Abstract. In this paper we present some results for φ-recurrent trans-Sasakian manifolds. We find conditions for such manifolds to be of constant curvature. Finally we give an example of a 3-dimensional φ-recurrent trans-Sasakian manifold.

1. Introduction

A class of almost contact metric manifolds known as trans-Sasakian manifolds was introduced by J. A. Oubina [6] in 1985. This class contains α-Sasakian, β-Kenmotsu and co-symplectic manifolds. An almost contact metric structure on a manifold $M$ is called a trans-Sasakian structure if the product manifold $M \times \mathbb{R}$ belongs to the class $W_4$, a class of Hermitian manifolds which are closely related to a locally conformal Kähler manifolds. Trans-Sasakian manifolds were studied extensively by J. C. Marrero [5], M. M. Tripathi [8], U. C. De [2, 3, 4] and others. M. M. Tripathi [8] proved that trans-Sasakian manifolds are always generalized quasi-Sasakian.

U. C. De et al. [2] generalized the notion of local φ-symmetry and introduced the notion of φ-recurrent Sasakian manifolds. In the present paper we study φ-recurrent trans-Sasakian manifolds. In Section 3, we prove that a conformally flat φ-recurrent trans-Sasakian manifold is a manifold of constant curvature. In the same section trans-Sasakian manifolds with η-parallel Ricci-tensor are considered and we prove that the scalar curvature of such a manifold is a constant. In Section 4, it is proved that a φ-recurrent conformally flat trans-Sasakian manifold is η-Einstein. Finally we construct an example of a 3-dimensional φ-recurrent trans-Sasakian manifold. This verifies the results proved in Section 3.

2. Preliminaries

Let $M$ be a $(2n+1)$-dimensional almost contact metric manifold with an almost contact metric structure $(\phi, \xi, \eta, g)$, where $\phi$, $\xi$, $\eta$ are tensor fields on $M$ of types
(1,1), (1,0), (0,1) respectively and \( g \) is the Riemannian metric on \( M \) such that
\[
\begin{align*}
(a) & \quad \phi^2 = -I + \eta \otimes \xi, \\
(b) & \quad \eta(\xi) = 1, \\
(c) & \quad \phi(\xi) = 0, \\
(d) & \quad \eta \circ \phi = 0
\end{align*}
\]
(2.1)

The Riemannian metric \( g \) on \( M \) satisfies the condition
\[
\begin{align*}
g(\phi X, \phi Y) &= g(X, Y) - \eta(X)\eta(Y) \\
g(X, \phi Y) &= -g(\phi X, Y)
\end{align*}
\]
(2.2)

\( \forall X, Y \in TM \). An almost contact metric structure \((\phi, \xi, \eta, g)\) in \( M \) is called a trans-Sasakian structure \([1]\) if the product manifold \((M \times R, J, G)\) belongs to the class \( W_4 \), where \( J \) is the complex structure on \((M \times R)\) defined by
\[
J(X, \lambda \frac{dt}{dt}) = (\phi - \lambda \xi, \eta(X) \frac{dt}{dt})
\]
(2.4)

for all vector fields \( X \) on \( M \) and smooth functions \( \lambda \) on \((M \times R)\) and \( G \) is the product metric on \((M \times R)\). This may be expressed by the following condition \([1]\)
\[
(\nabla_X \phi)(Y) = \alpha(g(\phi X, Y)\xi - \eta(Y)\phi X) + \beta(g(\phi X, Y)\xi - \eta(Y)\phi X)
\]
(2.5)

where \( \alpha \) and \( \beta \) are smooth functions on \( M \).

From (2.5), we have
\[
(\nabla_X \xi) = -\alpha(\phi X) + \beta(X - \eta(X)\xi)
\]
(2.6)

\[
(\nabla_X \eta)(Y) = -\alpha(\phi X, Y) + \beta(\phi X, \phi Y).
\]
(2.7)

In a \((2n + 1)\)-dimensional trans-Sasakian manifold, from (2.5), (2.6), (2.7), we can derive \([3]\)
\[
R(X, Y)\xi = (\alpha^2 - \beta^2)(\eta(Y)X - \eta(X)Y) + 2\alpha\beta(\eta(Y)\phi X - \eta(X)\phi Y)
\]
\[
- (X\alpha)\phi Y + (Y\alpha)\phi X - (X\beta)\phi^2 Y + (Y\beta)\phi^2 X
\]
\[
S(X, \xi) = (2n(\alpha^2 - \beta^2) - \xi\beta)\eta(X) - (2n - 1)(X\beta) - (\phi X)\alpha.
\]
(2.8)

