ALL GENERAL SOLUTIONS OF FINITE EQUATIONS

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Abstract. Recently Prešić determined in [4] all reproductive solutions of the finite equation, where a finite equation is an equation the solution set of which is a subset of a given finite set. In this paper we determine all general solutions of such equations. Especially we also get all reproductive solutions.

Let $E$ be a given non-empty set and $q(x)$ be any $x$-equation ($x$ is an unknown element of $E$ and $q$ is a given unary relation of $E$) supposing that $q(x)$ has at least one solution.

Definition 1. Let $f : E \to E$ be a given function. The formula $x = f(t)$ represents a general solution of $x$-equation $q(x)$ if and only if $(\forall t \in E)q(f(t)) \land (\forall x \in E)(q(x) \implies (\exists t \in E)x = f(t))$.

Definition 2. Let $g : E \to E$ be a given function. The formula $x = g(t)$ represents a reproductive solution of $x$-equation $q(x)$ if and only if $(\forall t \in E)q(g(t)) \land (\forall t \in E)(q(t) \implies x = g(t))$.

Let $B = \{b_0, b_1, \ldots, b_m\}$ be a given set of $m + 1$ elements and $S = \{0, 1\}$. Define the operation $x^y$ by

$$x^y = \begin{cases} 1, & \text{if } x = y \\ 0, & \text{otherwise} \end{cases} \quad (x, y \in B \cup S).$$

The standard Boolean operations $+$ and $\cdot$ ("or" and "and") are described by the following tables:

$$\begin{array}{ccc}
+ & 0 & 1 \\
0 & 0 & 1 \\
1 & 1 & 1 \\
\end{array} \quad \begin{array}{ccc}
\cdot & 0 & 1 \\
0 & 0 & 0 \\
1 & 1 & 0 \\
\end{array}$$

Extend these operations to the partial operations of the set $B \cup S$ by

$$x + 0 = x, \quad 0 + x = x, \quad x \cdot 0 = 0, \quad x \cdot 1 = x, \quad 0 \cdot x = 0, \quad 1 \cdot x = x, \quad (x \in B \cup S).$$

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We consider the equation
\[ s_0x^{b_0} + s_1x^{b_1} + \cdots + s_mx^{b_m} = 0 \]  
where \( s_i \in \{0, 1\} \) are given elements and \( x \in B \) is unknown.

Obviously the equation (1) is consistent if and only if
\[ s_0s_1 \cdots s_m = 0. \]  

**Definition 3.** Let \((a_0, a_1, \ldots, a_m) \in S^{m+1}\). Then the set \(Z(a_0, \ldots, a_m)\) ("the zero-set of \((a_0, \ldots, a_m)\)") is defined by
\[ b_i \in Z(a_0, \ldots, a_m) \iff a_i = 0 \quad (i = 0, 1, \ldots, m). \]

For instance, if \( m = 3 \) we have
\[ Z(1, 1, 1) = \varnothing, \quad Z(1, 0, 1, 0) = \{b_1, b_3\}, \quad Z(0, 0, 0, 0) = \{b_0, b_1, b_2, b_3\}. \]

Let \( M = \{0, 1, 2, \ldots, m\} \).

**Definition 4.** Let \( s_0 \cdots s_m = 0 \). A function \( A : B \to B \) of the form
\[ A(x) = A_0(s_0, \ldots, s_m)x^{b_0} + \cdots + A_m(s_0, \ldots, s_m)x^{b_m} \]
is a repro-function if and only if each coefficient \( A_k(s_0, \ldots, s_m) \) is determined by some equality of the form
\[ A_k(s_0, \ldots, s_m) = b_k s_0^0 + \sum_{a_k \neq 0, a_0 \cdot \cdots \cdot a_m = 0} F_k(a_0, \ldots, a_m)s_0^{a_0} \cdots s_m^{a_m}, \]

where
\[ (\forall k \in M)(\forall a_0, \ldots, a_m \in S)(a_k \neq 0 \land a_0 \cdot \cdots \cdot a_m = 0 \iff F_k(a_0, \ldots, a_m) \in Z(a_0, \ldots, a_m)). \]

