ON SOLVABILITY OF BOUNDARY VALUE PROBLEMS ON AN INFINITY INTERVAL FOR NONLINEAR TWO DIMENSIONAL GENERALIZED AND IMPULSIVE DIFFERENTIAL SYSTEMS

Abstract. Sufficient conditions are given for the solvability of boundary value problems on an infinite interval for nonlinear two dimensional generalized and impulsive differential systems.

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Let $c \in \mathbb{R}$, $a_{ik} : \mathbb{R}_+ \rightarrow \mathbb{R}$ ($i, k = 1, 2$) be nondecreasing continuous from the left functions, and let $f_k : \mathbb{R}_+ \times \mathbb{R}^2 \rightarrow \mathbb{R}$ be a vector-function belonging to the Carathéodory class corresponding to the $a_{ik}$ for every $i, k \in \{1, 2\}$.

In this paper we investigate the question of existence of solutions for the two dimensional generalized differential system

$$dx_i(t) = f_1(t, x_1(t), x_2(t)) \cdot da_{i1}(t) + f_2(t, x_1(t), x_2(t)) \cdot da_{i2}(t) \quad \text{for} \quad t \in \mathbb{R}_+ \quad (i = 1, 2), \quad (1)$$

satisfying one of the following two conditions

$$x_1(0) = c, \quad \sup \left\{ |x_1(t)| + |x_2(t)| : t \in \mathbb{R}_+ \right\} < \infty \quad (2)$$

and

$$\sup \left\{ |x_1(t)| + |x_2(t)| : t \in \mathbb{R}_+ \right\} < \infty. \quad (3)$$

We give sufficient conditions for the existence of solutions of the boundary value problems (1), (2) and (1), (3). Analogous results are contained in [10], [11], [13]–[17] for ordinary differential and functional differential systems.

The theory of generalized ordinary differential equations enables one to investigate ordinary differential, impulsive and difference equations from a common point of view (see, e.g., [1]–[9], [12], [23], and references therein).

We realize the obtained result for the following second order system of impulsive equations

\[
\frac{dx_i}{dt} = f_i(t, x_1, x_2) \quad \text{for almost all } t \in \mathbb{R}_+ \setminus \{\tau_1, \tau_2, \ldots\} \quad (i = 1, 2),
\]

\[
x_i(\tau_k^+)-x_i(\tau_k^-)=\alpha_{ki}I_{ki}(x_i(\tau_k^-), x_i(\tau_k^-)) \quad \text{for } k \in \{1, 2, \ldots\} \quad (i = 1, 2),
\]

where 0 < \tau_1 < \tau_2 < \ldots, \tau_k \to \infty (k \to \infty) (we will assume \tau_0 = 0 if necessary), \alpha_{ki} \in \mathbb{R} (i = 1, 2; k = 1, 2, \ldots), f_i \in K_{loc}(\mathbb{R}_+ \times \mathbb{R}^2, \mathbb{R}) (i = 1, 2), \text{ and } I_{ki} : \mathbb{R}^2 \to \mathbb{R} (i = 1, 2; k = 1, 2, \ldots) \text{ are continuous operators.}

Throughout the paper the following notation and definitions will be used.

\(\mathbb{R} = ]-\infty, +\infty[, \mathbb{R}_+ = [0, +\infty[, [a, b] (a, b \in \mathbb{R}) \text{ is a closed segment.}\)

\(\mathbb{R}^{n \times m} \text{ is the set of all real } n \times m \text{-matrices } X = (x_{ij})_{i,j=1}^{n,m}.\)

\(\mathbb{R}^n = \mathbb{R}^{n \times 1} \text{ is the set of all real column } n\text{-vectors } x = (x_i)_{i=1}^n, \mathbb{R}_+^n = \mathbb{R}_+^{n \times 1}.\)

\(\text{diag}(\lambda_1, \ldots, \lambda_n) \text{ is the diagonal matrix with the diagonal elements } \lambda_1, \ldots, \lambda_n.\)

\(\mathcal{B}V([a, b], \mathbb{R}^{n \times m}) \text{ is the set of all matrix-functions of bounded variation } X : [a, b] \to \mathbb{R}^{n \times m} \text{ (i.e., such that } \mathcal{B}V(X) < +\infty).\)

