Abstract. Let $M$ be a compact manifold and $D$ a Dirac type
differential operator on $M$. Let $A$ be a $C^*$-algebra. Given a bun-
dle $W$ (with connection) of $A$-modules over $M$, the operator $D$
can be twisted with this bundle. One can then use a trace on
$A$ to define numerical indices of this twisted operator. We prove
an explicit formula for these indices. Our result does complement
the Mishchenko–Fomenko index theorem valid in the same situa-
tion. We establish generalizations of these explicit index formulas
if the trace is only defined on a dense and holomorphically closed
subalgebra $B$.

As a corollary, we prove a generalized Atiyah $L^2$-index theorem
if the twisting bundle is flat.

There are actually many different ways to define these numerical
indices. From their construction, it is not clear at all that they
coincide. A substantial part of the paper is a complete proof of
their equality. In particular, we establish the (well-known but not
well-documented) equality of Atiyah’s definition of the $L^2$-index
with a K-theoretic definition.

In case $A$ is a von Neumann algebra of type 2, we put special
emphasis on the calculation and interpretation of the center valued
index. This completely contains all the K-theoretic information
about the index of the twisted operator.

Some of our calculations are done in the framework of bivariant
KK-theory.