**Definition 5.** Let \( s_0 \cdots s_m = 0 \). A function \( A : B \to B \) of the form
\[ A(x) = A_0(s_0, \ldots, s_m)x^{b_0} + \cdots + A_m(s_0, \ldots, s_m)x^{b_m} \]
is a gener-function if and only if there is a function \( \psi : M \to \Delta M \) such that each coefficient \( A_k(s_0, \ldots, s_m) \) is determined by some equality of the form
\[ A_k(s_0, \ldots, s_m) = b_\psi(k) s_0^0 + \sum_{a_\psi(k) \neq 0, a_0 \cdot \cdots \cdot a_m = 0} F_\psi(k)(a_0, \ldots, a_m)s_0^{a_0} \cdots s_m^{a_m}, \]

where
\[ (\forall k \in M)(\forall a_0, \ldots, a_m \in S)(a_\psi(k) \neq 0 \land a_0 \cdot \cdots \cdot a_m = 0 \iff F_\psi(a_0, \ldots, a_m) \in Z(a_0, \ldots, a_m)). \]
Let (1) be denoted by \( g(x) = 0 \).

**Lemma 1.** Let \( A(x) = A_0(s_0, \ldots, s_m)x^{b_0} + \ldots + A_m(s_0, \ldots, s_m)x^{b_m} \) assuming that \( A_0(s_0, \ldots, s_m), \ldots, A_m(s_0, \ldots, s_m) \in B \). Then the formula

\[
(\forall x \in B) (g(x) = 0 \implies (\exists t \in B) x = A(t))
\]

holds if and only if there is a function \( \psi : M \xrightarrow{\sim} M \) such that each coefficient \( A_k(s_0, \ldots, s_m) \) is determined by the equality

\[
A_k(s_0, \ldots, s_m) = b_{\psi(k)} s_{\psi(k)}^0 + \sum_{a_{\psi(k)} \neq 0, a_0 \cdots a_m = 0} F_{\psi(k)}(a_0, \ldots, a_m) s_0^{a_0} \cdots s_m^{a_m},
\]

where

\[
(\forall k \in M)(\forall a_0, \ldots, a_m \in S)(a_{\psi(k)} \neq 0 \land a_0 \cdots a_m = 0 \implies F_{\psi(k)}(a_0, \ldots, a_m) \in B).
\]

The proof follows from the following equivalences.

\[
(\forall x \in B) (g(x) = 0 \implies (\exists t \in B) x = A(t))
\]

\[
\iff (\forall x \in B) (x \in Z(s_0, \ldots, s_m) \implies (\exists t \in B) x = A(t))
\]

\[
\iff (\forall x \in Z(s_0, \ldots, s_m)(\exists t \in B) x = A(t)
\]

\[
\iff (\exists f : Z(s_0, \ldots, s_m) \xrightarrow{\sim} B)(\forall x \in Z(s_0, \ldots, s_m)) x = A(f(x))
\]

(by the axiom of choice)

\[
\iff (\exists f : Z(s_0, \ldots, s_m) \xrightarrow{\sim} B)(\forall x \in Z(s_0, \ldots, s_m)) x = A(f(x)).
\]

(Assuming \( b_p \in Z(s_0, \ldots, s_m) \), \( b_r \in Z(s_0, \ldots, s_m) \), \( b_p \neq b_r \) and \( \bar{f}(b_p) = \bar{f}(b_r) = b_u \), we get from \( x = A_0(f(x))^{b_0} + \ldots + A_m(f(x))^{b_m} \) the following implications:

\[
x = b_p \implies b_p = A_u, \quad x = b_r \implies b_r = A_u \quad \text{i.e.} \quad b_p = b_r.
\]

Thus \( f \) is \( \xrightarrow{\sim} \).

\[
\iff (\exists f : B \xrightarrow{\sim} B)(\forall x \in Z(s_0, \ldots, s_m)) x = A(f(x))
\]