Further we have
\[
2\alpha\beta + \xi\alpha = 0.
\]
(2.10)

In a conformally flat manifold the curvature tensor \( R \) satisfies
\[
R(X, Y, Z, W) = \frac{1}{2n - 1} [S(Y, Z)g(X, W) + g(Y, Z)S(X, W) - S(X, Z)g(Y, W)
\]
\[
- g(X, Z)S(Y, W)] - \frac{r}{2n(2n - 1)}[g(Y, Z)g(X, W) - g(X, Z)g(Y, W)].
\]
(2.11)

From (2.8) we have
\[
R(\xi, X, Y, \xi) = (\alpha^2 - \beta^2 - \xi\beta)g(\phi X, \phi Y).
\]
(2.12)

Suppose \( \alpha \) and \( \beta \) are constants. Then from (2.9), (2.11), (2.12), we obtain
\[
S(X, Y) = (\frac{r}{2n} - (\alpha^2 - \beta^2))g(\phi X, \phi Y) - 2n(\alpha^2 - \beta^2)\eta(X)\eta(Y).
\]
(2.13)
Applying (2.13) in (2.11), we get

\[
R(X, Y)Z = \frac{1}{2n-1}[(\frac{r}{2n} - 2(\alpha^2 - \beta^2))(g(Y, Z)X - g(X, Z)Y) + (\frac{r}{2n} + (2n + 1))(\alpha^2 - \beta^2)\{(g(Y, Z)\eta(X)\xi - g(X, Z)\eta(Y)\xi) + (\eta(Y)\eta(Z)X - \eta(X)\eta(Z)Y)\}]. \tag{2.14}
\]

From (2.10), for constants \(\alpha\) and \(\beta\), we have

**Proposition 2.1.** A trans-Sasakian manifold with \(\alpha\) and \(\beta\) are constants is either \(\beta\)-Sasakian or \(\alpha\)-Kenmotsu or co-symplectic.

It is well known that \(\beta\)-Sasakian manifolds are quasi Sasakian and \(\alpha\)-Kenmotsu manifold are \(C(-\alpha^2)\) manifolds. Hence we have the following corollary.

**Corollary 2.1.** In a trans-Sasakian manifold \(M\) with \(\alpha\) and \(\beta\) are constants, one of the following holds.

(i) \(M\) is quasi Sasakian  
(ii) \(M\) is a \(C(-\alpha^2)\) manifold  
(iii) \(M\) is co-symplectic.

3. Conformally flat \(\phi\)-recurrent trans-Sasakian manifolds

**Definition 3.1** A trans-Sasakian manifold is said to be \(\phi\)-recurrent if

\[
\phi^2(\nabla_W R)(X, Y)Z = A(W) R(X, Y)Z, \tag{3.1}
\]

\(\forall\ X, Y, Z, W \in TM\).

Differentiating (2.14) covariantly with respect to \(W\), we get

\[
(\nabla_W R)(X, Y)Z = \frac{1}{2n-1}[(\frac{dr(W)}{2n})(g(Y, Z)X - g(X, Z)Y) + (\frac{dr(W)}{2n})(g(Y, Z)\eta(X)\xi - g(X, Z)\eta(Y)\xi) + (\eta(Y)\eta(Z)X - \eta(X)\eta(Z)Y)] + [(\frac{r}{2n} + 3(\alpha^2 - \beta^2))
\]

\[
(g(Y, Z)(\nabla_W \eta)(X)\xi - \eta(X)(\nabla_W \eta)\xi) - g(X, Z)(\nabla_W \eta)(Y)\xi - \eta(Y)(\nabla_W \eta)\xi) + (\nabla_W \eta)(Y)\eta(Z) + \eta(Y)(\nabla_W \eta)Z - (\nabla_W \eta)(X)\eta(Z) - \eta(X)(\nabla_W \eta)(Z)]. \tag{3.2}
\]

