable hypotheses. The sum of citing papers impact numbers for all (known) papers citing a work will be called citation impact number of the cited work and will be denoted by \( c_i \). By adding these numbers to Table 3 on page 81 we obtain Table 5 as a new version of a citation list.

Table 5. Citation list including data for citation impact numbers. The reduced impact numbers \( I_j = w^a_n c^i_j, j = 1, \ldots, n \), take into account the author contribution weights as well as the citation impact numbers.

<table>
<thead>
<tr>
<th>Number of cit.</th>
<th>Rank</th>
<th>Cit. impact number</th>
<th>Number of authors</th>
<th>Author weight</th>
<th>Reduced cit. number</th>
<th>Reduced impact number</th>
<th>Paper descr.</th>
</tr>
</thead>
<tbody>
<tr>
<td>( c_1 )</td>
<td>1</td>
<td>( c^1_1 )</td>
<td>( a_1 )</td>
<td>( w^a_1 )</td>
<td>( c^1_1 = w^a_1 c^1_1 )</td>
<td>( I_1 = w^a_1 c^1_1 )</td>
<td>( p_1 )</td>
</tr>
<tr>
<td>( c_2 )</td>
<td>1</td>
<td>( c^1_2 )</td>
<td>( a_2 )</td>
<td>( w^a_2 )</td>
<td>( c^1_2 = w^a_2 c^1_2 )</td>
<td>( I_2 = w^a_2 c^1_2 )</td>
<td>( p_2 )</td>
</tr>
<tr>
<td>( \vdots )</td>
<td>( \vdots )</td>
<td>( \vdots )</td>
<td>( \vdots )</td>
<td>( \vdots )</td>
<td>( \vdots )</td>
<td>( \vdots )</td>
<td>( \vdots )</td>
</tr>
<tr>
<td>( c_n )</td>
<td>( n )</td>
<td>( c^1_n )</td>
<td>( a_n )</td>
<td>( w^a_n )</td>
<td>( c^1_n = w^a_n c^1_n )</td>
<td>( I_n = w^a_n c^1_n )</td>
<td>( p_n )</td>
</tr>
</tbody>
</table>

If the reduced impact citation numbers \( I_j = w^a_n c^i_j, j = 1, \ldots, n \), can be introduced, then we can rearrange Table 5 by their descending order and call the number of a row of the so-obtained table the reduced impact citation rank of the paper sitting in it. In this way we obtain the Table 6 on the next page below as new modified version of Tables 5 and 4 on page 81.

In conclusion, we have three major forms of any citation list which are given via the Tables 2 on page 79, 4 on page 81 and 6 on the following page which are suitable for farther analysis of the data in them.

5. Bibliometric Indices (Metrics)

The bibliometric indications [53] are a known tool for measuring authors impact. Starting from 2005 there were introduced many new (bibliometric) indices, called also metrics, whose purpose is to measure the influence of an author on the ground

\(^8\)The standard case is to set the mentioned number equal to one which represents only the fact of citation. We consider this situation quite rough.
Table 6. Citation list arranged by descending order of citation impact numbers. Here \((m_1, \ldots, m_n), (r^1_m, \ldots, r^n_m)\) and \((r_1, \ldots, r_n)\) are permutations of \((1, \ldots, n)\) and by definition \(I_i \geq I_{i+1}\) for \(i = 1, \ldots, n - 1\).

<table>
<thead>
<tr>
<th>Reduced impact cit. number</th>
<th>Reduced cit. rank</th>
<th>Reduced number of citations</th>
<th>Rank &amp; Reduced rank</th>
<th>Number of cit.</th>
<th>Cit. impact number</th>
<th>Number of authors</th>
<th>Personal author weight</th>
<th>Paper description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(I_{m_1} = w_{m_1}^a c_{m_1}^i)</td>
<td>1</td>
<td>(c_{m_1}^a = w_{m_1}^a c_{m_1})</td>
<td>(r_{m_1} &amp; r^r_{m_1})</td>
<td>(c_{m_1})</td>
<td>(c_{m_1}^i)</td>
<td>(a_{m_1})</td>
<td>(u_{1}^a)</td>
<td>(p_{m_1})</td>
</tr>
<tr>
<td>(I_{m_2} = w_{m_2}^a c_{m_2}^i)</td>
<td>2</td>
<td>(c_{m_2}^a = w_{m_2}^a c_{m_2})</td>
<td>(r_{m_2} &amp; r^r_{m_2})</td>
<td>(c_{m_2})</td>
<td>(c_{m_2}^i)</td>
<td>(a_{m_2})</td>
<td>(u_{2}^a)</td>
<td>(p_{m_2})</td>
</tr>
<tr>
<td>(\vdots)</td>
<td>(\vdots)</td>
<td>(\vdots)</td>
<td>(\vdots)</td>
<td>(\vdots)</td>
<td>(\vdots)</td>
<td>(\vdots)</td>
<td>(\vdots)</td>
<td>(\vdots)</td>
</tr>
<tr>
<td>(I_{m_n} = w_{m_n}^a c_{m_n}^i)</td>
<td>(n)</td>
<td>(c_{m_n}^a = w_{m_n}^a c_{m_n})</td>
<td>(r_{m_n} &amp; r^r_{m_n})</td>
<td>(c_{m_n})</td>
<td>(c_{m_n}^i)</td>
<td>(a_{m_n})</td>
<td>(u_{n}^a)</td>
<td>(p_{m_n})</td>
</tr>
</tbody>
</table>
of citations of his/her works. These indices can be described as bibliometric and
their connection with the scientific impact of an author is indirect\(^9\) as it cannot
be revealed without knowing the content of the cited and citing papers. However,
the usage of these indices has brought significant advance in this area compared
to the previous analysis based, for instance, on author’s total number of published
works and their total number of citations. For example, in [32] are provided argu-
ments that “the number of citations or the mean number of citations per paper are
definitely not good predictors of promotion”.
This section aims to list a few bibliometric indices and to present some analysis on
their ground. It is not our goal here to present a “complete” list of (all) bibliometric
indices introduced until now as well as to point to their “good” and “bad” sides,
which are known and already described (see, e.g., [4], http://sci2s.ugr.es/hindex/bi-
blio.php and http://sci2s.ugr.es/hindex/).

5.1. The Hirsch Index

All of the new game started with 2005 paper of Hirsch [29] in which he defined
the \(h\)-index, called nowadays the Hirsch index, as follows.

A scientists has index \(h\) if \(h\) of his/her \(N_p\) published papers have at least \(h\)
citations each and the other \((N_p - h)\) papers have no more than \(h\) citations each.

(This is not the Hirsch original definition, but the one of September 2006 e-print.)
In terms of Table 1 on page 78, we have

\[
c_h \geq h \geq c_{h+1}
\]

i.e., \(h\) is the maximal rank such that the corresponding to it paper has no less than \(h\)
citations and the papers with greater ranks have maximum \(h\) citations. The author
of the present paper failed to find in the available to him literature arguments why
the Hirsch index was defined exactly in this way. It contains only discussions of the
pros and cons of the Hirsch index (see, for instance, the discussion of the Hirsch
index in [16, Section 1] and in [4,12,34]). Of course, the pros are a posteriori argu-
ments of the definition but they do not answer the question why it works (“well”)

\(^9\)It is based on statistical data analysis [13,14,51].
in some cases.\footnote{The Hirsch index is applicable also for groups of scientist united by a journal, country, institute/university etc. For instance, in the site http://www.scimagojr.com/ it is calculated for the journals and countries covered by the Scopus database with URL http://www.scopus.com. For instance, in [1] is presented a testing and comparison of three bibliometric indexes (including the \( h \) and the \( g \)-index defined in Subsection 5.2.2.) for the Italian universities.} The Hirsch index received a lot of attention and found many applications as it combines in a single number quality, productivity and impact of an author.

However, the combination of output (number of papers) and impact (citations) in a single number is more a strong limitation than an advantage as these two measures are better kept separately. One of the limitations of the \( h \)-index (and related measures) is that it is just size dependent indicator. In general it correlates with other size dependent bibliometric indices (like total number of citations and total number of publications) \[37\].

