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EXPLICIT SOLUTION OF THE FIRST BVP
OF THE ELASTIC MIXTURE FOR HALF-SPACE
Abstract. We consider the first BVP of elastic mixture theory for a transversally-isotropic half-space. The solution of the first BVP for the transversally-isotropic half-space is given in [1]. The present paper is an attempt to use this result for the BVP of elastic mixture theory for a transversally-isotropic elastic body. Using the potential method and the theory of integral equations, the uniqueness theorem is proved for a half-space and the first BVP previously is solved effectively (in quadratures), which has not been solved.

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The first BVP and the uniqueness theorem for a half-space. Let the plane $\alpha x_1 x_2$ be the boundary of a half-space $x_3 > 0$. Let the upper half-space be denoted by $D$ and the boundary of $D$ by $S$. Let the axis $\alpha x_3$ be directed vertically upwards and the normal be $n(0,0,1)$.

A basic homogeneous equation of statics of transversally-isotropic elastic mixture theory can be written in the form \[2\]

\begin{equation}
C(\partial x)U = \left(\begin{array}{cc}
C_{11}^{(1)}(\partial x) & C_{12}^{(1)}(\partial x) \\
C_{12}^{(1)}(\partial x) & C_{22}^{(1)}(\partial x)
\end{array}\right) \quad U = 0, \tag{1}
\end{equation}

where the components of the matrix $C^{(j)}(\partial x) = \|C_{pq}^{(j)}(\partial x)\|_{3x3}$ are given in the form

\begin{align*}
C_{pq}^{(j)} & = C_{qp}^{(j)}, \quad j = 1,2,3; \quad p,q = 1,2,3, \\
C_{11}^{(j)}(\partial x) & = c_{11}^{(j)} \frac{\partial^2}{\partial x_1^2} + c_{66}^{(j)} \frac{\partial^2}{\partial x_2^2} + c_{44}^{(j)} \frac{\partial^2}{\partial x_3^2}, \\
C_{12}^{(j)}(\partial x) & = (c_{11}^{(j)} - c_{66}^{(j)}) \frac{\partial^2}{\partial x_1 \partial x_2}, \\
C_{k3}^{(j)}(\partial x) & = (c_{13}^{(j)} + c_{44}^{(j)}) \frac{\partial^2}{\partial x_k \partial x_3}, \quad k = 1,2, \\
C_{22}^{(j)}(\partial x) & = c_{66}^{(j)} \frac{\partial^2}{\partial x_1^2} + c_{11}^{(j)} \frac{\partial^2}{\partial x_2^2} + c_{44}^{(j)} \frac{\partial^2}{\partial x_3^2}, \\
C_{33}^{(j)}(\partial x) & = c_{44}^{(j)} \left(\frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2}\right) + c_{33}^{(j)} \frac{\partial^2}{\partial x_3^2}.
\end{align*}

$c_{pq}^{(j)}$ are the constants characterizing physical properties of the mixture and satisfying certain inequalities obtained due to positive definiteness of the potential energy. $U = U^T(x) = (u', u'')$ is a six-dimensional displacement vector-function, $u'(x) = (u'_1, u'_2, u'_3)$ and $u''(x) = (u''_1, u''_2, u''_3)$ are partial displacement vectors. Throughout this paper "$T$" denotes transposition.

**Definition.** A vector-function $U(x)$ defined in the domain $D$ is called regular if it has integrable continuous second derivatives in $D$ and $U(x)$ itself and its first derivatives are continuously extendable at every point of the boundary of $D$, i.e. $U(x) \in C^2(D) \cap C^1(D)$ and satisfies the following conditions at infinity

\begin{equation}
U(x) = O(|x|^{-1}), \quad \frac{\partial U}{\partial x_k} = O(|x|^{-2}), \quad |x|^2 = x_1^2 + x_2^2 + x_3^2, \quad k = 1,2,3.
\end{equation}

For the equation (1) we pose the following BVP. Find a regular function $U(x)$ satisfying the equation (1) in $D$ if on the boundary $S$ the displacement vector $U$ is given in the form

\begin{equation}
U^+ = f(z), \quad z \in S. \tag{2}
\end{equation}
where \((\cdot)^+\) denotes the limiting value from \(D\) and \(f\) is a given vector.