\((f \text{ is an extension of } \bar{f})\)

\[
\iff (\exists f : B \xrightarrow{\sim} B)(\forall x \in B)(x \in Z(s_0, \ldots, s_m) \implies x = A(f(x)))
\]

\[
\iff (\exists f : B \xrightarrow{\sim} B)(\forall x \in B)(s_0 x^{b_0} + \ldots + s_m x^{b_m})
\]

\[
\implies x = A_0(s_0, \ldots, s_m)(f(x))^{b_0} + \ldots + A_m(s_0, \ldots, s_m)(f(x))^{b_m} \land (\forall k \in M)A_k(s_0, \ldots, s_m) \in B)
\]
\[\iff (\exists f : B \xrightarrow{\sim} B)(\forall k \in M)(s_0 b_k^{b_0} + \cdots + s_m b_k^{b_m} = 0)
\implies b_k = A_0(s_0, \ldots, s_m)(f(b_k))^{b_0} + \cdots + A_m(s_0, \ldots, s_m)(f(b_k))^{b_m} \land A_k(s_0, \ldots, s_m) \in B\]
\[\iff (\exists \varphi : M \xrightarrow{\sim} M)(\forall k \in M)(s_k = 0)
\implies b_k = A(\varphi(k))(s_0, \ldots, s_m) \land A_k(s_0, \ldots, s_m) \in B\]

(\varphi is defined by (\forall c,d \in M)(\varphi(c) = d \iff f(b_c) = b_d))

\[\iff (\exists \psi : M \xrightarrow{\sim} M)(\forall k \in M)\left(A_{\psi(k)}(s_0, \ldots, s_m)
= b_{\psi(k)} s_0^{a_0 + \sum_{a_0 \neq 0 \ldots a_m = 0} F_k(a_0, \ldots, a_m)s_0^{a_0} \cdots s_m^{a_m}}
\land (\forall a_0, \ldots, a_m \in S)(a_0 \neq 0 \land a_0 \cdots a_m = 0 \implies F_k(a_0, \ldots, a_m) \in B)\right)\]

\iff (\exists \psi : M \xrightarrow{\sim} M)(\forall k \in M)\left(A_k(s_0, \ldots, s_m)
= b_{\psi(k)} s_0^{a_0 + \sum_{C} F_{\psi(k)}(a_0, \ldots, a_m)s_0^{a_0} \cdots s_m^{a_m}}
\land (\forall a_0, \ldots, a_m \in S)(C \implies F_{\psi(k)}(a_0, \ldots, a_m) \in B)\right)\]

(\psi is \varphi^{-1}, C is the conjunction \(a_{\psi(k)} \neq 0 \land a_0 \cdots a_m = 0\) and \(\sum_C\) means the sum over all \((a_0, \ldots, a_m) \in S^{m+1}\) such that \(C\) is satisfied).

**Theorem 1.** Let \(A(x) = A_0(s_0, \ldots, s_m)x^{b_0} + \cdots + A_m(s_0, \ldots, s_m)x^{b_m}\) and \(A_0(s_0, \ldots, s_m), \ldots, A_m(s_0, \ldots, s_m) \in B\). If
\[s_0 x^{b_0} + \cdots + s_m x^{b_m} = 0\] is a consistent equation then the formula \(x = A(t)\) (\(t\) is any element of \(B\)) represents a general solution of (1) and only if the function \(A\) is a gener-function.

**Proof.** It we denote (1) by \(g(x) = 0\) we have
\[(\forall x \in B)g(A(x)) = 0 \land (\forall x \in B)(g(x) = 0 \implies (\exists t \in B)x = A(t))\]
\[\iff (\forall x \in B)s_0(A_0(s_0, \ldots, s_m)x^{b_0} + \cdots + A_m(s_0, \ldots, s_m)x^{b_m})^{b_0} + \cdots + s_m(A_0(s_0, \ldots, s_m)x^{b_0} + \cdots + A_m(s_0, \ldots, s_m)x^{b_m})^{b_m} = 0\]
\[\land (\exists \psi : M \xrightarrow{\sim} M)(\forall k \in M)\left(A_k(s_0, \ldots, s_m)
= b_{\psi(k)} s_0^{a_0 + \sum_{C} F_{\psi(k)}(a_0, \ldots, a_m)s_0^{a_0} \cdots s_m^{a_m}}\right)\]
\[ \forall a_0, \ldots, a_m \in S \left( C \implies F_{\psi(k)}(a_0, \ldots, a_m) \in B \right) \]