By our opinion, one of the ideas behind the \( h \)-index is the selection of some of the “top cited” papers of an author and to take their number as a measure of his/her publications impact which is confirmed a posteriori by the results in \[59\].\footnote{Alternately, one can take as a measure, for instance, the number of papers with at least \( N \) citations or the number of citations of all papers with rank greater or equal to \( M \) for some integers \( N \) and \( M \). However, the numbers \( N \) and \( M \) are arbitrary to a great extend irrespectively of are they constant or not with respect to all authors. Example of such a measure is the “Einstein index” (see http://www.science20.com/hammock_physicist/who_todays_einstein_exercise_ranking_scientists-75928) characterized by \( M = 3 \).} From this point of view the Hirsch index has two significant advantages: i) it adapts to any particular author, hence being author-dependent and ii) it naturally defines the top cited papers as ones whose number of citations is no less that it.

There can be defined many indices that have the same properties as the Hirsch index. For example, we can define an \( f \)-modified Hirsch index \( h^f \) for some function \( f: \mathbb{R}^+ \to \{1, \ldots, n\} \) (in the notation of Table 1 on page 78) via

\[ c_{h^f} \geq f(h^f) \geq c_{h^f+1} \tag{4} \]

for particular choices of \( f \): for example, \( f(h^f) = h^f + 1 \) and \( f(h^f) = h^f - 2 \) lead to different indices\footnote{The Hirsch index is selected by \( f(h^f) = h^f \).} whose usefulness can be determined only by making particular calculations for particular authors. Without going into details we shall say that the results strongly depend on \( f \) and generally are not “stable” with respect to the choice of \( f \). Similarly, if we take a function \( g: \{1, \ldots, n\} \to \mathbb{R}^+ \), which may be the one inverse to \( f \) if it exists, then we can rewrite (4) as

\[ c_{g(h^f)} \geq h^f \geq c_{g(h^f)+1} \tag{5} \]
which introduces other modification $h_g$ of the Hirsch index. The particular choice $g(h_g) = 10h$ reproduces the $w$-index [65]. Similarly can be obtained the $k$- and $w$-indices as defined in [5].

5.2. Modifications of the Hirsch Index

The Hirsch index does not reflect many important data contained in a citation list. This has lead to the introduction of a lot of its variants each of which tries to take into account some features which the original Hirsch index misses to reflect. An excellent review on the Hirsch index and many its variants can be found in [4]. A list of 37 versions of the Hirsch index is contained in [14, Table 1 on page 349] (see also [13]) which paper contains also a quit complete list of relevant references. In [59] are analyzed and calculated 20 versions of the Hirsch index. Below we shall pay attention to some of the modifications of the Hirsch index that are closer to the aims of this work.

5.2.1. Multiple Authorship

The Hirsch index $h$ is insensitive to how many authors have the papers in Table 1 on page 78. But this index aims to represent the contribution of a particular author whose citation list is considered. So, if some or all of the first $h$ papers in Table 1 on page 78 have more than one author, then it is evident that in the $h$-index is incorporated also the work of authors different form the one whose list of citations is investigated. The correction of this unfairness with respect to the other authors (whose work is assigned to other person(s)) leads to a class of indices that reflect the number of authors of the cited papers. For definition and analysis of such indices are suitable citation lists in a form given by Table 2 on page 79.

The $h_m$ index introduced by Schreiber [57] is defined via equation (5) with the choice $g = r^{-1}_\text{eff}: \mathbb{R}^+ \rightarrow \{1, \ldots, n\}$ for

$$r^{-1}_\text{eff}: r \mapsto r^{-1}_\text{eff}(r) = \sum_{i=1}^{r} \frac{1}{a_i}$$

where $r \in \{1, \ldots, n\}$, $r^{-1}_\text{eff}$ is treated as an effective rank of the paper $p_r$ and we use the notation of Table 2 on page 79. We should mention that here is used the hypothesis of equal contribution of all authors of a multiple author paper which is behind the fractional counting. In [58] the $h_m$-index is calculated for 26 particular cases, which shows strong correlation with the $h$-index but the arrangement of the authors according to the both indices is generally quite different.
In the more general case, when personal authors weights are known (see Table 3 on page 81), the function $g$ in (5) should be chosen as $g = r^{-1}_w$ with

$$r^{-1}_w: r \mapsto r^{-1}_w(r) = \sum_{i=1}^{r} w_i^a$$

which reduces to (6) for $w_i^a = 1/a_i$ and leads to the author-weighted $h^a_w$-index.

Thus we have

$$c_{r_{\text{eff}}}^{-1}(h_m) \geq h_m \geq c_{r_{\text{eff}}}^{-1}(h_{m+1})$$

(8)

$$c_{r_w^{-1}(h_w^a)} \geq h_w^a \geq c_{r_w^{-1}(h_w^a+1)}$$

(9)

The values $w_i \equiv 1$ reduce $h_w^a$ to the original Hirsch $h$-index.

The $h_I$-index [10] corrects the $h$-index by dividing it by the mean number of authors of papers selected by the $h$-index:

$$h_I = h/\bar{a}, \quad \bar{a} := \left( \sum_{i=1}^{h} a_i \right)/h$$

(10)

in the notation of Table 2 on page 79.

In the Publish or Perish program user manual\textsuperscript{13} is defined the normalized Hirsch index $h_I$\textsubscript{norm} (Individual normalized Hirsch index) which is defined similarly to the Hirsch index with the difference that now is used Table 4 on page 81 and it is supposed that $w_i^a = 1/a_i$, i.e., (cf. (3))

$$c_{h_I,\text{norm}}^{-a} \geq h_I,\text{norm} \geq c_{h_I,\text{norm}}^{-a+1}.$$

(11)

In words, the papers are ordered by the descending order of the citations divided by the corresponding number of authors and then the (normalized) Hirsch index is calculated. The author of these lines shares the opinion that the $h_I,\text{norm}$-index reflects the author achievements considerably better than the original Hirsch index and the $h_m$-index.

The below introduced by (18) AWCRpA-index also takes care of the number of authors of the cited papers.

### 5.2.2. Taking into Account Missed Citations

The only information about the number of citations contained in the Hirsch index $h$ is that their total number is no less than $h^2$ (see (3)). It is clear that the more

\textsuperscript{13}See http://www.harzing.com/pophelp/metrics.htm.
citations a paper has, the more weight it should be given and vice versa.\textsuperscript{14} The $g$-index \cite{21} and the $e$-index \cite{66} aim to correct this situation with the Hirsch index.

The $g$-index of an author with citations list like Table 1 on page 78 is the unique largest number $g$ such that the total number of citations of the first $g$ papers is greater than or equal to $g^2$. Its aim is to give more weight to papers with more citations and thus improving the $h$-index.

The $e$-index also gives more attention to highly cited works and also helps to make difference between authors with similar Hirsch indices but different citations numbers. Using again the notation of Table 1 on page 78, we have

$$e = \sqrt{\frac{1}{h} \sum_{i=1}^{h} (c_i - h)} = \sqrt{\frac{1}{h} \sum_{i=1}^{h} c_i - h^2} \quad (12)$$

where $h$ is the Hirsch index of the author. The $e$-index is complementary to the $h$-index as it gives/measures some of the citations missed by the Hirsch index.

Similar aims persuade also: the $h^2$-index, the $A$-index ($= \frac{1}{h} \sum_{i=1}^{h} c_i$), the $R$-index ($= \sqrt{Ah}$), the $h_w$-index, and the $hg$-index ($= \sqrt{gh}$).

The citations outside the $h$-index core are taken into account also in the indices introduced in the following sub-subsections.

\section*{5.2.3. The Time Dependence}

Until now we have not touched the problem for the dependence of the citations on the time. The simples way to fill this gap is the introduction of the age of the cited papers.

Suppose we have a citation list in a form of Table 1 on page 78 and $t_i$ is the age of the paper $p_i$, $i = 1, \ldots, n$, counting from its first publication. Then the $AR$-index is

$$AR = \sqrt{\frac{1}{h} \sum_{i=1}^{h} c_i/t_i} \quad (13)$$

with $h$ being the Hirsch index of the considered author. The $AR$-index may decreases with time.

\textsuperscript{14}Unfortunately the Hirsch and Hirsch-like indices completely lost the low cited papers with non-zero citations, e.g. the ones with less than $h$ citations in a case of the $h$-index.