\(\mathcal{B}V_{loc}(\mathbb{R}, \mathbb{R}^{n \times m}) \text{ is the set of all matrix-functions } X : \mathbb{R} \to \mathbb{R}^{n \times m} \text{ for which } \mathcal{B}V(X) < +\infty \text{ for every } a, b \in \mathbb{R} (a < b).\)

\(s_j : \mathcal{B}V([a, b], \mathbb{R}) \to \mathcal{B}V([a, b], \mathbb{R}), (j = 0, 1, 2) \text{ are the operators defined, respectively, by}\)

\[
s_1(x)(a) = s_2(x)(a) = 0,
\]

\[
s_1(x)(t) = \sum_{a < \tau \leq t} d_1 x(\tau) \quad \text{and} \quad s_2(x)(t) = \sum_{a \leq \tau < t} d_2 x(\tau) \quad \text{for } a < t \leq b
\]

and

\[
s_0(x)(t) = x(t) - s_1(x)(t) - s_2(x)(t) \quad \text{for } t \in [a, b].
\]

\(\mathcal{A} : \mathcal{B}V_{loc}(\mathbb{R}, \mathbb{R}) \times \mathcal{B}V_{loc}(\mathbb{R}, \mathbb{R}) \to \mathcal{B}V_{loc}(\mathbb{R}, \mathbb{R}) \text{ is the operator defined by}\)

\(\mathcal{A}(x,y)(0) = 0,\)
\[ A(x, y)(t) = y(t) + \sum_{0 < \tau \leq t} d_1 x(\tau) \cdot (1 - d_1 x(\tau))^{-1} d_1 y(\tau) - \sum_{0 \leq \tau < t} d_2 x(\tau) \cdot (1 + d_2 x(\tau))^{-1} d_2 y(\tau) \quad \text{for } t > 0, \]
\[ A(x, y)(t) = y(t) - \sum_{t < \tau \leq 0} d_1 x(\tau) \cdot (1 - d_1 x(\tau))^{-1} d_1 y(\tau) + \sum_{t \leq \tau < 0} d_2 x(\tau) \cdot (1 + d_2 x(\tau))^{-1} d_2 y(\tau) \quad \text{for } t < 0 \]

for every \( x \in BV_{loc}(\mathbb{R}, \mathbb{R}) \) such that for every \( x \in BV_{loc}(\mathbb{R}, \mathbb{R}) \) such that
\[ 1 + (-1)^j d_j x(t) \neq 0 \quad \text{for } t \in \mathbb{R} \quad (j = 1, 2). \]

If \( g : [a, b] \to \mathbb{R} \) is a nondecreasing function, \( x : [a, b] \to \mathbb{R} \) and \( a \leq s < t \leq b \), then
\[
\int_a^t x(\tau) \, dg(\tau) = \int_{[s,t]} x(\tau) \, ds_0(g)(\tau) + \sum_{s < \tau \leq t} x(\tau) d_1 g(\tau) + \sum_{s \leq \tau < t} x(\tau) d_2 g(\tau),
\]
where \( \int_{[s,t]} x(\tau) \, ds_0(g)(\tau) \) is the Lebesgue–Stieltjes integral over the open interval \([s,t]\) with respect to the measure \( \mu_0(s_0(g)) \) corresponding to the function \( s_0(g) \).

If \( a = b \), then we assume
\[ \int_a^b x(t) \, dg(t) = 0. \]

If \( g(t) \equiv g_1(t) - g_2(t) \), where \( g_1 \) and \( g_2 \) are nondecreasing functions, then
\[ \int_s^t x(\tau) \, dg(\tau) = \int_s^t x(\tau) \, dg_1(\tau) - \int_s^t x(\tau) \, dg_2(\tau) \quad \text{for } s \leq t. \]

\( L([a, b], \mathbb{R}; g) \) is the set of all functions \( x : [a, b] \to \mathbb{R} \) measurable and integrable with respect to the measures \( \mu(g_i) \) (\( i = 1, 2 \)), i.e., such that
\[ \int_a^b |x(t)| \, dg_i(t) < +\infty \quad (i = 1, 2). \]

A matrix-function is said to be continuous, nondecreasing, integrable, etc., if each of its components is such.