\[
|f_k| < AR, \quad R = \sqrt{z_1^2 + z_2^2} \leq 1, \quad |f_k| < AR^{-\alpha},
\]
\[
\alpha > 0, \quad R > 1, \quad k = 1, \ldots, 6, \quad A = const > 0.
\]

**The Uniqueness Theorem.** Let us prove that the first homogeneous BVP has only a trivial solution. Note that if \(U\) is a regular solution of the equation (1) and satisfies the following conditions at infinity

\[
U(x) = O(|x|^{-\alpha}), \quad P(\partial x, n)U = O(|x|^{-1-\alpha}), \quad \alpha > 0,
\]

then we have the formula

\[
U(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left[ (P(\partial y, n)\Gamma)^* u^+ - \Gamma(y-z)(P(\partial y, n)u)^+ \right] dy_1 dy_2, \quad x \in D,
\]

where \(P(\partial y, n)U\) is the generalized stress vector

\[
(P(\partial y, n)U)_k = c_{44}^{(1)} \frac{\partial u_k'}{\partial x_3} + c_{44}^{(3)} \frac{\partial u_k'}{\partial x_3} + \delta^{(1)} \frac{\partial u_k'}{\partial x_3} + \delta^{(3)} \frac{\partial u_k'}{\partial x_3}, \quad k = 1, 2,
\]

\[
(P(\partial y, n)U)_3 = \beta^{(1)} \left( \frac{\partial u_1'}{\partial x_1} + \frac{\partial u_2'}{\partial x_2} \right) + \beta^{(3)} \left( \frac{\partial u_1''}{\partial x_1} + \frac{\partial u_2''}{\partial x_2} \right) +
\]

\[
+ c_{33}^{(1)} \frac{\partial u_3'}{\partial x_3} + c_{33}^{(3)} \frac{\partial u_3'}{\partial x_3} +\]

\[
(P(\partial y, n)U)_k = c_{44}^{(3)} \frac{\partial u'_k}{\partial x_3} + c_{44}^{(3)} \frac{\partial u'_k}{\partial x_3} +
\]

\[
+ \delta^{(4)} \frac{\partial u_3'}{\partial x_{k-3}} + \delta^{(2)} \frac{\partial u_3'}{\partial x_{k-3}}, \quad k = 4, 5,
\]

\[
(P(\partial y, n)U)_6 = \beta^{(4)} \left( \frac{\partial u_1'}{\partial x_1} + \frac{\partial u_2'}{\partial x_2} \right) + \beta^{(2)} \left( \frac{\partial u_1''}{\partial x_1} + \frac{\partial u_2''}{\partial x_2} \right) +
\]

\[
+ c_{33}^{(3)} \frac{\partial u''_3}{\partial x_3} + c_{33}^{(3)} \frac{\partial u''_3}{\partial x_3},
\]

\[
\beta^{(j)} + \delta^{(j)} = \alpha^{(j)}_{13}, \quad j = 1, 2, 3, \quad \beta^{(4)} + \delta^{(4)} = \alpha^{(3)}_{13},
\]

\[
c_{13}^{(j)} + c_{44}^{(j)} = \alpha^{(j)}_{13}.
\]

\(\Gamma(y-x)\) is the symmetric matrix of the fundamental solution of the equation (1)

\[
\Gamma(x-y) = \begin{pmatrix}
\Gamma^{(1)} & \Gamma^{(3)} \\
\Gamma^{(3)} & \Gamma^{(2)}
\end{pmatrix},
\]