\textsuperscript{15}See \cite[Table 2 on page 829]{33} and the references given therein.
The contemporary $h$-index $h^c$ [60, Section 2] is defined similarly but instead of the number $c_i$ of citations of the paper $p_i$ is used the score

$$S^c(i) = \frac{\gamma c_i}{(1 + t_i)^{\delta}}$$

(14)

where $\gamma$ and $\delta$ are constants and $t_i$ is the paper age in years (counted from its publication); often is taken $\gamma = 4$ and $\delta = 1$. An author has index $h^c$ if $h^c$ of his/her papers have a score not less than $h^c$ and the remaining ones have a score not greater than $h^c$. In particular, if we arrange a citation list by descending values of $S^c(i)$, then (cf. (3))

$$S^c(h^c) \geq h^c \geq S^c(h^c + 1).$$

(15)

If the score (14) is modified as $S^c(i) = \gamma \sum_{t \in c_i} 1/(1 + t)^{\delta}$ we obtain the trend $h$-index [60, Section 2].

In the program Publish or Perish are introduced three other indices that depend on the age of the cited work. The age-weighted citation rate is

$$AWCR = \sum_{i=1}^{n} \frac{c_i}{t_i}$$

(16)

where $c_i$ and $t_i$ are the citations and the age of the $i$-th paper and the sum is over all published papers, and the age-weighted index is

$$AW = \sqrt{AWCR} = \sqrt{\sum_{i=1}^{n} \frac{c_i}{t_i}}.$$  

(17)

Note that (17) differs from (13) by the inclusion of citations outside of the $h$-core. If the paper $p_i$ has $a_i$ authors, then the per-author modification of (16) is

$$AWCRpA = \sum_{i=1}^{n} \frac{c_i}{(t_i a_i)}.$$  

(18)

5.3. Comments

As we have seen, there were introduced quite a number of bibliometric indices. Their properties are well known and discussed at length in the cited references and the ones given in them. The general opinion is that different indices represent different measures of author’s published works and in many cases are complimentary to each other. This points to the complexity of the problem of giving an evaluation of authors impact by using citation lists.

5.4. Generation of New Indices

In Subsection 5.1 we pointed that to functions

\[ f : \mathbb{R}^+ \rightarrow \{1, \ldots, n\}, \quad g : \{1, \ldots, n\} \rightarrow \mathbb{R}^+ \]

(we use the notation of Table 1) there correspond respectively indices \( h_f \) and \( h_g \) with values in \( \{1, \ldots, n\} \) such that

\begin{align*}
    c_{h_f} & \geq f(h_f) \geq c_{h_f+1} & (19a) \\
    c_{g(h_g)} & \geq h_g \geq c_{g(h_g)+1}. & (19b)
\end{align*}

Here we implicitly supposed that the functions \( f \) and \( g \), which may be inverse to each other, are such that \( h_f \) and \( h_g \) exist and are unique which puts some restrictions on these functions. These are more or less trivial versions of the Hirsch index (cf. (3)) regardless that their particular properties and interpretation may be quite different depending on the particular choices of \( f \) and \( g \).

When Hirsch-like indices are utilized, only part of the author’s papers are taken into account. An important moment is that the number of these papers is author-dependent. Often, as in the case of the Hirsch index, this selection is done by the rank (sequential number) of the papers in a citation list in which the papers are arranged by descending number of citations (possibly normalized by some factors/weights). However, there are infinite number of ways to make similar selections on the base of other principles.\(^{17}\)

Define the (arithmetic) mean of the non-vanishing reduced numbers of citation by (we use the notation of Table 4 on page 81)

\[ \bar{c}_a = \frac{\sum_{i=1}^{n} c_i}{\sum_{i \in \{1, \ldots, n\}, c_i \neq 0}}. \]  

(20)

Now we can define a new index, say \( \tilde{h}^a \), via (cf. (3))

\[ \tilde{h}^a = \max_{r \in \{1, \ldots, n\}} \{ r : c_r^a \geq \bar{c}^a \} \]

(21)

i.e., \( \tilde{h}^a \) selects the papers with at least \( \bar{c}^a \) citations and it equals to the maximal reduced rank between papers with this property. Evidently, we can replace \( \bar{c}^a \) with other mean values, e.g. with the geometric mean value of all papers with

\(^{17}\)Take, for instance, a citation list of a form of Table 4. For \( w_i \equiv 1 \) it is a base for defining the \( h \)-index and for \( w_i = 1/a_i \) is a base for the introduction of the \( h_{I,norm} \)-index.
non-vanishing citations, and will obtain in this way a new index like $\bar{h}^a$ above. One can even use the mean square deviation

$$\delta = \sqrt{\sum_{c_i^a \geq \bar{c}^a} (c_i^a - \bar{c}^a)^2}$$

to define highly cited papers by $c_i^a \geq \bar{c}^a + \delta$ and use this inequality in the r.h.s. of (21) to define a new index.

Another way for generation of new Hirsch-like indices is to redefine the existing ones, usually based on Tables 1, 2 and 3, by indices based on Tables 4, 5 and 6. We do not want to go into details of this process as it is quite clear and evident and the real problem is how useful the new-obtained indices will be, which can be solved only by making particular calculations for particular persons. In any case, our opinion is that indices based on Tables 4 on page 81 and 6 on page 84 should be better than the original ones.

From theoretical point of view it can be invented an infinite number of “indices” that will reflect different aspects of a citation list. The discussed in the literature bibliometric indices confirm this opinion.

5.5. Which is the Best Index?

An analysis of some bibliometric indices [13, 14, 16] reveals that any one of them has its pros and cons and is useful in some cases and gives unsatisfactory consequences in other ones. All this points that there cannot be pointed the “best index” unless there are well defined criterion(s) what it must satisfy, what is expected from it and what is the area of its application. For example, if we are interested simply of the impact of a paper, then, e.g., the $h$-index is better then the $h_m$ and $H_{I,\text{norm}}$ indices, but if we aim to evaluate the author personal (individual) impact, then the $h_m$ and $H_{I,\text{norm}}$ indices are more adequate than the Hirsch index. Similarly, we have an intuitive understanding of “highly cited” papers of an author but without a rigorous definition of this concept we cannot do much. The same is the situation with the “low cited” papers with non-vanishing number of citations. Besides, there is a problem why some or all of the “low cited” papers are excluded from the scope of some of the bibliometric indices like the Hirsch index and most of the Hirsch-like ones.

The above points to the complexity of the problem of citation analysis and author evaluation/impact based on it. As we said, we share the opinion that the known approaches to it reveal only some its aspects and no one of them gives a “complete”
answer. Besides, we agree in general with Hirsch [29, page 4] that “a single number can never give more than a rough approximation to an individual’s multifaced profile”, but this concerns a more general problem than the one investigated in this work.

5.6. What to Do Next?

Tens of bibliometric indices are in current usage [14]. The process of invention and testing of new indices can be continued with a hope that the “best” index will be found.

The final goal is to be found quantitative measures for evaluation and comparison of authors and their impact. At the moment we consider the case when the information for realization of this aim are the citation lists of the authors. In this respect we notice that citation impact is strongly influenced by the following factors [26, page 61]: i) the subject matter and within the subject, the “level of abstraction”, ii) the paper’s age, iii) the paper’s “social status” (through the author(s) and the journal), iv) the document type and v) the observation period. All of them have to be taken into account when evaluating the scientific impact of scientists.

There are two global characteristics of a citation list like the one presented by Table 1 that are often used: the total number \( n \) of published papers and the total number

\[
c := \sum_{i=1}^{n} c_i
\]  

(22)

of their citations. To them can be added the author coefficient of citation performance

\[
E = \frac{\left( \sum_{c_i \neq 0} 1 \right)}{n} = \frac{\left( \sum_{c_i \neq 0} 1 \right)}{\left( \sum_{c_i} 1 \right)}
\]  

(23)

which is the ratio of the number of papers with non-vanishing citations and the number of all papers. From these numbers can be made qualitative conclusions concerning authors like: the greater \( n \), the more productive/active an author is and the greater \( c \), the more is his/her impact on (other) authors. Of course, the coefficient of performance (23) is a rigorous measure but it concerns only a single author and cannot be used to measure the authors impact on other authors; it only measures now much of his/her works have non-vanishing usage by (other) authors.

The total number of citations \( c \) shows in how many papers the author’s works have been mentioned/used. But, since we aim to make conclusions concerning only the author, not his/her co-authors, if any, this number in the general case does not give adequate measure of the author without counting the number of authors of each
paper. For the purpose the index of citations
\[ c_I = \sum_{i=1}^{n} c_i w_i \]  

(24)
is considerably better characteristic. Here we have used the notation of Table 3 on page 81. Recall that, \( w_i = 1/a_i \) if all authors are suppose to have equal contributions in the paper \( p_i \). The \( c_I \)-index shows how many papers have been influenced by the author’s personal contribution in his/her published works as a whole.

