NEWS AND NOTICES

Prof. RNDr. BŘETISLAV NOVÁK, DrSc. (1938–2003)
WOULD BE SEVENTY

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Born in the city of Pardubice (former Czechoslovakia, now Czech Republic) on March 2, 1938, Břetislav Novák spent the first 18 years of his life in the near-by historical town of Chrudim. It was there where he attended the elementary school (1944–1949) and both the lower and upper secondary schools (1949–1956). His mathematical talent started to demonstrate itself outwardly already here, in particular in

B. Novák in 1983, photo by J. Lukeš
the mathematical competitions called Mathematical Olympiads. Already in 1955, at the age of 17, he won the 6th place in the all-state round of the Olympiad, and a year later, in 1956, he became the absolute winner of the competition. It was therefore not surprising that B. Novák decided to study mathematics at the Faculty of Mathematics and Physics of the Charles University in Prague (in years 1956–1961).

It was there where B. Novák met Professor Vojtěch Jarník, a renowned mathematician and outstanding teacher whose unequalled four-volume textbook on differential and integral calculus still influences generations of students. Professor Jarník became his teacher, academic advisor of both his diploma and CSc. theses (cf. [50], [51]), and later on also his colleague in research. B. Novák graduated at the Faculty of Mathematics and Physics of the Charles University in 1961, cum laude, and strongly influenced by Jarník decided to devote his career to number theory. He successively obtained the scientific and scientific-pedagogic degrees of RNDr. (= Rerum Naturalium Doctor) and CSc. (= Candidatus Scientiarum, an equivalent to PhD., cf. [51]), both in 1967, docent (Associate Professor, cf. [52]), in 1972, DrSc. (= Doctor Scientiarum, cf. [53]), in 1980, and profesor (Professor) in 1982.

The whole life-path of Břetislav Novák was connected with his Alma Mater, Faculty of Mathematics and Physics of the Charles University in Prague. Still a student he was appointed a half-time tutor in the years 1959–1961, while in the years 1961–1964 he held the position of a tutor, and in the years 1964–1972 the position of a lecturer. In 1972, after his successful habilitation, he was appointed Associate Professor, and in 1982 he was appointed Full Professor of Mathematical Analysis, a position which he held until his death in 2003.

Břetislav Novák visited many universities and research institutes during his professional career. From his long-termed positions let us mention the most prestigious ones, the winter term stays at the Moscow State University in 1962/63, and at the University of Illinois at Urbana-Champaign in years 1972/73.

Břetislav Novák was also very active in organizing mathematical life at the Faculty, and in the whole Czechoslovak and later (after splitting of Czechoslovakia) the Czech mathematical community (cf. [76]). He served as Head of the Department of Mathematical Analysis in the period 1971–1990, and also as Vice-Dean of the Faculty of Mathematics and Physics in 1973–1976 and 1980–1989. He also devoted a great deal of his efforts to the activities within the Union of Czechoslovak (and Czech) Mathematicians and Physicists whose member he had become already in 1956. In 1978 he was elected member of the Central Committee of the Union. In the years 1987–1990 he was Chairman of the Union, and in the period 1993–1996 Chairman of the Mathematical Section of the Union. Břetislav was honored with the award of the Union of the first degree, in 1981 he was elected the distinguished member of the Union, and in 1993 the honorary member of the Union.
Last but not least, since 1966 he was also active as member of the Society for the history of sciences and technology, of the Scientific Board of the Czech Technical University in 1977–1979, of the Scientific Board for Mathematics of the Czechoslovak Academy of Sciences (1977–1990), of the National Committee for Mathematics (since 1977), and as member of many other committees, boards and panels.

Břetislav also devoted a lot of his personal energy and effort to improving the viability and operativeness of mutual communication between the research and teaching communities in the process of the transfer and implementation of modern mathematical ideas and results into the teaching and pedagogical processes in general. He is a coauthor of several textbooks devoted to secondary school mathematics and his lecture notes [47] on number theory remain three decades after their publishing the only Czech-written source for some important techniques of classical analytic number theory.