If \( G = (g_{ik})_{i,k=1}^{l,n} : [a, b] \to \mathbb{R}^{l \times n} \) is a nondecreasing matrix-function and \( D \subset \mathbb{R}^{n \times m} \), then \( L([a, b], D; G) \) is the set of all matrix-functions \( X = (x_{kj})_{k,j=1}^{n,m} : [a, b] \to D \) such that \( x_{kj} \in L([a, b], \mathbb{R}; g_{ik}) \) (\( i = 1, \ldots, l; k = 1, \ldots, n \)).
1, \ldots, n; j = 1, \ldots, m);

\int_t^s dG(\tau) \cdot X(\tau) = \left( \sum_{k=1}^{n} \int_s^t x_{kj}(\tau) dg_{kj}(\tau) \right)_{i,j=1}^{1,m} \text{ for } a \leq s \leq t \leq b,

S_j(G)(t) \equiv (s_j(g_{jk})(t))_{i,k=1}^{1,n} (j = 0, 1, 2).

If \( D_1 \subset \mathbb{R}^n \) and \( D_2 \subset \mathbb{R}^{n \times m} \), then \( K([a, b] \times D_1, D_2; G) \) is the Carathéodory class, i.e., the set of all mappings \( F = (f_{kj})_{k,j=1}^{n,m} : [a, b] \times D_1 \rightarrow D_2 \) such that for each \( i \in \{1, \ldots, l\} \), \( j \in \{1, \ldots, m\} \) and \( k \in \{1, \ldots, n\} \):

a) the function \( f_{kj}(\cdot, x) : [a, b] \rightarrow D_2 \) is \( \mu(g_{ik}) \)-measurable for every \( x \in D_1 \);

b) the function \( f_{kj}(t, \cdot) : D_1 \rightarrow D_2 \) is continuous for \( \mu(g_{ik}) \)-almost every \( t \in [a, b] \), and sup \( \{ |f_{kj}(t, x)| : x \in D_0 \} \in L([a, b], \mathbb{R}; g_{ik}) \) for every compact \( D_0 \subset D_1 \).

If \( G_j : [a, b] \rightarrow \mathbb{R}^{l \times n} (j = 1, 2) \) are nondecreasing matrix-functions, \( G = G_1 - G_2 \) and \( X : [a, b] \rightarrow \mathbb{R}^{n \times m} \), then

\[ \int_t^s dG(\tau) \cdot X(\tau) = \int_t^s dG_1(\tau) \cdot X(\tau) - \int_t^s dG_2(\tau) \cdot X(\tau) \text{ for } s \leq t, \]

\[ S_k(G) = S_k(G_1) - S_k(G_2) \quad (k = 0, 1, 2), \]

\[ L([a, b], D; G) = \bigcap_{j=1}^{2} L([a, b], D; G_j), \]

\[ K([a, b] \times D_1, D_2; G) = \bigcap_{j=1}^{2} K([a, b] \times D_1, D_2; G_j). \]

\( L_{loc}(\mathbb{R}, D; G) \) is the set of all matrix-functions \( X : \mathbb{R} \rightarrow D \) such that its restriction on \([a, b] \) belongs to \( L([a, b], D; G) \) for every \( a \) and \( b \) from \( \mathbb{R} \) \((a < b)\).

\( K([a, b] \times D_1, D_2; G) \) is the set of all matrix-functions \( F = (f_{kj})_{k,j=1}^{n,m} : \mathbb{R} \times D_1 \rightarrow D_2 \) such that its restriction on \([a, b] \) belongs to \( K([a, b], D; G) \) for every \( a \) and \( b \) from \( \mathbb{R} \) \((a < b)\).

If \( G(t) \equiv \text{diag}(t, \ldots, t) \), then we omit \( G \) in the notation containing \( G \).

The inequalities between the vectors and between the matrices are understood componentwise.

