COHERENT STATES ASSOCIATED TO THE JACOBI GROUP - A VARIATION ON A THEME BY ERICH KÄHLER

STEFAN BERCEANU

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Abstract. Using the coherent states attached to the complex Jacobi group – the semi-direct product of the Heisenberg-Weyl group with the real symplectic group – we study some of the properties of coherent states based on the manifold which is the product of the $n$-dimensional complex plane with the Siegel upper half plane.

1. Introduction

In this paper we continue the investigation of the Jacobi group [7, 8, 16] – the semi-direct product of the Heisenberg-Weyl group and the symplectic group – started in [4, 5], using Perelomov’s coherent states (CS). The Jacobi group is an important object in connection with Quantum Mechanics, Geometric Quantization, Optics, etc., [2, 9, 14, 15, 18, 20].

Applying the methods developed in [3], in [4] we have constructed generalized CS attached to the Jacobi group $G^f_1 = H_1 \times SU(1, 1)$, based on the homogeneous Kähler manifold $D^f_1 = H_1 / \mathbb{R} \times SU(1, 1) / U(1) = \mathbb{C}^1 \times D_1$. Here $D_1$ denotes the unit disk $D_1 = \{ w \in \mathbb{C}; |w| < 1 \}$, and $H_n$ is the $(2n + 1)$-dimensional real Heisenberg-Weyl group with Lie algebra $\mathfrak{h}_n$. In [4] we have also emphasized that, when expressed in appropriate coordinates on the manifold $\chi^f_1 = \mathbb{C} \times \mathcal{H}_1$, $\mathcal{H}_1 = \{ v \in \mathbb{C}; \text{Im}(v) > 0 \}$, the Kähler two-form $\omega_1$ is identical with the one considered by Kähler-Berndt [6, 7, 10–12].

In [5] we have considered coherent states attached to the Jacobi group $G^f_n = H_n \times Sp(n, \mathbb{R})$, based on the manifold $D^f_n = \mathbb{C}^n \times D_n$, where $D_n$ is the Siegel ball $D_n = \{ W \in M(n, \mathbb{C}); W = W^t, 1 - WW^t > 0 \}$. In this paper we calculate the Kähler two-form $\omega_n'$ on the manifold $\chi^f_n = \mathbb{C}^n \times \mathcal{H}_n$, where $\mathcal{H}_n$ is the Siegel upper half plane obtained by the Cayley transform of the Siegel ball $D_n$. This $\omega_n'$ is a “$n$”-dimensional generalization of Kähler-Berndt’s two-form $\omega_1'$ on $\chi^f_1$ to the corresponding one on $\chi^f_n$. The physical relevance of these results follows from