Notice, the weights \( w_i \) can also be used to make the effectiveness (23) more accurate, viz. by making it to represent the author individual contributions
\[ E^I = \frac{\left( \sum_{c_i \neq 0} w_i \right) / \left( \sum_{i=1}^{n} w_i \right)}{n \sum_{i=1}^{n} w_i} = \frac{n_{\neq 0}^I}{n^I} \]  

(25)

where the numbers
\[ n^I := \sum_{i=1}^{n} w_i \]  

(26)
\[ n_{\neq 0}^I := \sum_{i \in \{1, \ldots, n\}, c_i \neq 0} w_i \]  

(27)
can be interpreted as respectively effective (individual) number of author’s published works and individual number of papers with non-zero number of citations.

The index (24) can be called individual-to-works impact index. Similarly, we can introduce individual-to-author index \( c_I^A \) which shows how many authors have been influenced by the publish papers of an author. For the purpose we notice that in \( c_I \) every citing paper is counted exactly one time, which represents the simple fact of citation. Instead of this we can count the number of authors of each citing paper that are acquainted with the cited work, if these numbers are known. So, the analogue of the total citation number (24) now is
\[ c_i^j := \sum_{i=1}^{n} c_{ij}^i \]  

(28)
where \( c_{ij}, j = 1, \ldots, n \), is the sum of all authors of all papers citing the paper \( p_j \) and which authors are acquainted with the cited paper \( p_j \).\(^{18}\) Besides, to take into account the contribution of the author whose citation list is considered, we have to

\(^{18}\)Recall, if for some citing paper cannot be determined the number of authors acquainted with the cited paper, then we set this number equal to one, which represents only the fact of citation.
introduce in (28) also his/her weight $w_j$ in the creation of the paper $p_j$. Hence the number

$$c_i^j := \sum_{j=1}^{n} c_j^i w_i = \sum_{j=1}^{n} I_j$$

(29)

is a global measure of the individual impact of an author on (other) authors citing his/her papers. The global characteristics like (22)–(29) miss a lot of local information and in this respect some of the existing or new bibliometric indices may provide essential complimentary information. However, here are needed particular calculation which is not a purpose of this work.

6. To What Should be Paid Attention?

Until now we have looked on the citations from pure bibliometrics point of view. However our aim at this stage is the usage of citation analysis for making conclusions for the scientific impact/achievements of a scientist without going into the scientific content of the cited and citing papers. In this respect there are important arguments that are not purely bibliometric.

6.1. Self-Citations

When an author cites a paper and he/she is between the authors of this paper, we say that this is a self-citation for this author. Often this definition is broaden by saying that a citation is a self-citation if the intersection of the authors of the cited and citing works is not empty.

As we said earlier in Subsection 2.5, the self-citations may be included in the citation lists and treated on equal footing with the rest of author’s citations when the self–citations are made for pure scientific reasons, in particular when an author uses his/her earlier works in subsequent own publications.

One of the problems with the self-citations [26, Section 4.2] is that they can easily be manipulate and, if this is the case, this artificially brings more citations. So, in this context, the problem is more a moral than a scientific one and our believe is that most of the authors are fair in this respect, do not cite their own papers without need and cite them on equal footing with other works.

Good reasons why the self-citation should not be count are given at the beginning of [56, Section V] and we agree with them. Analysis of the self-citations and their influence on some bibliometric indices can be found in many papers like [9, 18, 25,
It seems that the general opinions are that the self-citation should be excluded when evaluating the scientific impact of an author. The main reason for this is that it is more important the influence of author’s works on other scientists and their papers than on the author himself/herself. Other important fact confirming the exclusion of self-citations is that they can relatively easy be manipulated in favour of one or other author; as pointier in [55] “not only the author’s own self-citations have a substantial effect in reducing the Hirsch index appreciably, but also the self-citations of the co-authors are usually quite significant and reduce the Hirsch index further”.

In this respect it seems reasonable to be introduced an upper limit on the number of self-citation which should be regarded as natural/ordinary citation. This limit is reasonable to be connected with the total number of author’s publications and possibly with the number of their citations. If the self-citations exceed this limit, the self-citation should be reduced to it or neglected at all.

In general, the approaches to self-citations are [52, page 21]: excluding them, not- ing them and trying to give a suitable their interpretation and ignoring their effect by assuming their even distribution.

An example of a bibliometric index that takes into account the self-citations is the discounted $h$-index ($dh$-index, called $V$-index in an early version) [24] which is the $h$-index multiplied by the square root of one minus the ratio of the number of self-citations and the total number of citations.

### 6.2. The Number of Authors

Since we wont to make conclusions about a particular author whose citation list is analyzed, the number of authors of each his/her paper should be counted and taken into account. More precisely, his/her contribution/weight in any cited paper in his/her citation list should be presented and used for the citation analysis. If such an information is missing, we assume that his/her contribution in a paper is one divided by the number of all paper’s authors.

Another thing is the number of authors of citing works. If it is insignificant for some problem, then such a citation simply adds the number one to the number of citations to the cited paper. But if this number is important, then instead of counting this paper once, we should consider replacing this weight 1 by the number of its authors or, more precisely, the number of its authors that are acquainted (and using) the cited paper, if the last number can be determined.
6.3. Highly/Low Cited Papers

It is generally accepted that the more/less citations a paper has, the more/less weight to its impact should be assigned. However, in the general case there is no a definition or a criterion of highly/low cited paper except that this seems to be author-dependent concept and the first/last paper in Table 1 is regarded as highly/low cited one (it is possible that the both may coincide). A discussion of this problem with some proposals for its solution is given in [26, pages 71–72].

When we are dealing with Hirsch-like indices, then the highly cited papers are the ones in their cores, while the ones outside the cores are low-cited and are not taken into account at all. Besides, in some cases, the more citations has a paper in the core, the more weight it has in the corresponding indices.

6.4. Different Editions/Versions of a Published Work

From bibliometric point of view any separate publication of a work is a different published paper no matter if there is a difference between the publications. But there is also a different point of view. For example, two identical editions of a book are different publications but form scientific position the second edition simply supplies more copies of the book as it does not contain different content. Similarly, a lot of works are published in journals and then appear in an identical form in collections of papers. Besides, there are books/monographs translated in different languages which, excluding some (introductory) remarks and comments by the publishers/translators, are identical by their content. Evidently these are different publications that are copies of one and the same work from scientific point of view. Moreover, a work can have essentially different publications like preprints, electronic prints, conference reports and journal versions that have identical content and differ possibly only in the presentation of the material (e.g. differently numbered equations and permutations of parts of the text).

So, we face a problem: in a citation list may be presented different publications which are identical from scientific view point. Our opinion is that such publications should be identified under a single work title.

Unfortunately there is not a strict criterion when two different publications should be considered identical. For instance, adding essentially new results to a previously published work can be considered as resulting into a new work, but a renumbering/remutating the sections and/or equations in it does not change it for the Science.
6.5. Different Types of Publications

In Section 2 we talked about reasons for citation in different kinds of publications. Now we can look on this material from two more viewpoints: are there reasons for assigning different weights for citation in or citing from different types of publications? Here and below by publication types we shall understand such as: monograph/book, textbook, booklet, originals research article, review paper, collection of original or already published papers, handbook, encyclopedia, simple list of papers on some subject(s), and so on.

To begin with, it must be mentioned that the publication type (partially) reflects the paper content; e.g. it is quite rare a research paper to appear in an encyclopedia, but a review work may be published in a textbook, handbook or encyclopedia.

Consider first citing papers of different types. Are there reasons to assign different weights to them without knowing their particular content? In Section 2 we paid attention to some reasons for citation in some publication types. These reasons are generally type-dependent and a priority of some of them can be given only via additional hypothesis. Consider for instance a citation in an original research work. If one wants simply to cite in it a particular result, e.g. an equation, then a random choice of a paper containing it is sufficient. However, if the author wants not only to mention work containing the result, but also to pay a tribute to author(s) that have firs found it, then a priority will be given to papers that historically mentioned the result for the first time. So, our opinion is that without knowing the content of a citing paper we cannot assign in an abstract way different weights to the cited papers in a sense that when a paper is cited in different types of publications we cannot assign different weights to such a citation without an additional information.