Břetislav Novák died on August 18, 2003, in Prague, and all of us who knew him will remember him as an outstanding mathematician, teacher and good colleague.

Mathematical interests of Břetislav Novák

Břetislav Novák was interested in a wide range of topics in mathematical analysis, with particular preference to the theory of functions of complex variable and its applications. Nonetheless, it was the number theory, especially the analytic one, which became his main subject.

That the number theory will accompany him through his life began to manifest itself already in his teenage leisure time activities. Young Břetislav as an enthusiastic participant of Mathematical Olympiads, and probably under the influence of his secondary school teachers, got in touch with Czech mathematical journals. In his first short published paper [1] he characterized quadratic polynomials with integer coefficients taking at integer points only values of the form $6m \pm 1$. The core of his proof was the observation that if a quadratic polynomial with integer coefficients takes only values of the form $6m \pm 1$ at six consecutive integer values of the argument, then its values at all integer points are of the form $6m \pm 1$.

In his second paper [2] written shortly after the first he improved this argument proving that if an arbitrary polynomial with integer coefficients takes only values of the form $6m \pm 1$ at three consecutive integer values of the argument, then all its values at integer points are of the form $6m \pm 1$.

From the very beginning of his career at the university he was interested in the theory of lattice points in multi-dimensional ellipsoids (cf. [50], [51], [52], [53]), under direct influence of his scientific advisor, Professor Vojtěch Jarník.
Břetislav’s main target was to find the precise asymptotic estimates for the so-called “generalized lattice residue”, that is the difference between the weighted number of lattice points in an $n$-dimensional ellipsoid and its volume. The main difficulty here is the abysmal difference between the so called rational and irrational ellipsoids. The first interesting result for irrational ellipsoids had been proved by A. Walfisz in 1927. The way to the definitive solution was paved by V. Jarník in a series of papers starting in the next year 1928. Břetislav mastered Jarník’s methods and in his paper on “Gitterpunkte mit Gewichten” around 1968 he actually opened the way for the definitive analysis of the influence of weights. For a more detailed account on the history and the contribution of Břetislav to this important problem we refer the reader to F. Fricker’s monograph [74]. To get a more precise idea about the problem let us outline some basic facts, see [75].

Let $Q(\mu_1, \mu_2, \ldots, \mu_r), r > 4$, be a positively definite quadratic form with integral coefficients and a determinant $D$. Let $M_1, M_2, \ldots, M_r$ be positive integers, $b_1, b_2, \ldots, b_r$ integers, and let $\alpha_1, \alpha_2, \ldots, \alpha_r$ be real numbers.

For $x > 0$ consider a function $A(x)$,

$$A(x) = \sum e^{2\pi i (\alpha_1 \mu_1 + \alpha_2 \mu_2 + \ldots + \alpha_r \mu_r)}$$

where the sum runs over such integers $\mu_1, \mu_2, \ldots, \mu_r$ for which

$$Q(\mu_1, \mu_2, \ldots, \mu_r) \leq x \quad \text{and} \quad \mu_j \equiv b_j \mod M_j, \quad j = 1, 2, \ldots, r.$$

If $M_1 = M_2 = \ldots = M_r = 1$ and $\alpha_1 = \alpha_2 = \ldots = \alpha_r = 0$ then $A(x)$ gives the number of integer points inside the corresponding ellipsoid. The main term of $A(x)$ is equal to

$$I(x) = \frac{x^{r/2} e^{2\pi i (\alpha_1 b_1 + \alpha_2 b_2 + \ldots + \alpha_r b_r)}}{\sqrt{D} M_1 M_2 \ldots M_r \Gamma \left( \frac{r}{2} + 1 \right)} x^{r/2} \delta$$

where $\delta = 1$ if all numbers $\alpha_1 M_1, \alpha_2 M_2, \ldots, \alpha_r M_r$ are integers, and $\delta = 0$ otherwise.

B. Novák was interested in good estimates of the remainder $R(x)$,

$$R(x) = A(x) - I(x).$$

He proved a number of theorems on the behaviour of $R(x)$ for different types of systems of $(\alpha_1, \alpha_2, \ldots, \alpha_r)$. (Some of them he proved during his stay at the Moscow University in the years 1962–1963.)