where

\[
\Gamma^{(j)}(x-y) = \sum_{k=1}^{6} \|\Gamma^{(j)(k)}_{pq}\|_{3x3}, \quad j = 1, 2, 3, \quad \Gamma^{(j)(k)}_{pq} = \Gamma^{(j)(k)}_{qp},
\]
\[
\Gamma_p^{(k)} = \delta_p \frac{A_{11}^{(k)}}{r_k} + A_{12}^{(k)} \frac{\partial^2 \Phi_k}{\partial x_p \partial x_q}, \quad p = 1,2; \quad q = 1,2;
\]
\[
\delta_{pq} = 1, \quad p = q, \quad \delta_{pq} = 0, \quad p \neq q,
\]
\[
\Gamma_p^{(k)} = A_{11}^{(k)} \frac{\partial^2 \Phi_k}{\partial x_p \partial x_3}, \quad \Gamma_{13}^{(k)} = A_{33}^{(k)} \frac{r_k}{r_k}, \quad \Gamma_{33}^{(k)} = \delta_p \frac{A_{14}^{(k)}}{r_k} + A_{12}^{(k)} \frac{\partial^2 \Phi_k}{\partial x_p \partial x_q},
\]
\[
\Gamma_p^{(k)} = A_{16}^{(k)} \frac{\partial^2 \Phi_k}{\partial x_p \partial x_3}, \quad \Gamma_{13}^{(k)} = A_{36}^{(k)} \frac{r_k}{r_k}, \quad \Gamma_{33}^{(k)} = A_{34}^{(k)} \frac{\partial^2 \Phi_k}{\partial x_p \partial x_3},
\]
\[
\Gamma_p^{(k)} = \delta_p \frac{A_{14}^{(k)}}{r_k} + A_{12}^{(k)} \frac{\partial^2 \Phi_k}{\partial x_p \partial x_q},
\]
\[
\Gamma_p^{(k)} = A_{16}^{(k)} \frac{\partial^2 \Phi_k}{\partial x_p \partial x_3}, \quad p = 1,2, \quad \Gamma_{13}^{(k)} = \frac{A_{16}}{r_k}.
\]

The coefficients \(A_{pq}^{(k)}\) are defined as follows:
\[
A_{11}^{(k)} = (-1)^k (c_{44}^{(2)} - c_{66}^{(2)} a_k) r_0', \quad A_{14}^{(k)} = -(-1)^k (c_{44}^{(3)} - c_{66}^{(3)} a_k) r_0',
\]
\[
A_{12}^{(k)} = A_{14}^{(k)} / a_k, \quad A_{24}^{(k)} = A_{14}^{(k)} / a_k, \quad A_{15}^{(k)} = \frac{A_{14}^{(k)}}{a_k},
\]
\[
A_{44}^{(k)} = (-1)^k (c_{44}^{(1)} - c_{66}^{(1)} a_k) r_0', \quad k = 1,2, \quad \Gamma_{12}^{(k)} = [r_0(a_1 - a_2)]^{-1},
\]
\[
A_{12}^{(k)} = \frac{\delta_k}{a_k} [q_{44}^{(3)} + a_k t_{12} - a_k^2 t_{11} + c_{11}^{(2)} q_{44}^{(2)}],
\]
\[
A_{42}^{(k)} = \frac{\delta_k}{a_k} [q_{44}^{(3)} + a_k t_{13} - a_k^2 t_{12} - c_{11}^{(2)} q_{44}^{(2)}],
\]
\[
A_{45}^{(k)} = \frac{\delta_k}{a_k} [-q_{44}^{(1)} + a_k t_{23} - a_k^2 t_{33} + c_{11}^{(1)} q_{44}^{(2)}],
\]
\[
A_{33}^{(k)} = \delta_k [q_{44}^{(3)} - a_k t_{42} + a_k^2 t_{44} - c_{44}^{(2)} q_{44}^{(2)}],
\]
\[
A_{36}^{(k)} = \delta_k [q_{44}^{(3)} - a_k t_{62} + a_k^2 t_{66} + c_{44}^{(3)} q_{44}^{(2)}],
\]
\[
A_{66}^{(k)} = \delta_k [q_{44}^{(1)} - a_k t_{52} + a_k^2 t_{55} - c_{44}^{(1)} q_{44}^{(2)}],
\]
\[
A_{13}^{(k)} = \delta_k [v_{13} - v_{11} a_k + v_{12} a_k^2], \quad A_{16}^{(k)} = \delta_k [w_{13} - w_{12} a_k + w_{11} a_k^2],
\]
\[
A_{34}^{(k)} = \delta_k [v_{23} - v_{21} a_k + v_{22} a_k^2], \quad A_{46}^{(k)} = \delta_k [w_{34} - w_{14} a_k + w_{24} a_k^2],
\]
\[
\delta_k = d_k (a_1 - a_k) (a_2 - a_k) b_0^{-1}, \quad k = 3, \ldots, 6,
\]
where \(a_k\) are the positive roots of the characteristic equations:
\[
(r_0 a^2 - c_0 a + q_1)(b_0 a^4 - b_1 a^3 + b_2 a^2 - b_3 a + b_4) = 0,
\]
\[
r_0 = c_0^{(1)} c_{66}^{(2)} - c_0^{(2)}, \quad c_0 = c_0^{(1)} c_{44}^{(2)} + c_0^{(1)} c_{66}^{(2)} - 2 c_0^{(3)} c_{44}^{(3)}.
\]
The coefficients \(d_k, b_k, v_{ij}, w_{ij}, t_{ij}\) are given in [3]. The singular matrix \([P(\partial y, n) \Gamma^* = \sum_{k=1}^6 (M_{pq}^{(k)})_{6\times 6}\), which is obtained from \(P(\partial x, n) \Gamma(x - y)\) by
transposition of the columns and rows and the variables \(x\) and \(y\), has the form