Consider now cited papers of different types. It is a generally accepted opinion, e.g. in annual reports and personal CVs, that books/monographes weight more then other publication types, a chapter in a book is heavier than a research article and so on. But what are the reasons for such a rating? Nowadays a new result normally appears first as a research (journal) article, (electronic) preprint, conference report or in some combinations of these publications types and in this respect one cannot give more weight to some of them. However, on one hand, a research article is often considered more “stable” and reliable than a preprint or conference report, but, on other hand, a preprint, especially if it is electronic, spreads quite quickly and reaches the audience before a journal article as a result of which it is a common practice a preprint to be cited without mentioning its journal version, if any. Further, if an author of a new already published result continuous to work on it, then it is possible that he/she will write a review paper, chapter of a book or a whole
book that contains this result and its developments. If this happens, then these publications are often more cited than the original ones as they normally explain the result more widely and in connection with other items. In this way, the last type of publication receive more weight in a form of citations and there is not a need this to be done by other means. Of course, e.g., publishing a book or a review paper is considered as authors good achievement but it may not contain results belonging to its author(s), however, again, there is not a need to assign to it more weight as the only objective criterion is the usage of the work by other authors which, in our case, is reflected by its citations.

In conclusion, we share the opinion that without knowing the content of a citing/cited work it should not be assigned different weights to different types of publications.

6.6. Quality of Publication Carrier

It is a general opinion that it is significant where a particular work is published. Consider several examples

- There are peer review and not peer review journals. The former are considered as a better place for publication. Besides, some of the journals in the first group are distinguished at present by their impact factors (IF) [35, Section 5.1.1] (see also the discussion of the IF in [26, Section 4.4]), the greater the IF, the better the journal.

- There are peer review books and ones simply published by their authors by paying for that. Moreover, the books can be rated by the prestige of the publishing company that produced them.

- The electronic preprints in the arXiv database with URL http://arXiv.org are not so valued as their journal versions regardless that in the most cases both have almost identical content\(^{19}\) The reason for this being that the e-prints do not pass real peer review process.

- Nevertheless that a lot of conference reports pass peer judgement, they are less valued than, e.g., journal articles or chapters in books.

These examples show that there is needed of a quantitative measure for comparing the publication carriers of the scientific works. If such a measure is available, then

\(^{19}\)In this respect it must be noted that there are electronic preprints with world-class excellent results that never appear in other publications.
it can be combined with the citation analysis for making better conclusions about papers and/or their authors. An example of such a measure is worked out in [40]. It is called “weighted sum indicator” and is defined by

\[ W(c, t) = cA_t + (1 - c)IF \]  

where \( A_t \) is the number of citations of a work for \( t \) years after publication, \( IF \) is the impact factor of the journal in which the work was published (evaluated for the year of publication), and \( c \) is a constant weight with \( 0 \leq c \leq 1 \). Unfortunately this indicator is sensible only for works published as journal articles in journals for which the impact factor rating is defined.

Let us note that the impact factor of a journal is not a good measure of the quality of a paper published in this journal [2]. One of the reasons for this being that the citations of the works published in a journal determine the journal IF, not vice versa.

### 6.7. Time Dependence

A published paper lives as long as people remember (and use) it and the often they recall about it, the better it is. In the context of this work we can paraphrase this as: a paper lives as people cite it and the more citations it obtains, the more useful it is. Accepting this point of view, we see that the time evolutions of the citations of a work or an author should be taken into account in the citation analysis.

The time distribution of paper’s citations can tell us a lot about the interest for it. Consider some clear examples.

i) After publication a paper receives a lot of citations for a short period of time, say 3–12 months, and then its citation stops. This can be interpreted as essential result(s) contained in the paper but it is likely to be in a rapidly developing field, where exists a flow of new results which leads to fast redirection of the readers attention. ii) Suppose a paper receives relatively constant non-zero citations for an unit time for a long period, say 40–70 years. This point to fundamental result(s) in the work that are used extensively for a long time. iii) Other possibility is that the paper’s citations per unit time grows with the time, reaches some maximum, then continuously falls down to zero and after that the paper gets random citations from time to time. This is approximately the mean statistical situation in which the paper content attracts the readers attention for some time, then new works in its field appear and the attention begins to fall down until the original paper is almost forgotten, but some scientists accidentally find something interesting in (e.g. for

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20 Usually this happens with “good” bookes/monographs.
historical reasons) and cite it. iv) There are papers with delayed citations which for some time after publication receive no or very little citation and after this period begin accumulating citations. These works are known as “sleepers” (they are of the “sleeping beauty” type). A good analysis of papers of this type is given in [39]. Such papers remain (relatively) unnotice for some period of time after which there is a splash of interest to them. They are called “Sleeping Beauties in science” [50]. Examples of such works are ones that are too novice and far away of the time they are publish to be correctly understood and evaluated.

This list of possible time distribution of paper’s citations can be continued, but the important for us inference from it is that this distribution carries information about the usage of the work by its readers. Here is hard to make general conclusions. However, excluding some rare exceptional works (which sometimes appear in new editions continuously), the tendency is that the rate of appearance of new citations decreases with time and tends to zero. The objective reason for this is the progress of Science as a result of which new results and publications appear with increasing speed with time. However, there are exceptions of this observation. They ordinary belong to the works of recognized and famous scientist. We shall mention here only the name of Sir Isaac Newton whose works continue to receive citations continuously centuries after they were first published which point to his great footprint in the Science heritages.\(^\text{21}\)

The time-depending bibliometric indices (14)–(18) considered in Subsection 5.2.3. adopt (with exception of (17)) inverse proportionality of paper’s age (measured in years), which is in conformity with the above mentioned tendency. But in these indices appear the citation speed \(c_i(t)/t_i\) of the paper \(p_i\) for the whole its age \(t_i\). We may suppose that if \(c_i(t)\) are the citations of \(p_i\) for time \(t\) after its publication, then the speed \(\frac{dc_i(t)}{dt}\) is more objective feature of the paper \(p_i\). Since \(c_i(t)\) is not a continuous function of \(t\), the derivative in \(\frac{dc_i(t)}{dt}\) is not defined. For this reason it can be approximated and replaced by

\[
\dot{c}_i(t, \tau) := \frac{c_i(t + \tau) - c_i(t)}{\tau}
\]

for a fixed time period \(\tau\), which is sensible to be set equal to, e.g., one year. The graph of this function contains information of interest as for us. Of course, the speed characterizes only the paper \(p_i\), not the whole citation list of an author. From

\(^{21}\)This is true with respect to the works of the historians of science and partially for the active researchers in the corresponding fields. The reason is that Newton works have been canonized and are incorporated in the body of knowledge, in particular in physics, which phenomenon is known as “obliteration by incorporation” [42,43]. The same is valid for the most famous scientists, e.g. Albert Einstein.
the speeds $\dot{c}_i(t, \tau), i = 1, \ldots, n$, can be constructed different characteristics ("indices") for a given citation list, e.g. "Hirsch speed", defined as the Hirsch index but the role of citations being replaced by these speeds, arithmetic mean speed, $\sum_{i=1}^{n} \dot{c}_i(t, \tau)/n$, and so on.

It is likely that instead of the above speeds a more adequate global characteristic of a citation list is the (global) citation speed

$$\dot{c}(t, \tau) := \frac{c(t + \tau) - c(t)}{\tau} = \sum_{i=1}^{n} \dot{c}_i(t, \tau)$$

where $c(t) := \sum_{i=1}^{n} c_i(t, \tau)$ is the number of all citations of the author’s papers at time moment $t$ (for a period of time $\tau$). Again, it is reasonable to set $\tau$ to one year, but such a choice is based more on statistical and phycological reasons, than on some abstract arguments. Local/global extremums of $\dot{c}(t, \tau)$ and other peculiarities of its graph can suitably be interpreted, but we shall not speculate on this item.

The speeds (31) and (32) characterize a citation list but generally not the author to which it belongs. There are analogues that take into account the individual contribution of the author like:

$$\dot{c}_i^a(t, \tau) := \dot{c}_i(t, \tau) \cdot w_i^a$$

$$\dot{c}^a(t, \tau) := \sum_{i=1}^{n} \dot{c}_i^a(t, \tau)$$

where $w_i^a$ is the weight of author’s contribution to the paper $p_i$, which we assume to be $w_i^a = 1/a_i$, $a_i$ being the number of authors of $p_i$, in a lack of information about $w_i^a$.

If $h(t)$ is an author’s h-index at a time $t > 0$ (usually measured in years of author career), then in [15] is argued that the Hirsch-rate (h-rate) given by the average speed $h(t)/t$ is an good characteristic of the authors and suitable for their comparison.

In conclusion, the time distribution of the citations of an author’s published works carries an information that should not be neglected in citation analysis and scientific evolution of that author.

