CORRIGENDUM TO "PRODUCTS OF PSEUDORADIAL SPACES"

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In our paper (Math. Pannonica 6/1 (1995), 29 – 38) the proofs of Ths. 3.2 and 3.3 are wrong. They should be substituted as follows; in order to make as little changes as possible Ths. 3.2 and 3.3 will be labeled 3.3 and 3.3', while the next Th. 3.2 is a proposition of independent interest.

Theorem 3.2. Let \((X_\alpha)_{\alpha \in \omega_1}\) be a family of compact Hausdorff spaces and for each \(\alpha \in \omega_1\) let \(|X_\alpha| < 2^{\omega_2}\). Then every closed subset \(F\) of the cartesian product \(\prod_{\alpha \in \omega_1} X_\alpha\) has a point \(p\) such that \(\chi(p,F) \leq \omega_1\).

Proof. Let \(F\) be a closed subset of \(X = \prod_{\alpha \in \omega_1} X_\alpha\). Let \(\pi_\alpha : X \to X_\alpha\) be the projection on the \(\alpha\)-th factor of the product. Since \(|X_0| < 2^{\omega_2}\), then, by the Čech – Pospíšil theorem, there is a point \(x_0 \in \pi_0(F) \subset X_0\) such that \(\chi(x_0,\pi_0(F)) \leq \omega_1\). Let \(F_1 = (\{x_0\} \times \prod_{1 \leq \alpha < \omega_1} X_\alpha) \cap F\). Clearly \(F_1\) is a \(G_{\omega_1}\) set in \(F = F_0\) and there is \(x_1 \in \pi_1(F_1) \subset X_1\) such that \(\chi(x_1,\pi_1(F_1)) \leq \omega_1\).

Suppose we have found points and closed sets \(x_\gamma\) and \(F_\gamma\), where \(x_\gamma \in \pi_\gamma(F_\gamma) \subset X_\gamma\) such that \(F_\gamma\) is a \(G_{\omega_1}\) subset of \(F\) for every \(\gamma < \alpha\), \(\alpha < \omega_1\) and \(F_\gamma \subset F_{\gamma'}\) whenever \(\gamma' \leq \gamma\). If \(\alpha\) is limit, take \(F_\alpha = \bigcap_{\gamma < \alpha} F_\gamma\). By compactness \(F_\alpha \neq \emptyset\).
If $\alpha = \delta + 1$ define $F_\alpha = \{(x_\gamma)_{\gamma \leq \delta} \times \prod_{\alpha \leq \beta < \omega_1} X_\beta\} \cap F$. In both cases $F_\alpha$ is a $G_{\omega_1}$ subset of $F$ and, moreover, we can select a point $x_\alpha \in \pi_\alpha(F_\alpha) \subset X_\alpha$ such that $\chi(x_\alpha, \pi_\alpha(F_\alpha)) \leq \omega_1$. Now the point $p = (x_\alpha)_{\alpha < \omega_1}$ satisfies $\{p\} = \cap_{\alpha < \omega_1} F_\alpha$ and therefore it is a $G_{\omega_1}$ point in $F$. This shows that $\chi(p, F) \leq \omega_1$. \( \diamond \)

This theorem can be considered as a generalization of the \u{C}ech–Posp{\v{s}}il\u{v} theorem in the case that the compact space $X$ is given as a product of a family of compact spaces.

**Theorem 3.3.** Let $\mathcal{F} = (X_n)_{n < \omega_0}$ be a family of Hausdorff compact pseudoradial spaces and $|X_n| < 2^{\omega_2}$ for every $n < \omega_0$. Then $\prod \mathcal{F}$ is pseudoradial.

**Proof.** In fact, by the previous Th. 3.2, if $F$ is a closed subset of the product space, then there is a point $p \in F$ such that $\chi(p, F) \leq \omega_1$. Each of the $X_n$ is a CSC space, and also $\prod \mathcal{F}$ is a CSC space. So, by Th. 3.1, it is also a pseudoradial space. \( \diamond \)

Concerning the equality $h = \mu$, it should be remarked that it was completely proven only recently in [1].

The following (consistently) more general result can then be proved

**Theorem 3.3'.** Suppose that $h \leq \omega_2$ holds. Then, if $\mathcal{F}$ is a family of strictly less than $h$ compact Hausdorff pseudoradial spaces each one having cardinality $< 2^{\omega_2}$, the cartesian product $\prod \mathcal{F}$ is pseudoradial.

**Proof.** In fact, the proof follows the same scheme as in the previous theorem. If $F$ is a closed subset of $\prod \mathcal{F}$ there is a point $p \in F$ such that $\chi(p, F) \leq \omega_1$, since $|\mathcal{F}| < h \leq \omega_2$ means $|\mathcal{F}| \leq \omega_1$, and the product is still a CSC space; so, by Th. 3.1, it is pseudoradial. \( \diamond \)

Obviously if $c = \omega_1$ Th. 3.3 reduces to Th. 3.2'.

**Reference**