(by Lemma 1)

\[ \iff \left( \forall i, k \in M \right)s_i A_k^b(s_0, \ldots, s_m) = 0 \]

\[ \land \left( \exists \psi : M \xrightarrow{1 \leftarrow} M \right) \left( \forall k \in M \right) A_k(s_0, \ldots, s_m) \]

\[ = b_{\psi(k)} s_{\psi(k)} \sum_C F_{\psi(k)}(a_0, \ldots, a_m) s_{0}^{a_0} \cdots s_{m}^{a_m} \]

\[ \land \left( \forall a_0, \ldots, a_m \in S \right) \left( C \implies F_{\psi(k)}(a_0, \ldots, a_m) \in B \right) \]

(This part of the proof is based on the following general facts:

If \( a_0, \ldots, a_n, b \), are any elements of \( B \) then

1. \( a_0 x^{b_0} + \cdots + a_n x^{b_n} = a_0 x^{b_0} + \cdots + a_n x^{b_n} \) (for all \( x \in B \))

2. \( \forall x \in B \), \( a_0 x^{b_0} + \cdots + a_n x^{b_n} = 0 \implies \left( \forall \psi \in M \right) a_i = 0. \)

\[ \iff \left( \exists \psi : M \xrightarrow{1 \leftarrow} M \right) \left( \forall i, k \in M \right) s_i A_k^b(s_0, \ldots, s_m) = 0 \]

\[ \land \left( \forall a_0, \ldots, a_m \in S \right) \left( C \implies F_{\psi(k)}(a_0, \ldots, a_m) \right) \]

\[ \land A_k(s_0, \ldots, s_m) = b_{\psi(k)} s_{\psi(k)} \sum_C F_{\psi(k)}(a_0, \ldots, a_m) s_{0}^{a_0} \cdots s_{m}^{a_m} \]

\[ \iff \left( \exists \psi : M \xrightarrow{1 \leftarrow} M \right) \left( \forall i, k \in M \right) \left( s_i \left( b_{\psi(k)} s_{\psi(k)} \right) \right) \]

\[ + \sum_C F_{\psi(k)}(a_0, \ldots, a_m) s_{0}^{a_0} \cdots s_{m}^{a_m} \]

\[ \land \left( \forall a_0, \ldots, a_m \in S \right) \left( C \implies F_{\psi(k)}(a_0, \ldots, a_m) \right) \]

\[ \land A_k(s_0, \ldots, s_m) = b_{\psi(k)} s_{\psi(k)} \sum_C F_{\psi(k)}(s_0, \ldots, s_m) s_{0}^{a_0} \cdots s_{m}^{a_m} \]

\[ \iff \left( \exists \psi : M \xrightarrow{1 \leftarrow} M \right) \left( \forall i, k \in M \right) \left( s_i \left( \sum_C F_{\psi(k)}(a_0, \ldots, a_m) s_{0}^{a_0} \cdots s_{m}^{a_m} \right) \right) = 0 \]

\[ \land \left( \forall a_0, \ldots, a_m \in S \right) \left( C \implies F_{\psi(k)}(a_0, \ldots, a_m) \right) \]

\[ \land A_k(s_0, \ldots, s_m) = b_{\psi(k)} s_{\psi(k)} \sum_C F_{\psi(k)}(a_0, \ldots, a_m) s_{0}^{a_0} \cdots s_{m}^{a_m} \]