\[
[P(\partial x)\Gamma(x - y)]^* = \sum_{k=1}^{6} \begin{pmatrix} M^{(1k)} & M^{(3k)} \\ M^{(2k)} & M^{(4k)} \end{pmatrix},
\]

where the elements of the matrix \(M^{(jk)} = ||M^{(jk)}||_{3x3}, j = 1, 2, 3, 4\), are written as

\[
M^{(1k)}_{pq} = \delta_{pq} R^{(1k)}_{11} \frac{\partial}{\partial x_3 r_k} + R^{(1k)}_{12} \frac{\partial^3 \Phi_k}{\partial x_3 \partial y_p \partial r_k},
\]

\[
M^{(3k)}_{pq} = R^{(1k)}_{31} \frac{\partial}{\partial x_3 r_k}, \quad M^{(3k)}_{3p} = R^{(3k)}_{11} \frac{\partial}{\partial x_3 r_k}, \quad M^{(3k)}_{33} = R^{(3k)}_{33} \frac{\partial}{\partial x_3 r_k},
\]

\[
M^{(2k)}_{pq} = \delta_{pq} R^{(2k)}_{11} \frac{\partial}{\partial x_3 r_k} + R^{(2k)}_{32} \frac{\partial^3 \Phi_k}{\partial x_3 \partial x_3 \partial x_3}, \quad M^{(4k)}_{pq} = R^{(3k)}_{12} \frac{\partial}{\partial x_3 r_k}, \quad M^{(4k)}_{33} = R^{(3k)}_{33} \frac{\partial}{\partial x_3 r_k},
\]

\[
M^{(2k)}_{pq} = \delta_{pq} R^{(4k)}_{11} \frac{\partial}{\partial x_3 r_k} + R^{(4k)}_{32} \frac{\partial^3 \Phi_k}{\partial x_3 \partial x_3 \partial x_3}, \quad M^{(4k)}_{pq} = R^{(4k)}_{12} \frac{\partial}{\partial x_3 r_k}, \quad M^{(4k)}_{33} = R^{(4k)}_{33} \frac{\partial}{\partial x_3 r_k},
\]

The coefficients \(R^{(k)}_{pq}\) satisfy the following conditions

\[
\sum_{k=1}^{2} R^{(k)}_{11} a_k = \sum_{k=3}^{6} R^{(k)}_{23} a_k = \sum_{k=3}^{6} R^{(k)}_{66} a_k = \sum_{k=1}^{2} R^{(k)}_{44} a_k = 1,
\]

\[
\sum_{k=1}^{2} R^{(k)}_{12} a_k = \sum_{k=1}^{6} R^{(k)}_{12} a_k = 0,
\]

\[
\sum_{k=1}^{2} R^{(k)}_{33} a_k = \sum_{k=1}^{2} R^{(k)}_{33} a_k = 0.
\]
and, after elementary calculations the coefficients $R^{(k)}_{13}, \ldots, R^{(k)}_{64}$ take the form