We may assume that all vector fields \(X, Y, Z, W\) are orthogonal to \(\xi\). Then (3.2) takes the form

\[
(\nabla_W R)(X, Y)Z = \frac{1}{2n-1}[(\frac{dr(W)}{2n})(g(Y, Z)X - g(X, Z)Y) + (\frac{r}{2n} + 3(\alpha^2 - \beta^2))\{g(Y, Z)(\nabla_W \eta)(X) - g(X, Z)(\nabla_W \eta)(Y)\}]. \tag{3.3}
\]

Applying \(\phi^2\) to both sides of (3.3), we get

\[
A(W) R(X, Y)Z = \frac{1}{2n-1}[(\frac{dr(W)}{2n})(g(Y, Z)X - g(X, Z)Y)]
\]
i.e.
\[ R(X, Y)Z = \frac{1}{2n(2n - 1)} \frac{d^2r(W)}{dW^2} (g(Y, Z)X - g(X, Z)Y). \]
Putting \( W = e_i \) in the above equation, where \( \{e_i\} \) is an orthonormal basis of the tangent space at any point of the manifold and taking summation over \( i \), \( 1 \leq i \leq 2n + 1 \), we obtain
\[ R(X, Y)Z = \lambda(g(Y, Z)X - g(X, Z)Y), \]
where \( \lambda = \left( \frac{d^2r(e_i)}{dW^2} \right) \) is a scalar. Since \( A \) is non zero, \( \lambda \) will be a constant. Therefore \( M \) is of constant curvature \( \lambda \). Thus we can state that

**Theorem 3.1.** A conformally flat \( \phi \)-recurrent trans-Sasakian manifold of dimension greater than 3 is a manifold of constant curvature provided \( \alpha \) and \( \beta \) are constants.

Since three dimensional Riemannian manifolds are conformally flat, we have

**Corollary 3.1.** A three dimensional \( \phi \)-recurrent trans-Sasakian manifold is a manifold of constant curvature.

Now from Proposition 2.1 and the above corollary, we have

**Corollary 3.2.** A three dimensional \( \phi \)-recurrent \( \beta \)-Sasakian manifold (or \( \alpha \)-Kenmotsu manifold or co-symplectic manifold) is a manifold of constant curvature.

By virtue of (2.1)(a) and (3.1), we have
\[ -(\nabla_WR)(X, Y)Z + \eta((\nabla_WR)(X, Y)Z)\xi = A(W)R(X, Y)Z \]
from which we get
\[ -g((\nabla_WR)(X, Y)Z, U) + \eta((\nabla_WR)(X, Y)Z)\eta(U) = A(W)R(X, Y, Z, U). \tag{3.4} \]
Putting \( X = U = e_i \) and summing over \( i = 1, \ldots, 2n + 1 \), we get
\[ -(\nabla_WS)(Y, Z) + \sum \eta((\nabla_WR)(e_i, Y)Z)\eta(e_i) = A(W)S(Y, Z). \tag{3.5} \]
The second term of (3.5) by putting \( Z = \xi \) takes the form \( g((\nabla_WR)(e_i, Y)\xi, \xi)g(e_i, \xi) \).

Consider
\[ g((\nabla_WR)(e_i, Y)\xi, \xi) = g(\nabla_WR(e_i, Y)\xi, \xi) - g(R(\nabla_WR(e_i, Y)\xi) \xi) - g(R(e_i, \nabla_WR)\xi, \xi) \tag{3.6} \]
at \( P \in M \).

