GROUP THEORY IN THE PROBLEMS OF MODELING
AND CONTROL OF MULTI-BODY SYSTEMS

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Abstract. This work is a review of our research activity during the last ten years concerning the problems of modeling and control of multi-body mechanical systems. Because the treatment of the above topics is quite sensitive with respect to the different parameterizations of the rotation group in three dimensional space SO(3) and because the properties of the parameterization more or less influence the efficiency of the dynamic model, here the so called vector-parameter is used for parallel considerations. The consideration of the mechanical system in the configurational space of pure vector-parameters with a group structure opens the possibilities for the Lie group theory to be applied in the problems of the dynamics and control. The sections in this paper present independent parts of an unified scientific approach.

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