(we have used the identity \( s_{p^i} a_{b_p} = 0 \))

\[ \iff \left( \exists \psi : M \xrightarrow{1 \leftarrow} M \right) \left( \forall i, k \in M \right) \left( \sum_{a_0, \ldots, a_m \in S} a_i s_{0}^{a_0} \cdots s_{m}^{a_m} \right) \]
\[
\cdot \sum_{C} F_{\psi(k)}(a_0, \ldots, a_m)s_{a_0}^0 \cdots s_{a_m}^m = 0
\]
\[
\land \langle \forall a_0, \ldots, a_m \in S \rangle (C \implies F_{\psi(k)}(a_0, \ldots, a_m) \in B)
\]
\[
\land A_k(s_0, \ldots, s_m) = b_{\psi(k)} s_{a_0}^0 + \sum_{C} F_{\psi(k)}(a_0, \ldots, a_m)s_{a_0}^a \cdots s_{a_m}^m
\]
(because of the identity \(s_i = \sum_{(a_0, \ldots, a_m) \in S^{m+1}} a_i s_{a_0}^a \cdots s_{a_m}^m\))

\[\iff (\exists \psi : M \xrightarrow{1 \rightarrow} M)(\forall i, k \in M)\left(\sum_{C} a_i F_{\psi(k)}(a_0, \ldots, a_m)s_{a_0}^0 \cdots s_{a_m}^m = 0\right)\]
\[
\land \langle \forall a_0, \ldots, a_m \in S \rangle (C \implies F_{\psi(k)}(a_0, \ldots, a_m) \in B)
\]
\[
\land A_k(s_0, \ldots, s_m) = b_{\psi(k)} s_{a_0}^0 + \sum_{C} F_{\psi(k)}(a_0, \ldots, a_m)s_{a_0}^a \cdots s_{a_m}^m
\]

\[\iff (\exists \psi : M \xrightarrow{1 \rightarrow} M)(\forall i, k \in M)\left((\forall a_0, \ldots, a_m \in S) (C \implies a_i b_{n(k)}^i = 0)\right)\]
\[
\land \langle \forall a_0, \ldots, a_m \in S \rangle (C \implies F_{\psi(k)}(a_0, \ldots, a_m) \in B)
\]
\[
\land A_k(s_0, \ldots, s_m) = b_{\psi(k)} s_{a_0}^0 + \sum_{C} F_{\psi(k)}(a_0, \ldots, a_m)s_{a_0}^a \cdots s_{a_m}^m
\]

(we have used the following facts:

(I) \((\forall k \in M) F_{\psi(k)} \in B \iff (\forall k \in M)(\exists j \in M) F_{\psi(k)} = b_j;\)

(II) \((\forall k \in M)(\exists j \in M) F_{\psi(k)} = b_j \iff (\exists h : M \to M)(\forall k \in M) F_{\psi(k)} = b_{h(k)},\)

by the axiom of choice)

\[\iff (\exists \psi : M \xrightarrow{1 \rightarrow} M)(\forall k \in M)\left( (\forall i \in M)(\forall a_0, \ldots, a_m \in S) (C \implies a_i b_{h(k)}^i = 0) \right)\]
\[
\land \langle \forall a_0, \ldots, a_m \in S \rangle (C \implies F_{\psi(k)}(a_0, \ldots, a_m) \in B)
\]
\[
\land A_k(s_0, \ldots, s_m) = b_{\psi(k)} s_{a_0}^0 + \sum_{C} F_{\psi(k)}(a_0, \ldots, a_m)s_{a_0}^a \cdots s_{a_m}^m
\]

\[\iff (\exists \psi : M \xrightarrow{1 \rightarrow} M)(\forall k \in M)\left( (\forall a_0, \ldots, a_m \in S) (C \implies a_{h(k)} = 0) \right)\]
\[
\land \langle \forall a_0, \ldots, a_m \in S \rangle (C \implies F_{\psi(k)}(a_0, \ldots, a_m) \in B)\]
All general solutions of finite equations

\[ \land A_k(s_0, \ldots, s_m) = b_{\psi(k)} s^0_{\psi(k)} + \sum_C F_{\psi(k)} (a_0, \ldots, a_m) s_0^{a_0} \cdots s_m^{a_m} \]

\[ \iff \exists \psi : M \xrightarrow{1 \rightarrow 1} M (\forall k \in M) \left( (\forall a_0, \ldots, a_m \in S) \right. \]
\[ (C \implies b_h(k) \in Z(a_0, \ldots, a_m)) \land (\forall a_0, \ldots, a_m \in S) (C \implies F_{\psi(k)} (a_0, \ldots, a_m) \in B) \]
\[ \land A_k(s_0, \ldots, s_m) = b_{\psi(k)} s^0_{\psi(k)} + \sum_C F_{\psi(k)} (a_0, \ldots, a_m) s_0^{a_0} \cdots s_m^{a_m} \]

