Characterizations of groups generated by Kronecker sets

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Abstract. In recent years, starting with the paper [B-D-S], we have investigated the possibility of characterizing countable subgroups of the torus $T = \mathbb{R}/\mathbb{Z}$ by subsets of $\mathbb{Z}$. Here we consider new types of subgroups: let $K \subseteq T$ be a Kronecker set (a compact set on which every continuous function $f : K \to T$ can be uniformly approximated by characters of $T$), and $G$ the group generated by $K$. We prove (Theorem 1) that $G$ can be characterized by a subset of $\mathbb{Z}^2$ (instead of a subset of $\mathbb{Z}$). If $K$ is finite, Theorem 1 implies our earlier result in [B-S]. We also prove (Theorem 2) that if $K$ is uncountable, then $G$ cannot be characterized by a subset of $\mathbb{Z}$ (or an integer sequence) in the sense of [B-D-S].