COMPLEXITY FOR INFINITE WORDS ASSOCIATED WITH QUADRATIC NON-SIMPLE PARRY NUMBERS*

L'UBOMÍRA BALKOVÁ

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Abstract. Studying of complexity of infinite aperiodic words, i.e., the number of different factors of the infinite word of a fixed length, is an interesting combinatorial problem. Moreover, investigation of infinite words associated with \( \beta \)-integers can be interpreted as investigation of one-dimensional quasicrystals. In such a way of interpretation, complexity corresponds to the number of local configurations of atoms.

1. Introduction

To study the structure of an infinite word \( u \) on a finite alphabet \( A \) and to measure the diversity of patterns occurring in this word, it is useful to define complexity of \( u \). It is a function \( C(n) \) which with every \( n \in \mathbb{N} \) associates the number of different words of length \( n \) contained in \( u \). The simplest infinite word is a constant sequence \( z^\omega \) with \( z \in A \). There exists only one word of each length, therefore \( C(n) = 1 \) for all \( n \in \mathbb{N} \). One extreme of the opposite side is a random sequence for which, almost surely, the complexity \( C(n) = (\#A)^n \). Between these two extremes, one can find infinite eventually periodic words for which the complexity \( C(n) \leq n \) for all \( n \in \mathbb{N} \), and the simplest aperiodic words, called Sturmian words, with the complexity \( C(n) = n + 1 \) for all \( n \in \mathbb{N} \).

Some kinds of infinite aperiodic words can serve as models for one dimensional quasicrystals, i.e., materials with long-range orientational order and sharp diffraction images of non-crystallographic symmetry. To understand the physical properties of these materials, it is important to describe their combinatorial properties. For instance, complexity corresponds to the number of local configurations of atoms.

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