### 6.8. Web Resources

With the development of the computer networks, in particular the Internet (the Web), the scientists publish documents on them and use such documents on equal footing with the ones printed on paper. The most obvious example of this kind for
any physicists nowadays is the arXiv database with URL http://arXiv.org whose
documents and the citations to/from them are included in the free web service
Google Scholar with URL http://scholar.google.com/.

But even the last sentence reveal one of the major problems with the web resources,
viz. their addresses, i.e., where the corresponding files can be found. To be more
specific, let us talk about the uniform resource locator (URL), known as web ad-
dress, of a document, say http://arXiv.org/ (this is the main site page), which is
like the physical location of a standard library of paper documents.22 The problem
is that the web address may be changed easily and unpredictably, e.g. the initial
address of the arXiv was http://xxx.lanl.gov/, which immediately leads to disap-
pearance of the connected with it resources at the original (web) location in the
general case. For reliable data resources this should happen quite rarely and the
previous address should be active some time after the change and should contain
proper information; happily, the initial address of the arXiv database is still ac-
tive and contains current copy of the arXiv many years after the address change
has occurred. So, when citing a web document we must know and quote its web
address but we cannot be sure that the cited address will be valid at a next mo-
moment when one reads this information; the Internet is full with thousands of dead
(hyper)links. This is in contrast to citing paper documents when information like
article from journal X, volume y, pages z1−z2 will never change.23 Therefore,
when citing web documents, we have to be sure that they have stable and reliable
web addresses.

The problem with unstable Web addresses is partially solved with the Digital Ob-
ject Identifier (DOI) which is a string uniquely identifying an object like electronic
document. The DOI’s are more stable than the URLs. The DOI’s are maintained
by International DOI Foundation (IDF) (see http://www.doi.org/).24 The DOI’s are
a standard for the articles in the most international (paper or electronic) journals.
However, not all of the documents in the Internet have DOI25, so the problem with
unstable web linking of documents is open in general.

22This analogy is quite rough as the web address may remain unchanged while the physical carrier
of the information (or its IP address) changes. Besides, the particular location of the physical carrier
is insignificant.

23This information is constant also for web (on-line) journals but if, e.g., the journal web site
disappears, then all of the journal files will disappear too if they reside in this site.

24A digital object (DO) architecture consists of (see http://www.cnri.reston.va.us/activities.html):
the DO Repository (for storage and access) (see http://www.dorepository.org), the Handle System
(the resolution system, for unique and persistent identification) (see http://www.handle.net), and the
DO Registry (for discovery, see http://www.doregistry.org).

25At the end of 2014 the number of indexed DOI’s is about 71 millions. The site
http://search.crossref.org/, in particular the page http://search.crossref.org/help/status gives a good
idea of the documents with DOI’s in the Internet.
Other problem with citing web documents is their content. We are used to having paper documents to have constant content after their publication and this content can be changed with subsequent documents making additions, corrections etc. to the original text. The same procedure works and for web documents but the problem is that the content of the original and subsequent documents may be changed at any moment by anyone having suitable knowledge (e.g. of web design) and access to the corresponding server. Of course, this does not happen on reliable web sites like http://arXiv.org/, but if, for instance, the document is on the personal web site of an author, then he/she can change anything on it, in particular if he/she finds an error in some text it is easier to correct the original document without any announcement than to write a correction with suitable explanation to the original document. Now the conclusions is that, when citing web documents we have to be sure that they have constant and reliable content; possible changes in it should be done only by independent different subsequent documents or in its revised versions in which case is supposed that all previous versions are available and unchanged after their first publication.

The above problem can be solved if there will be created organization in which will be archived copies of the cited documents at the moment they are cited. An example is the web site http://www.webcitation.org/. Such organizations may play a role similar to ordinary libraries for paper documents.

As noted in [27, page 92], the major differences “between print media and the Web is that time plays a different role on the Web” and “the possibility of an almost continuous change of contents on the Web”. In particular [27, page 93]: “Most bibliometric processes are cumulative since publications (except for the extremely rare cases of retractions) and citations are irreversible and bibliographic links cannot be removed if they have once been established. By contrast, the Web is in terms of both, content and links in permanent change.” Besides, the web documents are in general non quality-controlled refereed products as, in principle, information can be published online by anybody. For these and other reasons the bibliometric indications have to be applied with some caution to the Web rescourses (if they are applicable at all in this case) and in general new measures are needed for the analysis of web linking of documents.

From the viewpoint of web (hyper-)linking one can look on usual citing of paper documents as on links between documents. Such an approach can be considered as an aspect of a kind of social network and suggest the usage of some web metrics, known as sites statistics indicators, to citation analysis. For instance, as the downloads of a web document can indicate its usage (this factor is quite disputable [35, Section 5.2]), the loans of documents of an ordinary library can serve the same purpose (with the corresponding doubts).
At any rate, at present more and more scientists use and cite web documents on the same base as they do with other resources. For this reason the web citations should be count on the same footing as citations in paper documents.

In [36] are reviewed some measures (web indicators) concerning usage of web documents. The more traditional ones (Webometric indicators, usually found by the web search engines) are: web citation, web links (hyperlinks between web documents) and web mentions of Uniform Resource Locators (URLs) and titles. The altmetric indicators are intended to measure usage of some information in Internet. A site statistics may provide information about the usage of the site data like downloads, views and readers. Other types of statistics may include recommendations, citations, posts and video presentation in blogs, social and scientific networks.

7. Citations and Scientific Achievements

Until now we dealt with bibliometric data connected with author’s citation lists. The aim of this section is to try to make conclusions concerning author’s scientific achievement and impact. Recall, at this stage we have completely ignored the cited and citing papers contents as such data is not presented in a citation list.

So, let us have an author’s citation list and some its bibliometric descriptions. What can we say about the author’s contribution in the Science and can on this base be compared different scientists?

It is a general opinion that the more citations an author (or a particular his/her paper) has, the more is his/her (its) impact. But how big? The problems seems open from quantitative position.

The different bibliometric metrics reflect different sides of the problem. Once these metrics are defined (usually without any a priori arguments), they are a posteriori confirmed or rejected by gathering statistics for them. e.g. evaluating them for a number of scientists with recognized administrative and/or scientific positions and comparing the metrics values with their positions. It seems that no one of the existing single number metrics describes the scientific achievement of an authors in a satisfactory way, which is in conformity with the ideas of [54] that this cannot be done in this way.

A possible measure of an author (or a paper) scientific impact may serve the time distribution of his/her (or it) citations, in particular his/her (it) citation life, i.e., the period after which citations stop (which does not mean that they will not appear in future). In general, the longer the citation life, the bigger is the scientific impact. But how big? The problem seems open from quantitative point of view. Also, it seems that the more uniform/even and large the distribution is, the bigger the
impact may be but, again, a quantitative measure is missing. Besides, the peaks in the distribution point to a temporary raising interest to an author or his/her paper(s).

Obviously, there is a connection between citations and their bibliometric measures with the scientific impact of an author or particular his/her paper(s). At present this connection is far from being well investigated and the known results in this field of research are mainly based on statistical analysis [13, 14]. By the last we mean that after some bibliometric measure (index, metric) is chosen, it is calculated for a selection of scientists (e.g. from departments of an institute/university, for winners of some prize), then the results are compared with the known administrative and/or scientific positions of the scientists and, finally, on this base are drawn conclusions for the adequateness, in particular the pros and cons, of the measure chosen.26 If the measure receives sufficient number of pros in some field, then it is accepted in this field, but it is often applied to other fields which sometimes shows inadequate results. Exactly this is the case with the Hirsch and Hirsch-like indices when they are used for evaluation of authors scientific impact or for comparing scientists.

Let us say a few words on the total number of author’s published works and their total number of citations. The total number of published works is a measure of author’s productivity, not of his/her scientific impact. In this respect a more adequate measure is the coefficient of citation performance (23) or the published works with non-zero citations. Besides, to be more precise, one should count only the author’s contribution, in which case the individual effectiveness (25) and the individual publications with non-zero citations given via (27) should be considered.27

8. Taking into Account Papers Content

For pure bibliometric purposes the content of citing and cited works does not matter. But when one begins to interpret and use bibliometrics for scientific evaluation of authors and their works, the content begins playing essential role and in a lot of cases it is more important than citation metrics. Unfortunately, in this field the problems are more than the solutions.