**Theorem 1.** For all systems $(\alpha_1, \alpha_2, \ldots, \alpha_r)$ we have

$$R(x) = O(x^{r/2-1}) \quad \text{as} \quad x \to \infty.$$
Theorem 2. If at least one of the numbers $\alpha_1, \alpha_2, \ldots, \alpha_r$ is irrational, then

\[ R(x) = o(x^{r/2-1}) \quad \text{as} \quad x \to \infty. \]

The estimate in Theorem 2 is definitive and cannot be improved.

Theorem 3. Let $\varphi(x) > 0$ and $\varphi(x) \to 0$ as $x \to \infty$. Then there exists a system $(\alpha_1, \alpha_2, \ldots, \alpha_r)$ such that (1) holds, and simultaneously

\[ R(x) = \Omega(x^{r/2-1} \varphi(x)) \quad \text{as} \quad x \to \infty. \]

Theorem 4. For almost all systems $(\alpha_1, \alpha_2, \ldots, \alpha_r)$ (in the sense of Lebesgue measure in $\mathbb{R}_r$) we have for every $\varepsilon > 0$ the estimate

\[ R(x) = O(x^{r/4} \log^{2r+1+\varepsilon} x) \quad \text{as} \quad x \to \infty. \]

These results were published in [3], their publication having been recommended by A.N. Kolmogorov.

As already mentioned, the theory of the lattice points in multi-dimensional ellipsoids forms the main skeleton of B. Novák’s work. He was one of the leading world specialists in the field. One of the last papers (cf. [37]) on the subject was B. Novák’s contribution at the International conference on analytical methods in number theory and analysis held in Moscow in 1981. The conference was organized on the occasion of the 90th birthday of I.M. Vinogradov. In this important contribution B. Novák summarized the results of his research and formulated a list of unsolved problems.

B. Novák was also interested in other areas of the number theory. Let us recall some of them in which he achieved some interesting results.

In 1997 B. Novák (together with A.A. Karatsuba) was engaged in arithmetic problems with integers having an even (odd) number of 1’s in their binary representation. He proved results on the distribution of such numbers in short arithmetic progressions, on the distribution of quadratic residues and non-residues modulo increasing prime moduli in sets of such numbers, and also results on the distribution of primitive roots and indices among them. Some of these results were published in Matematicheskie Zametki (Math. Notes, cf. [40]).

In the last years of his life B. Novák was engaged in a problem the formulation of which was motivated by a problem posed at the 31st International Mathematical Olympiad held in Beijing (July 8–July 19, 1990). The problem was proposed by Romania and asked to determine all integers $n > 1$ such that $n^2$ divides $2^n + 1$. It
follows from the solution that the least prime divisor of a solution \( n \) is equal to 3, and that actually the only solution is \( n = 3 \). B. Novák began to study properties and the distribution of integers \( n \) dividing \( 2^n + 1 \). Subsequently, the integers of this form were called in Russia the “Novák numbers” (cf. [75]), and the set of such integers was denoted by \( N_B \) with the reference to B. Novák’s initials.

If \( N_B(x) \) denotes the number of Novák’s numbers not exceeding \( x \), i.e.,

\[
N_B(x) = \#\{n \leq x; \ n \in N_B\}, \quad x > 2,
\]

then B. Novák proved a number of results about its asymptotic behaviour (cf. [41]), e.g. he proved that

\[
N_B(x) = O\left(x/\sqrt{\log x}\right) \quad \text{for } x \to \infty.
\]

Till his death he was tirelessly engaged in finding the best estimate for this counting function. With help of computer he found the first million of such numbers, and also formulated a conjecture that for any \( \varepsilon > 0 \) we have

\[
N_B(x) = O\left(x^{\varepsilon}\right) \quad \text{for } x \to \infty.
\]

This conjecture is still unproved and its proof would provide a substantial contribution to the theory of numbers.

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**Theses**


**Other publications**


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**OTHER REFERENCES**


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