A vector-function \( x = (x_i)^2 \in B V_{loc}(\mathbb{R}^+, \mathbb{R}^2) \) is said to be a solution of the system (1) if

\[ x_1(t) = x_1(s) + \int_s^t f_1(\tau, x_1(\tau), x_2(\tau)) \cdot da_{i1}(\tau) + \]

\[ + \int_s^t f_2(\tau, x_1(\tau), x_2(\tau)) \cdot da_{i2}(t) \text{ for } 0 \leq s \leq t \quad (s, t \in \mathbb{R}) \quad (i = 1, 2). \]
If \( s \in \mathbb{R} \) and \( \beta \in \text{BV}_{\text{loc}}(\mathbb{R}, \mathbb{R}) \) are such that
\[
1 + (-1)^j d_j \beta(t) \neq 0 \quad \text{for} \quad (-1)^j (t - s) < 0 \quad (j = 1, 2),
\]
then by \( \gamma_\beta(\cdot, s) \) we denote the unique solution of the Cauchy problem
\[
d\gamma(t) = \gamma(t) \, d\beta(t), \quad \gamma(s) = 1.
\]
It is known (see [9], [12]) that
\[
\gamma_\beta(t, s) = \exp \left( \xi_\beta(t) - \xi_\beta(s) \right) \prod_{s < \tau \leq t} \text{sgn} \left( 1 - d_1 \beta(\tau) \right) \times \prod_{s \leq \tau < t} \text{sgn} \left( 1 + d_2 \beta(\tau) \right) \quad \text{for} \quad t > s,
\]
\[
\gamma_\beta(t, s) = \gamma_\beta^{-1}(s, t) \quad \text{for} \quad t < s,
\]
where
\[
\xi_\beta(t) = s_0(\beta)(t) - s_0(\beta)(0) - \sum_{0 < \tau \leq t} \ln \left| 1 - d_1 \beta(\tau) \right| + \sum_{0 \leq \tau < t} \ln \left| 1 + d_2 \beta(\tau) \right| \quad \text{for} \quad t > 0,
\]
\[
\xi_\beta(t) = s_0(\beta)(t) - s_0(\beta)(0) + \sum_{t < \tau \leq 0} \ln \left| 1 - d_1 \beta(\tau) \right| - \sum_{t \leq \tau < 0} \text{sgn} \left| 1 + d_2 \beta(\tau) \right| \quad \text{for} \quad t < 0.
\]

**Remark 1.** Let \( \beta \in \text{BV}([a, b], \mathbb{R}) \) be such that
\[
1 + (-1)^j d_j \beta(t) > 0 \quad \text{for} \quad t \in [a, b] \quad (j = 1, 2).
\]
Let, moreover, one of the functions \( \beta, \xi_\beta \) and \( A(\beta, \beta) \) be nondecreasing (nonincreasing). Then the other two functions will be nondecreasing (non-increasing) as well.

Let \( \delta > 0 \). We introduce the operators
\[
\nu_{1+}(\xi)(t) = \sup \left\{ \tau \geq t : \xi(\tau) \leq \xi(t+) + \delta \right\}
\]
and
\[
\nu_{-1+}(\eta)(t) = \inf \left\{ \tau \leq t : \eta(\tau) \leq \eta(t-) + \delta \right\},
\]
respectively, on the set of all nondecreasing functions \( \xi : \mathbb{R} \rightarrow \mathbb{R} \) and on the set of all nonincreasing functions \( \eta : \mathbb{R} \rightarrow \mathbb{R} \).

\( \tilde{C}([a, b], D) \), where \( D \subset \mathbb{R}^{n \times m} \), is the set of all absolutely continuous matrix-functions \( X : [a, b] \rightarrow D \);

\( \tilde{C}_{\text{loc}}(\mathbb{R}^+ \setminus \{\tau_1, \tau_2, \ldots\}, D) \) is the set of all matrix-functions \( X : \mathbb{R}^+ \rightarrow D \) whose restriction to an arbitrary closed interval \([a, b]\) from \( \mathbb{R}^+ \setminus \{\tau_1, \tau_2, \ldots\}\) belongs to \( \tilde{C}([a, b], D) \);

\( L([a, b], D) \) is the set of all matrix-functions \( X : [a, b] \rightarrow D \), measurable and integrable.
\[ L_{\text{loc}}(\mathbb{R}^+ \setminus \{\tau_1, \tau_2, \ldots\}, D) \] is the set of all matrix-functions \( X : \mathbb{R}^+ \rightarrow D \) whose restriction to an arbitrary closed interval \([a, b]\) from \(\mathbb{R}^+ \setminus \{\tau_1, \tau_2, \ldots\}\) belongs to \(\tilde{C}([a, b], D)\).

