Abstract. Let $E/k$ be an elliptic curve over a number field. We obtain some quantitative refinements of results of Hindry–Silverman, giving an upper bound for the number of $k$-rational torsion points, and a lower bound for the canonical height of non-torsion $k$-rational points, in terms of expressions depending explicitly on the degree $d = [k : \mathbb{Q}]$ of $k$ and the Szpiro ratio $\sigma$ of $E/k$. The bounds exhibit only polynomial dependence on both $d$ and $\sigma$. 