Using (2.8), (2.1)(d) and \( g(X, \xi) = \eta(X) \), we obtain
\[ g(R(e_i, \nabla_WR)\xi, \xi) = g((\alpha^2 - \beta^2)(\eta(\nabla_WR)e_i - \eta(e_i)(\nabla_WR)) + 2\alpha\beta\eta(\nabla_WR)\phi e_i - \eta(e_i)\phi(\nabla_WR)) + (\nabla_WR)\phi \phi \phi e_i - (e_i)(\alpha)(\phi(\nabla_WR)) - (e_i)(\beta)(\phi^2(\nabla_WR)) + (\nabla_WR)(\beta)(\phi^3) e_i = 0. \tag{3.7} \]
By virtue of \( g(R(e_i, Y)\xi, \xi) = g(R(\xi, \xi)e_i, Y) = 0 \) and (3.7), (3.6) reduce to
\[
g((\nabla WR)(e_i, Y)\xi, \xi) = g(\nablaWR(e_i, Y)\xi, \xi) - g(R(e_i, Y)\nabla W\xi, \xi). \tag{3.8}
\]
Since \( (\nabla Xg) = 0 \), we have \( g((\nabla WR)(e_i, Y)\xi, \xi) + g(R(e_i, Y)\xi, \nabla W\xi) = 0 \), which implies
\[
g((\nabla WR)(e_i, Y)\xi, \xi) = -g(R(e_i, Y)\xi, \nabla W\xi) - g(R(e_i, Y)\nabla W\xi, \xi). \tag{3.9}
\]
Using (2.6) and by the skew symmetry of \( R \), we get
\[
g((\nabla WR)(e_i, Y)\xi, \xi) =
\]
\[
g(R(e_i, Y)\xi, -\alpha(\phi W) + \beta(W - \eta(W)\xi)) + g(R(e_i, Y) - \alpha(\phi W) + \beta(W - \eta(W)\xi), \xi)
\]
\[
= g(R(-\alpha(\phi W) + \beta(W - \eta(W)\xi)Y, e_i), \xi) + g(R(\xi, -\alpha(\phi W) + \beta(W - \eta(W)\xi))Y, e_i).
\]
Multiplying the above equation by \( \eta(e_i) = g(\xi, e_i) \) and summing over \( i = 1, \ldots, 2n + 1 \), we get
\[
\sum \eta((\nabla WR)(e_i, Y)Z)g(e_i, \xi) =
\]
\[
g(R(\xi, -\alpha(\phi W) + \beta(W - \eta(W)\xi))Y, e_i)g(e_i, \xi) =
\]
\[
= \{ g(R(-\alpha(\phi W) + \beta(W - \eta(W)\xi)Y, \xi))
\]
\[
+ g(R(\xi, -\alpha(\phi W) + \beta(W - \eta(W)\xi))Y, \xi) \} = 0.
\]
Replacing \( Z \) by \( \xi \) in (3.5) and using (2.9) we get
\[
-(\nabla WS)(X, \xi) = A(W)\{ 2n(\alpha^2 - \beta^2)\eta(X) \} \tag{3.10}
\]
provided \( \alpha \) and \( \beta \) are constants. Now from
\[
(\nabla XS)(Y, \xi) = \nabla X S(Y, \xi) - S(\nabla X Y, \xi) - S(Y, \nabla X \xi).
\]
Using (2.6) and (2.9), for constant \( \alpha \) and \( \beta \), we have
\[
(\nabla XS)(Y, \xi) = 2n(\alpha^2 - \beta^2)[(\nabla X \eta)(Y) + \beta \eta(X)\eta(Y)] + S(Y, \alpha \phi X - \beta X). \tag{3.11}
\]
From (3.11), (3.3) and (2.7), we obtain
\[
(\nabla XS)(Y, \xi) = 2n(\alpha^2 - \beta^2)[\beta g(X, Y) - \alpha g(X, \phi Y)] + S(Y, \alpha \phi X - \beta X). \tag{3.12}
\]
From (3.10) and (3.12), we have
\[
-A(X)\{ 2n(\alpha^2 - \beta^2)\eta(Y) \} = 2n(\alpha^2 - \beta^2)[\beta g(X, Y) - \alpha g(X, \phi Y)] + S(Y, \alpha \phi X - \beta X).
\tag{3.13}
\]
Replacing \( Y \) by \( \phi Y \) in (3.13) and using (2.2), we obtain
\[
2n(\alpha^2 - \beta^2)[\beta g(X, \phi Y) + \alpha g(X, Y) - \alpha \eta(X)\eta(Y)] + \alpha S(\phi Y, \phi X) - \beta S(\phi Y, X) = 0
\]
i.e.
\[
-\alpha S(\phi Y, \phi X) + \beta S(\phi Y, X) = 2n(\alpha^2 - \beta^2)[\beta g(X, \phi Y) + \alpha g(X, \phi Y)]. \tag{3.14}
\]
Interchanging \( Y \) and \( X \) in (3.14) and by using the skew symmetry of \( \phi \), we obtain
\[
-\alpha S(\phi X, \phi Y) = 2n\alpha (\alpha^2 - \beta^2)(g(\phi X, \phi Y)).
\] (3.15)

By skew symmetry of \( \phi \) and using (2.9), we obtain \( S(\phi X, \phi Y) = -S(\phi^2 X, Y) = S(X, Y) - 2n(\alpha^2 - \beta^2)\eta(X)\eta(Y) \). Substituting this in (3.15), we get
\[
S(X, Y) = ag(X, Y) + \eta(X)\eta(Y),
\] (3.16)
where \( a = 2n(\alpha^2 - \beta^2) \), i.e. \( M \) is \( \eta \)-Einstein. Thus we have

**Theorem 3.2** A \( \phi \)-recurrent conformally flat trans-Sasakian manifold is \( \eta \)-Einstein provided \( \alpha \) and \( \beta \) are constants.