If \(D_1 \subset \mathbb{R}^n\) and \(D_2 \subset \mathbb{R}^{n \times m}\), then \(K([a, b] \times D_1, D_2)\) is the Carathéodory class, i.e., the set of all mappings \(F = (f_{kj})_{k,j=1}^{n,m} : [a, b] \times D_1 \rightarrow D_2\) such that for each \(i \in \{1, \ldots, l\}\), \(j \in \{1, \ldots, m\}\) and \(k \in \{1, \ldots, n\}\): a) the function \(f_{kj}(\cdot, x) : [a, b] \rightarrow D_2\) is measurable for every \(x \in D_1\); b) the function \(f_{kj}(t, \cdot) : D_1 \rightarrow D_2\) is continuous for almost all \(t \in [a, b]\), and sup \(\{|f_{kj}(\cdot, x)| : x \in D_0\} \in L([a, b], \mathbb{R}; g_{ik})\) for every compact \(D_0 \subset D_1\).

\(K_{\text{loc}}(\mathbb{R}^+ \times D_1, D_2)\) is the set of all mappings \(F : \mathbb{R}^+ \times D_1 \rightarrow D_2\) whose restriction to an arbitrary closed interval \([a, b]\) from \(\mathbb{R}^+ \setminus \{\tau_1, \tau_2, \ldots\}\) belongs to \(K([a, b] \times D_1, D_2)\).

By a solution of the impulsive system (3), (4) we understand a continuous from the left vector-function \(x \in \tilde{C}_{\text{loc}}(\mathbb{R}^+ \setminus \{\tau_1, \tau_2, \ldots\}) \cap \text{BV}_{\text{loc}} s(\mathbb{R}^+, \mathbb{R}^m)\) satisfying both the system (1) for a.a. \(t \in \mathbb{R}^+ \setminus \{\tau_k\}_{k=1}^{m_0}\) and the relation (2) for every \(k \in \{1, 2, \ldots\}\).

**Theorem 1.** Let
\[
0 \leq d_2(a_{i1}(t) + a_{i2}(t)) < |\eta_{ii}|^{-1} \quad \text{for } t \in \mathbb{R}^+ \quad (i = 1, 2),
\]
\[
1 + \sigma_id_2a_{ii}(t) > 0 \quad \text{for } t \in \mathbb{R}^+ \quad (i = 1, 2),
\]
\[
\sigma_if_{ik}(t, x_1, x_2) \text{sgn } x_i \leq \eta_{i1}|x_1| + \eta_{i2}|x_2| + q_k(t)
\]
for \(\mu(a_{ik})\)-almost all \(t \in \mathbb{R}^+\) and \(x_1, x_2 \in \mathbb{R}\) \((i, k = 1, 2)\),

and let the real part of every eigenvalue of the matrix \((\eta_{ii})_{i=1}^2\) be negative, where \(\sigma_1 = 1, \sigma_2 = -1\) \((\sigma_1 = \sigma_2 = -1)\), \(\eta_{i1}, \eta_{i2} \in \mathbb{R}; \eta_{ii} < 0 \quad (i = 1, 2),\)
\(q_k \in L_{\text{loc}}(\mathbb{R}^+, \mathbb{R}; a_{1k}) \cap L_{\text{loc}}(\mathbb{R}^+, \mathbb{R}; a_{2k}) \quad (k = 1, 2)\). Let, moreover,
\[
\sigma_i \lim_{t \to \infty} \inf_{\tau \leq t} (\xi_{\sigma_ia_{ii}}(t) - \xi_{\sigma_ia_{ii}}(0)) > \delta > 0 \quad (i = 1, 2)
\]
for some \(\delta > 0\),
\[
\sup \left\{ \int_{\nu_i(t)}^{\nu_i(t)} |q_k(\tau)| ds_0(a_{ik})(\tau) + \sum_{t < \tau \leq \nu_i(t)} (1 + \sigma_id_2a_{ii}(t))^{-1} |q_k(\tau)| ds_0(a_{ik})(\tau) : t \in \mathbb{R}^+ \right\} < \infty \quad (i, k = 1, 2),
\]
\[
s_{it} = \left| \int_0^{\infty} \gamma_{\sigma_ia_{ii}}(t, s) dA(\sigma_ia_{ii}, a_{il})(s) \right| < \infty \quad (i \neq l; \ i, l = 1, 2)
\]
and
\[
s_{1}s_{2} < 1,
\]
where \(\nu_i(t) \equiv \nu_{\sigma_i,\delta}(-\xi_{\sigma_ia_{ii}})(t) \quad (i = 1, 2)\). Then the problem (1), (2) ((1), (3)) is solvable.
where the functions \( \sigma \) to (6)