Suppose in a citing paper is said and proved that the cited one contains plagiarism(s) and nothing more. For the bibliometrics this simply adds one more citation for the author(s) of the cited work but the common sense tells us that here is something terribly wrong. Our suggestions is to count such citation (if the stated in

26 In this connection we want to point to the paper [51] in which the h-index is calculated for 147 university chemistry research groups and the results are compared with standard bibliometric measures and of peer review judgment and a correlation in this respect is observed.

27 Note that if we count only the author’s contribution, then the number \( n \) of all his/her publications should be replaced by the effective/individual number of publications (26).
them is true of course) with negative sign, i.e., by subtracting their number from the other author’s citations.

Next, let a citing paper points to error(s), wrong result(s), etc. in the cited work. Now the situation is not so simple as it may look and the assessment depends on (subsequent) details. If it is said that the cited work is so wrong that it cannot be corrected (e.g. a theory contradicting to experiments), then such a citations are reasonably to be neglected, i.e., they have not to be counted in the citations list. Contrary to this possibility, if the wrong result(s) are not only pointed but also corrected and then strictly proved in the citing paper, then this means that the cited work has influence on the citing one with inspiration of finding new result(s) in which case the citation may be treated as an ordinary one. There are many other possibilities, like simple pointing to error(s) in which case the citation may be omitted, but we do not want to speculate on them.

There are also “neutral” citations in which nothing is said about a cited paper. Examples are simple lists of works on some item(s), mentioning of the cited paper in introduction/conclusion in connection with some problem or in a general list of references on the subject of the citing paper. The purpose of such a citation is to point readers attention to the cited paper without giving opinion on it. This sort of citations are completely covered by the bibliometrics and should be count as ordinary citations.

Consider now the most difficult problem when a citing work makes particular use (of part(s)) of the cited one. This is the most creative reason for citation as the cited paper has directly influence the citing one in a positive way. It is intuitively clear that to such citations is fair to be given more weight than, e.g., the neutral ones. This weight should surely depend in the particular usage of the cited paper but the problem for its quantitative measure is open. Let us mention some possibilities:

- following/developing method(s) introduce in the cited work
- application of particular result(s) from the cited work
- testing result(s) from the cited work for particular events/data
- using result(s)/idea(s) from the cited work as a ground for further research.

It is quite obvious, any one of these and many more situations gives arguments for assigning to such kind of citations greater weight than e.g. ordinary citations or negative citations (e.g. revealing a plagiarism). But, as we already said, a quantitative measure for such weight is missing.
In Subsection 6.5 we have presented arguments that the different types of publications should not be distinguished for pure bibliometrics reasons. But what happens if the content of the works is taken into account?

The main output of a research paper are new results, ideas, concepts, methods, etc. and these are the main reasons for citing them.

The main output of (scientific) popular works are presentations of known and established knowledge in a way that can be understand by wider range of people, e.g. non-scientists or scientists from other fields of research. So, these papers can be regarded as review works written for non-specialists in the material they cover.

The general purpose of a (scientific) review paper is a systematic detailed presentation of material from published research papers, usually on some fixed topic. Often review papers give unified notation system, present and compare different aproaches/ideas, contain proofs, discuss pros and cons and are easier to read than the original works they cite. These and other features attract more readers and in this way they contribute to the spread and acceptance/non-acceptence of the information from the papers they review.

The books/monographs are considered as the most “heavy” and reliable sort of publications. Normally they are the most detailed and different-sided presentations of the topic(s) they cover and contain suitable references which in some cases are quite intensive. For these and other reasons a good book can be used by other authors for many years by putting aside review and original works on its topic(s).

The textbooks are books written for educational purposes and hence usually presuppose less preliminary knowledge compared to monographs on the same subject. Besides, they normally include material that is accepted with certainty as a true by the scientific community and only partially concern latest scientific news. However, often good textbooks can be regarded as monographs and vice versa.

The textbooks may be cited more rarely than monographs, but often they leave a greater footprint on their readers by giving them basic background for further research/development.

We can continue to list and partially analyze other publication types like handbook, encyclopedia, chapter in book/collecton and so on, but the above material is enough to confirm the opinion that to different publication types may be assigned different weights. However, a quantitative way for doing this is not known.

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28 We exclude from our considerations the “self-published” and similar books (or other publications) when an author pays a company to publish the books without any realistic peer review process. In this connection it should be mentioned that the reputation and respect of the publishing company among the scientists is also essential.
Until this section we talked about external aspects of the citation process with respect to the cited papers. The most important thing of any paper is its content. Its citation and all connected with a paper ratings and scientific impact are consequence of its content. But the content of any particular paper is specific and, besides, its evaluation strongly depends on the particular readers of the paper (e.g. of their education, knowledge and even psychological conditions).\textsuperscript{29} The final decision on author (respectively paper) scientific impact and value is formed by the scientific community on the base of the opinions of persons acquainted with the content of his/her published works (respectively the given paper). At present is not known a way to formalize this human-dependent process.\textsuperscript{30}

For these and many other reasons it is a great responsibility of the peers to evaluate the scientific results, impact, achievements and position of a person. In particular this concerns the decisions for giving prizes, honorable and scientific degrees, etc.

9. Implicit Citations or Citations without Citing

When evaluating the achievements of a scientist there are also other objective criterions than the ones based on citations.

Possibly the most important and recognized tribute to a scientist is by directly connecting his/her name with some scientific formulation like experiment, law, constant, equation/formula, observation, idea, hypothesis, theory, etc. There are thousands of such examples, for instance: Minkowski-Morley experiment, Newton’s (first, second, third, gravity) law, Boltzman constant, Schrödinger equation, Galilei moons, Mössbauer effect, Dirac large number hypothesis and Einstein theory (e.g. of gravitation). In this way we not only recognize the scientific impact of a scientists but also pay tribute to his/her personal work and role in the Science. In this way when we say/write, e.g., Plank constant we implicitly have in mind the contribution(s) of Max Plank to quantum physics in the particular case and there is not need to cite his paper(s) on early quantum physics which any one familiar with Plank constant can find and cite easily. In this sense here we have an implicit citation of scientist’s work(s) whose weight is certainly more than a simple citation of a particular his/her paper, but a quantitative measure of this weight is missing. Other way of scientific impact is via symbols, notations, concepts, names of different results/objects introduced by known scientists(s) which do not have his/her

\textsuperscript{29} Other problem is how a published work finds its way to its readers.

\textsuperscript{30} We do not want to speculate how such a process can be manipulated. The history reveals that manipulations in this field are short-lasting and after some time the scientific truth takes its place. It is enough to recall here the church manipulations of the famous investigations of Galileo Galilei.
(their) name(s) in written version. Examples are the plus sign “+” (the teacher Michael Stifel, 1544), the speed of light (in vacuum) constant $c$ (W. Weber and R. Kohlrausch, 1858), the “equivalence principle” in gravity theories (A. Einstein 1907, but the origin can be seen in G. Galilei experiment demonstrating the independence of the gravity acceleration of bodies in vacuum from their masses), the “classical electromagnetism” theory (in its present day understanding) due mainly to J. C. Maxwell (but behind this theory stay also many other scientists), Special/General theory of relativity (A. Einstein, 1905/1916), and “isospin” (W. Heisenberg 1912). By using such notations, names, concepts etc. we accept their importance and role in the Science and thus implicitly recognize the contributions of their inventors which can also be considered as an implicit citation of their works.

Behind many physics titles like kinetic gas theory, standard model (of elementary particles), quantum mechanics and classical/quantum field theory stay the work of many scientists. A check in the corresponding (historical) resources reveals to whom they are due to. Thus the usage of these titles (even without any explanations) is an implicit way to pay tribute to the persons that have contributed to their formulation and the physics content that they have.

Here should be mentioned also the concept of “obliteration by incorporation” [42] which usually affects famous scientists whose works are incorporated in the existing knowledge into such an extend that the working scientist accept them as a part of the common knowledge due to which the original works are rarely cited or not cited at all. Typical examples in this respect being the original papers of Isaac Newton and Albert Einstein.

When citing e.g. books, textbooks and review papers one often has in mind results, experiments, formulations etc. which do not belong to the author(s) of these publications and are only collected, summarized or reformulated by him/her (them). In this sense such citations pay tribute not only to the work of the direct author(s) of the publication but also indirectly to the persons who have made, for instance, the discovery describe in the cited paper. Namely the collection of a lot of material that generally does not belong to the authors of review works is one of the reasons that makes them convenient to cite without mentioning the original resources, which, on other hand, brings more citations to the review papers.