\[
\begin{align*}
R^{(k)}_{13} &= \delta^{(1)}_0 A^{(k)}_{33} + \delta^{(3)}_0 A^{(k)}_{36} + c^{(1)}_4 A^{(k)}_{13} + c^{(3)}_4 A^{(k)}_{43}, \\
R^{(k)}_{16} &= \delta^{(1)}_0 A^{(k)}_{36} + \delta^{(3)}_0 A^{(k)}_{66} + c^{(1)}_4 A^{(k)}_{16} + c^{(3)}_4 A^{(k)}_{46}, \\
R^{(k)}_{31} &= -a_k \beta^{(0)}_0 A^{(k)}_{12} - a_k \beta^{(0)}_0 A^{(k)}_{42} + c^{(1)}_3 A^{(k)}_{13} + c^{(3)}_3 A^{(k)}_{46}, \\
R^{(k)}_{34} &= -a_k \beta^{(0)}_0 A^{(k)}_{42} - a_k \beta^{(0)}_0 A^{(k)}_{45} + c^{(1)}_3 A^{(k)}_{43} + c^{(3)}_3 A^{(k)}_{46}, \\
R^{(k)}_{43} &= \delta^{(i)}_0 A^{(k)}_{33} + \delta^{(2)}_0 A^{(k)}_{36} + c^{(3)}_4 A^{(k)}_{43}, \\
R^{(k)}_{46} &= \delta^{(i)}_0 A^{(k)}_{36} + \delta^{(2)}_0 A^{(k)}_{66} + c^{(3)}_4 A^{(k)}_{46}, \\
R^{(k)}_{61} &= -a_k \beta^{(0)}_0 A^{(k)}_{12} - a_k \beta^{(0)}_0 A^{(k)}_{42} + c^{(3)}_3 A^{(k)}_{13} + c^{(3)}_3 A^{(k)}_{46}, \\
R^{(k)}_{64} &= -a_k \beta^{(0)}_0 A^{(k)}_{42} - a_k \beta^{(0)}_0 A^{(k)}_{45} + c^{(3)}_3 A^{(k)}_{43} + c^{(3)}_3 A^{(k)}_{46}, \quad k = 3, \ldots, 6.
\end{align*}
\]

We can easily prove that every column of the matrix $[P(\partial x, n)\Gamma]^{*}$ is a solution of the system (1) with respect to the point $x$ if $x \neq y$ and all elements $M^{(k)}_{pq}$ have a singularity of type $|x|^{-2}$.

We choose $\delta^{(j)}_0, \beta^{(j)}_0, j = 1, \ldots, 4$, so that

\[
\begin{align*}
\sum_{k=3}^{6} \frac{R^{(k)}_{13}}{\sqrt{a_k}} &= 0, & \sum_{k=3}^{6} \frac{R^{(k)}_{16}}{\sqrt{a_k}} &= 0, & \sum_{k=3}^{6} \frac{R^{(k)}_{31}}{\sqrt{a_k}} &= 0, & \sum_{k=3}^{6} \frac{R^{(k)}_{34}}{\sqrt{a_k}} &= 0, \\
\sum_{k=3}^{6} \frac{R^{(k)}_{43}}{\sqrt{a_k}} &= 0, & \sum_{k=3}^{6} \frac{R^{(k)}_{46}}{\sqrt{a_k}} &= 0, & \sum_{k=3}^{6} \frac{R^{(k)}_{61}}{\sqrt{a_k}} &= 0, & \sum_{k=3}^{6} \frac{R^{(k)}_{64}}{\sqrt{a_k}} &= 0.
\end{align*}
\]

After some simplification, we find from (10) that

\[
\Delta = \sum_{k=3}^{6} A^{(k)}_{12} \sqrt{a_k} \sum_{k=3}^{6} A^{(k)}_{45} \sqrt{a_k} - \left( \sum_{k=3}^{6} A^{(k)}_{42} \sqrt{a_k} \right)^2 =
\]

\[
= \sqrt{a_3 a_4 a_5 a_6} \left[ \sum_{k=3}^{6} \frac{A^{(k)}_{33}}{\sqrt{a_k}} \sum_{k=3}^{6} \frac{A^{(k)}_{36}}{\sqrt{a_k}} - \left( \sum_{k=3}^{6} \frac{A^{(k)}_{46}}{\sqrt{a_k}} \right)^2 \right] =
\]

\[
= \frac{b_0}{b_0} \left[ (\delta_{11} \delta_{22} + b_0 m_1 m_3) q_4 + q_1 b_4 + \delta_{22} b_0 m_2 \sqrt{a_3 a_4 a_5 a_6} \right]^{-1} + q_1 (\delta_{11} \delta_{22} + b_0 m_1 m_3 - k_1) + b_0 \delta_{11} m_2,
\]