**Corollary 3.3** A 3-dimensional \( \phi \)-recurrent trans-Sasakian manifold is \( \eta \)-Einstein provided \( \alpha \) and \( \beta \) are constants.

4. Trans-Sasakian manifolds with \( \eta \)-parallel Ricci tensor

Let us consider a trans-Sasakian manifold \( M \) of dimension \( 2n+1 \) with \( \eta \)-parallel Ricci tensor. Replacing \( Y \) by \( \phi Y \) and \( Z \) by \( \phi Z \) in (2.13), we obtain
\[
S(\phi X, \phi Y) = \left( \frac{r}{2n} - (\alpha^2 - \beta^2) \right)(g(X, Y) - \eta(X)\eta(Y)).
\] (4.1)

Differentiating (4.1) covariantly with respect to \( X \) we obtain
\[
(\nabla_X S)(\phi Y, \phi Z) = \left( \frac{dr(X)}{2n} \right)(g(Y, Z)X - \eta(Y)\eta(Z))
- \left( \frac{r}{2n} - (\alpha^2 - \beta^2) \right)\left\{ (\nabla_X \eta)(Y)\eta(Z) + \eta(Y)(\nabla_X \eta)(Z) \right\}. \] (4.2)

Suppose the Ricci tensor is \( \eta \)-parallel. Then we obtain
\[
\left( \frac{dr(X)}{2n} \right)(g(Y, Z) - \eta(Y)\eta(Z)) = \left[ (\frac{r}{2n} - (\alpha^2 - \beta^2)) \{ (\nabla_X \eta)(Y)\eta(Z) + \eta(Y)(\nabla_X \eta)(Z) \} \right].
\] (4.3)

Putting \( Y = Z = e_i \) in (4.3), where \( \{e_i\} \) is an orthonormal basis and summing over \( i = 1, \ldots, 2n+1 \), we obtain
\[
\frac{dr(X)}{2n} = \left( \frac{r}{2n} - 2(\alpha^2 - \beta^2) \right)(\nabla_X \eta)(\xi).
\] (4.4)

Since \( \eta(\xi) = 1 \), from (2.5), we have \( (\nabla_X \eta)(\xi) = 0 \). Thus from (4.4), we obtain \( dr(X) = 0 \) or \( r \) is a constant. Thus we have

**Theorem 4.1.** In a conformally flat trans-Sasakian manifold with \( \eta \)-parallel Ricci tensor, the scalar curvature is constant provided \( \alpha \) and \( \beta \) are constants.

**Corollary 4.1.** A 3-dimensional trans-Sasakian manifold with \( \eta \)-parallel Ricci tensor, the scalar curvature is constant provided \( \alpha \) and \( \beta \) are constants.
5. Example of $\phi$-recurrent trans-Sasakian manifolds

Consider three dimensional manifold $M = \{(x, y, z) \in \mathbb{R}^3 \setminus z \neq 0\}$, where $(x, y, z)$ are the standard coordinates of $\mathbb{R}^3$. The vector fields
\[ e_1 = \frac{x}{z} \frac{\partial}{\partial x}, \quad e_2 = \frac{y}{z} \frac{\partial}{\partial y}, \quad e_3 = \frac{\partial}{\partial z} \] (5.1)
are linearly independent at each point of $M$. Let $g$ be the Riemannian metric defined by
\[ g(e_1, e_1) = 1, \quad g(e_2, e_2) = 1, \quad g(e_3, e_3) = 1, \quad g(e_1, e_2) = 0, \quad g(e_1, e_3) = 0, \quad g(e_2, e_3) = 0. \] (5.2)
Let $\eta$ be the 1-form defined by $\eta(X) = g(X, \xi)$ for any vector field $X$. Let $\phi$ be the $(1, 1)$ tensor field defined by
\[ \phi(e_1) = e_2, \quad \phi(e_2) = -e_1, \quad \phi(e_3) = 0. \] (5.3)
Then by using the linearity of $\phi$ and $g$ we have $\phi^2 X = -X + \eta(X) \xi$, with $\xi = e_3$.