10. Peer Judgements

The written in the last two sections shows explicitly that when evaluating the impact of scientific works and their authors there are important factors that should be taken into account and that are completely out of the range of the bibliometrics.
At present these factors are only in the range of the peers. The peers are qualified experts in some field of Science that give their opinions on some scientific works in this field and their authors and possibly give some recommendations about them. The peers are supposed to know very well the considered scientific field and to be able to write corresponding reports (peer reviews) on papers in it which they evaluate the scientific research for originality, competence, significance, etc. The peer reviews play roles like advice system, quality control and error detection. On the base of the peer reports are taken further decisions like acceptance/non-acceptance of papers for publication, promotions, ratings of works and/or their authors, etc. These reports are of crucial importance for non-experts which form their opinions on this ground; in particular in this way is generally formed the public opinion on the scientists. The peer reviews are central for many key problems of Science like quality control and decision-making.

The work [11] is a comprehensive review of the peer review process: ground and background, purposes, advantages and disadvantages, problems, perspectives, reliability and fairness. It also contain a detailed bibliography on the item.

Of course, the peer judgements are human-depending activities and, respectively, may be influenced by non-objective reasons like the psychological condition of the particular peer, his/her personal relations with some author(s)/scientist(s), his/her interests and knowledge of the field of a reviewed material, etc. Such factors are partially remove by taking into account reviews written independently by more than one peer (usually two or three peers), but the personal elements of all of them remain and it is up to other persons, for example (managing) editors or super-peers, to try to eliminate them or/and to make more objective decisions.\(^{31}\)

Until now the peer judgements are not formalized in a form of some algorithms and it is unlikely that this will ever be done. The bibliometrics provides quantitative methods for analysis of the scientific and technological literature and in this sense it helps for revealing the impact of the scientific papers and their authors. It should be noted that a lot of these methods are based on statistical data analysis as a consequence of which the results point to tendencies or statistical laws the automatic application of which to particular papers/authors may lead to wrong conclusions. One of the roles of the peers is to decide upon the applicability of these results to particular situations. On the opposite, the bibliometric results may be used to trace statistically the validity of the peer reports. All this points that

\(^{31}\) In [3] is drawn the conclusion that the bibliometric approach gives more adequate results in the natural and formal sciences than in the other ones. Besides, in this paper we see also arguments that with respect to some criterions (e.g. time and cost) the bibliometrics may be preferable with respect to peers or informed peer review process.
peer reviews and the bibliometric evaluations should be regarded as complimentary to each other and used simultaneously for obtaining better assessments of the scientists and their papers.

11. Conclusion

The evaluation of the scientific value and impact of the works of a scientist is important for many purposes, in particular for comparing with other scientists, promotion and recruitment, prize awarding, fellowship, and funding. The main methods for such an evaluation are peer review judgments, based on the opinions of a group of experts, and citations analysis. Of course, a combination of the both methods is possible and seems a better choice.

In the paper [51] is calculated the h-index for 147 university chemistry research groups and the results are compared with standard bibliometric measures and of peer review judgment. It is important that a correlation in this respect is observed which, in particular, means that the h-index follows in general the peer judgment.

In [16] is analyzed the h-index in different situations and its relations with standard bibliometric characteristics like total number of the papers and their total number of citations, citations per paper, highly cited papers (with no less than 15 citations), impact factor(s) based on impact factors of the journals in which an author has published, etc. The general conclusion is that the Hirsch index is a good thing but it alone cannot be a “complete” measure of a scientists and it should be complimentary to other bibliometric measures. To overcome its disadvantages were introduced many other indices each of which has its pros and cons [4] but no one of them cannot pretend to be an ultimate measure of an author impact.

In this respect we want to note that the Hirsch index is not adequate when some or all of the papers in its core have more than one author as it assigns all of the work of these authors to one of them and, respectively, the achievements of this work are also assigned to the author whose citation list is considered. The senseless of this situation is evident if we take an $n$-author paper with $n \geq 2$ and calculate the Hirsch index for any one of the authors of this paper. If it falls into the core of the Hirsch index for all authors, then any one of them can claim the “fame” of this paper belongs to him/her which will mean that the whole “fame” of the paper is $n \times 100\%$ instead of $100\%$.

In [45] is performed a deep investigation of the influence of the evaluation of person’s work on his/her academic career. In particular, attention is paid to the so-called “invisible work”, like teaching and administrative work, which often is not taken into account.
Let us note that the automatic calculation of bibliometric indices based on Internet databases generally depends on the database [7]. Besides, the Internet databases do not capture all existing citations [47]. In general the results of the application of the bibliometrics depends on the used data sets and the captured publications (which is always limited). Besides, bibliometrics cannot measure procedures like reviewing, editing and mentoring. In this sense it has serious limitations. Similarly, the peer reviews have limitations too[33], but each of the both methods partially corrects the disadvantages of the other one. It is observed a correlation between assessment by different bibliometric indicators and quality judgment of peers [11, 51, 63]. This naturally suggest that [44, page 229] the bibliometrics can be used as a supplementary tool in peer review process as well as either of them can be used as validation and monitoring tool relative to the other one.

Besides documented via citations usage of published works, there can be a lot of other their usages that are not recorded, e.g. full or partial viewing/reading without citing, hearing about them on a seminar, conference or a private conversation etc. It is practically impossible to count and/or measure such events and to evaluate their impact but it is clear that they are due to the authors of the discussed papers and in this sense they contribute to the authors fame and impact on other scientists.

As noted in [38, Section 5] the gathering of page/site statistics of Internet pages with author works can be used for conclusions about the author. In fact, when certain web pages are a (partial) home of authors works like abstracts, partial/full text papers, lists of titles (possibly with further links), files with data etc., then from the statistics of such pages[34] can be made different conclusions. For instance, the page views and downloads of author works files can be interpreted as an interest of other people about the particular author and his/her works. Of course, from these raw data cannot be made unique conclusions, e.g. the fact that someone has viewed a particular page for one hour does not mean that he/she has read this page for one hour as he/she may simply doing other things and forgot to leave the particular page. However, in [38, Section 5] is reported a particular example when “high viewership does lead to high citations, and highly cited articles do not necessarily have high viewership”. Quantitively the usage of web documents is measured by different webometric indicators and altmetric indicators [36].

[33] An excellent review on the process and results of peer review research is presented in [11].
[34] Usually a web page/site statistics includes data like number of (unique) visitors, number of visited pages, number of downloaded files, the distribution of these numbers in time, as well as more detailed info like the particular visited pages and the time spend on them, list of downloaded files, user-dependent data (IP address, country, browser, etc.), and many more.
In this respect as a Web analogue of the standard citations may be considered the Internet (hyper)links to web pages that contain authors papers or/and relevant information about them. Such pages may be from author’s personal web site, databases with works like the e-print archive http://arxiv.org/, publisher or journal web site, and so on. Regardless that such links can be generated automatically by robots, from them can be made conclusions similar to the ones from the citations. It is clear that such an approach to author impact is in favour of authors that (extensively) use the capabilities of Internet but such are the present day realities and possibilities. As an argument in favour of such measure may serve the reported in [62] statistical result that “Web citation correlates with ISI citation and the average Web citation count of a journal correlates with the Journal’s Impact Factor” in biology and genetics. Besides “Web citations show a broader geographic coverage and capture a greater number and variety of users of journal articles”. However, the fact that Internet links and documents are unstable is a big problem. By “unstable” we understand that they may change within seconds or simply become not valid; e.g. a web page may disappear alone or with a part of the site that contains it and the content of a web page may be changed at any moment from the corresponding web designer/administrator. For this reason it is a good idea to be made a copy of a web resource when citing it as a proof for its existence and content at the moment when it was used.\textsuperscript{35}

The modern Science is due to a great extend to the research and its assessment by peers. The methods and tools of bibliometrics are an alternative to the work of peers. Since aspects such as the quality and impact of a paper are not yet formalised in a strict mathematical sense, the peer reviews remain leading in the final decisions on these items, but the bibliometric indicators reveal some objective their properties and tenencies. The both approaches seem to be overlapping and complimentary to each other which stimulate the further development of strict methods for assessment of papers and their authors.

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\textsuperscript{35} But the things do not end here as the copies of web pages can be manipulate extremely easy by anyone knowing some web design. As a real proof of the existence and the content of a web page (or a link) its copy should be archived in a publicly available and respected place the documents in which should have the same reputation and reliability as the paper books in an ordinary public library.
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