CLIFFORD ALGEBRA IMPLEMENTATIONS IN MAXIMA

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Abstract. This tutorial focuses on the packages clifford and cliffordan for the computer algebra system Maxima. Maxima is the open source descendant of the first computer algebra system and features a rich functionality from a large number of shared packages. The Maxima language is based on the ideas of functional programming, which is particularly well suited for transformations of formal mathematical expressions. While clifford implements Clifford algebras $\mathbb{C}_{\ell}^{p,q,r}$ of arbitrary signatures and order based on the elementary construction of Macdonald, cliffordan features geometric calculus functionality. Using clifford expressions containing geometric, outer and inner products can be simplified. Applications of clifford and cliffordan in linear algebra and calculus are demonstrated.

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