Further $g(\phi X, \phi Y) = g(X, Y) - \eta(X) \eta(Y)$ for any vector fields $X$ and $Y$. Hence for $e_3 = \xi$, the structure defines an almost contact structure on $M$. Let $\nabla$ be the Levi-Civita connection with respect to the metric $g$, then we have
\[ [e_1, e_2] = 0, \quad [e_1, e_3] = \frac{1}{z} e_1, \quad [e_2, e_3] = \frac{1}{z} e_2. \] (5.4)
The Riemannian connection $\nabla$ of the metric $g$ is given by
\[ 2g(\nabla_X Y, Z) = X g(Y, Z) + Y g(Z, X) - Z g(X, Y) \]
\[ - g(X, [Y, Z]) - g(Y, [X, Z]) + g(Z, [X, Y]). \] (5.5)
Using (5.5), we have
\[ 2g(\nabla_{e_1} e_3, e_1) = 2g\left(\frac{1}{z} e_1, e_1\right) + 2g(e_2, e_1) = 2g\left(\frac{1}{z} e_1 + e_2, e_1\right), \]
since $g(e_1, e_2) = 0$. Thus
\[ \nabla_{e_1} e_3 = \frac{1}{z} e_1 + e_2. \] (5.6)
Again by (5.5) we get,
\[ 2g(\nabla_{e_2} e_3, e_2) = 2g\left(\frac{1}{z} e_2, e_2\right) - 2g(e_1, e_2) = 2g\left(\frac{1}{z} e_2 - e_1, e_2\right), \]
since $g(e_1, e_2) = 0$. Therefore we have
\[ \nabla_{e_2} e_3 = \frac{1}{z} e_2 - e_1. \] (5.7)
Again from (5.5) we have
\[ \nabla_{e_3} e_3 = 0, \quad \nabla_{e_1} e_1 = -\frac{1}{z} e_1, \quad \nabla_{e_1} e_2 = 0, \]
\[ \nabla_{e_2} e_1 = 0, \quad \nabla_{e_2} e_2 = -\frac{1}{z} e_2, \quad \nabla_{e_3} e_1 = 0, \quad \nabla_{e_3} e_2 = 0. \] (5.8)
The manifold $M$ satisfies (2.5) with $\alpha = -1$ and $\beta = \frac{1}{2}$. Hence $M$ is a trans-Sasakian manifold. Using the relations (5.6), (5.7) and (5.8), the non-vanishing components of the curvature tensor are computed as follows:

$$R(e_1, e_3)e_3 = \frac{1}{z^2} e_1, \quad R(e_3, e_1)e_3 = -\frac{1}{z^2} e_1,$$

$$R(e_2, e_3)e_3 = \frac{1}{z^2} e_2, \quad R(e_3, e_2)e_3 = -\frac{1}{z^2} e_2.$$

(5.9)

The vectors $\{e_1, e_2, e_3\}$ form a basis of $M$ and so any vector $X$ can be written as $X = a_1 e_1 + a_2 e_2 + a_3 e_3$ where $a_i \in \mathbb{R}^+$, $i = 1, 2, 3$. From (5.9), we have

$$(\nabla_X R)(e_1, e_3)e_3 = -\frac{2a_3}{z^3} e_1$$

and

$$(\nabla_X R)(e_2, e_3)e_3 = -\frac{2a_3}{z^3} e_2.$$

Applying $\phi^2$ to both sides of the above equations and using (5.3), we obtain

$$\phi^2((\nabla_X R)(e_1, e_3)e_3) = A(X) R(e_1, e_3)e_3$$

and

$$\phi^2((\nabla_X R)(e_2, e_3)e_3) = A(X) R(e_2, e_3)e_3,$$

where $A(X) = \frac{2a_3}{z^3}$ is a non-vanishing 1-form. This implies that there exists a $\phi$-recurrent trans-Sasakian manifold of dimension 3.

From the non-vanishing curvature components as given in (5.9), we have

$$R(e_1, e_3)e_3 = \lambda (g(e_3, e_3)e_1 - g(e_1, e_3)e_3)$$

and

$$R(e_2, e_3)e_3 = \lambda (g(e_3, e_3)e_2 - g(e_2, e_3)e_3).$$

This verifies Corollary 3.2.

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