where

\[
q_1 = c^{(1)}_{11} c^{(2)}_{11} - c^{(3)}_{11}, \quad q_4 = c^{(1)}_{44} c^{(2)}_{44} - c^{(3)}_{44}, \quad b_0 = q_1 q_4,
\]

\[
m_1 = \sum_{k=3}^{6} \sqrt{a_k}, \quad m_2 = \sum_{p \neq q} \sqrt{a_p a_q},
\]

\[
m_3 = \sum_{p \neq q \neq j} \sqrt{a_p a_q a_j}, \quad p, q, j = 3, \ldots, 6.
\]
\[ \delta_{11} = c_{11}^{(2)} c_{44}^{(1)} + c_{44}^{(1)} c_{11}^{(2)} - 2 c_{11}^{(3)} c_{44}^{(3)} > 0, \]
\[ \delta_{22} = c_{33}^{(1)} c_{44}^{(2)} + c_{44}^{(1)} c_{33}^{(2)} - 2 c_{33}^{(3)} c_{44}^{(3)} > 0, \]
\[ k_1 + k_2 = 2 (\alpha_{13}^{(1)} - \alpha_{13}^{(2)}) - \alpha_{13}^{(1)} v_{11} - \alpha_{13}^{(2)} w_{14} - \alpha_{13}^{(3)} (w_{12} + v_{21}), \]
\[ k_1 = \frac{1}{c_{44}^{(2)}} \left[ c_{44}^{(2)} c_{13}^{(3)} - 2 c_{44}^{(2)} c_{44}^{(3)} c_{13}^{(3)} + c_{44}^{(3)} c_{13}^{(3)} + c_{44}^{(3)} \right]^2 + \frac{2 q_4}{c_{44}^{(2)}} \left[ c_{44}^{(2)} c_{13}^{(3)} - c_{44}^{(3)} c_{13}^{(3)} \right]^2 + \frac{q_4^2}{c_{44}^{(2)}} \alpha_{13}^{(2)}, \]
\[ B_0^{-1} = \prod_{p \neq q} (\sqrt{a_p} + \sqrt{a_q}), \quad p, q = 3, \ldots, 6. \]

Taking into account the inequalities obtained from the positive definiteness of the energy \( E(u, u) \), we conclude that \( \Delta \neq 0 \). When \( \delta_0^{(2)}, \delta_0^{(3)} \) are solutions of the system (10), we denote the vector \( P(\partial y, n) \) by \( N(\partial y, n) \). Then from (4), when \( U^+ = 0 \), we have

\[ U(x) = -\frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \Gamma(y - x) N(\partial y, n) U dy_1 dy_2. \]

Hence for the vector \( NU \) as \( x(x_1, x_2, x_3) \rightarrow z(z_1, z_2, 0) \) we find

\[ [N(\partial z, n)U]^+ + \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} N \Gamma(y - z)(NU)^+ dy_1 dy_2 = 0. \]

Note that \( N \Gamma(z - y) = 0, \quad z \in S \). Therefore \( (NU)^+ = 0 \), and from (4) we have \( U = 0, \quad x \in D \). Therefore the homogeneous equation has only the trivial solution. Thus we formulate the following

**Theorem.** *The first BVP has at most one regular solution.*

**The first BVP.** A solution of the first BVP will be sought in the domain \( D \) in terms of the double layer potential

\[ U(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} [N(\partial y, n) \Gamma(y - x)]^+ g(y) dy_1 dy_2, \quad (11) \]

where \( g \) is an unknown real vector. Taking into account the properties of the double layer potential and the boundary condition for determining \( g \), we obtain the following Fredholm integral equation of second kind:

\[ g(z) + \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} [N(\partial y, n) \Gamma(y - z)]^+ g(y) dy_1 dy_2 = f(z), \]
Taking into account the fact that \([NT] = 0, x_3 = 0\), from the latter equation we have \(g(z) = f(z)\) and (11) takes the form

\[
U(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} [N(\partial y, n)\Gamma(y - x)]* f(y) dy_1 dy_2.
\] (12)

Thus we have obtained the Poisson formula for the solution of the first BVP for the half-space. Note that (12) is valid if and only if \(f \in C^{1,\alpha}(S)\) and satisfies the condition \(f = O(A|x|^{\beta})\) at infinity, where \(A\) is a constant vector and \(\beta > 0\).

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References


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