and let the real part of every eigenvalue of the matrix

where \( a \) continuous from the left. In this case

the impulsive system (4) if and only if it is a solution of the system (1),

where \( a_{12}(t) = a_{21}(t) \equiv 0, \]

\[
\sigma_{ii}(t) \equiv t + \sum_{k: 0 \leq t_k < t} \alpha_{ki} \quad (i = 1, 2),
\]

\[
f_i(t_k, x_1, x_2) = I_{ki}(x_1, x_2) \quad \text{for} \quad x_1, x_2 \in \mathbb{R} \quad (i = 1, 2; \quad k = 1, 2, \ldots).
\]

It is evident that \( a_{ii} \quad (i = 1, 2) \) are nondecreasing if \( \alpha_{ki} \geq 0, \)

\( d_{2a_{ii}}(t_k) = \alpha_{ki} \) and \( d_{2a_{ii}}(t) = 0 \) \( t \neq t_k \quad (i = 1, 2; \quad k = 1, 2, \ldots). \)

Moreover, they are continuous from the left. In this case

\[
\xi_{i, a_{ii}} \equiv \sigma_i t + \sum_{k: 0 \leq t_k < t} \ln |1 + \sigma_i \alpha_{ki}| \quad (i = 1, 2).
\]

**Theorem 2.** Let

\[
0 \leq \alpha_{ki} < |\eta_{ii}|^{-1} \quad (i = 1, 2; \quad k = 1, 2, \ldots), \quad (7)
\]

\[
1 + \sigma_i \alpha_{ki} > 0 \quad (i = 1, 2; \quad k = 1, 2, \ldots), \quad (8)
\]

\[
f_i(t, x_1, x_2) \quad \text{sgn} \quad x_i \leq \eta_{i1} |x_1| + \eta_{i2} |x_2| + q_i(t)
\]

for almost all \( t \in \mathbb{R}^+ \) and \( x_1, x_2 \in \mathbb{R} \quad (i = 1, 2; \quad k = 1, 2, \ldots), \)

\[
I_{ki}(x_1, x_2) \quad \text{sgn} \quad x_i \leq \eta_{i1} |x_1| + \eta_{i2} |x_2| + q_i \quad 
\]

for \( x_1, x_2 \in \mathbb{R} \quad (i = 1, 2; \quad k = 1, 2, \ldots), \)

and let the real part of every eigenvalue of the matrix \( (\eta_{ii})_{i=1}^{2} \) be negative,

where \( \sigma_1 = 1, \quad \sigma_2 = -1 \quad (\sigma_1 = \sigma_2 = -1), \quad \eta_{i1}, \quad \eta_{i2} \in \mathbb{R}, \quad \eta_{ii} < 0 \quad (i = 1, 2), \quad q_i \in L_{loc}(\mathbb{R}^+, \mathbb{R}) \quad (i = 1, 2). \)

Let, moreover,

\[
\lim_{t \to \infty} \inf \left( t + \sigma_i \sum_{k: 0 \leq t_k < t} \ln(1 + \sigma_i \alpha_{ki}) \right) > \delta > 0 \quad (i = 1, 2) \quad (9)
\]

for some \( \delta > 0, \)

\[
\sup \left\{ \int_{t} |q_i(\tau)| d(\tau) + \sum_{k: 0 \leq t_k < t_i} (1 + \sigma_i \alpha_{ki})^{-1} |q_{ki}| : \quad t \in \mathbb{R}^+ \right\} < \infty
\]

\[(i = 1, 2), \]

where the functions \( \nu_i(t) \equiv \nu_{i, \delta}(-\xi_{i, a_{ii}}(t)) \quad (i = 1, 2) \) are defined according to (6). Then the problem (4), (5); (2) ((4), (5); (3)) is solvable.

**Remark 2.** By condition (7), the conditions (8) and (9) are fulfilled if \( \sigma_i = 1. \)
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