(Definition 3)

\[ \iff \exists \psi : M \xrightarrow{1 \rightarrow 1} M (\forall k \in M) \left( (\forall a_0, \ldots, a_m \in S) \right. \]
\[ (C \implies F_k(a_0, \ldots, a_m) \in Z(a_0, \ldots, a_m)) \land (\forall a_0, \ldots, a_m \in S) (C \implies F_{\psi(k)} (a_0, \ldots, a_m) \in B) \]
\[ \land A_k(s_0, \ldots, s_m) = b_{\psi(k)} s^0_{\psi(k)} + \sum_C F_{\psi(k)} (a_0, \ldots, a_m) s_0^{a_0} \cdots s_m^{a_m} \]

(because \((p_1 \iff p_2) \land (p_1 \iff p_3) \iff (p_1 \iff p_2 \land p_3)\) is a tautology)

\[ \iff \exists \psi : M \xrightarrow{1 \rightarrow 1} M (\forall k \in M) \left( (\forall a_0, \ldots, a_m \in S) \right. \]
\[ (C \implies F_k(a_0, \ldots, a_m) \in Z(a_0, \ldots, a_m)) \land A_k(s_0, \ldots, s_m) = b_{\psi(k)} s^0_{\psi(k)} + \sum_C F_{\psi(k)} (a_0, \ldots, a_m) s_0^{a_0} \cdots s_m^{a_m} \]

\[ \iff A \text{ is a gener-function.} \]

The following Theorem 2 can be obtained from Theorem 1 if we assume that \(\psi : B \rightarrow B\) is the identical mapping.

**Theorem 2.** [4] If
\[ s_0 x^{b_0} + \cdots + s_m x^{b_m} = 0 \quad (1) \]
is a consistent equation, then the formula
\[ x = A(p) \quad (p \text{ is any element of } B) \]
represents a reproductive solution of the equation (1) if and only if the function \( A \) is a repro-function.

**Example 1.** Let \( s_0 x^{s_0} + s_1 x^{s_1} + s_2 x^{s_2} = 0 \) be a consistent equation i.e. \( s_0 s_1 s_2 = 0 \). If \( \psi = \begin{pmatrix} 0 & 1 & 2 \\ 2 & 0 & 1 \end{pmatrix} \) then the formula \( x = A_0 P^{s_0} + A_1 P^{s_1} + A_2 P^{s_2} \) represents a general solution of \( s_0 x^{s_0} + s_1 x^{s_1} + s_2 x^{s_2} = 0 \) provided

\[
A_0 = b_0 s_0^{s_0} + F_0 (0, 0, 1) s_0^{s_0} s_1^{s_0} s_2^{s_0} + F_2 (0, 1, 1) s_0^{s_0} s_1^{s_0} s_2^{s_0} + F_2 (1, 0, 1) s_0^{s_0} s_1^{s_0} s_2^{s_0}
\]
\[
A_1 = b_0 s_0^{s_1} + F_0 (1, 0, 0) s_0^{s_1} s_1^{s_0} s_2^{s_0} + F_0 (1, 0, 1) s_0^{s_1} s_1^{s_0} s_2^{s_0} + F_0 (1, 1, 0) s_0^{s_1} s_1^{s_0} s_2^{s_0}
\]
\[
A_2 = b_1 s_1^{s_1} + F_1 (0, 1, 0) s_0^{s_1} s_1^{s_1} s_2^{s_0} + F_1 (0, 1, 1) s_0^{s_1} s_1^{s_1} s_2^{s_0} + F_1 (1, 1, 0) s_0^{s_1} s_1^{s_1} s_2^{s_0}
\]

i.e.

\[
A_0 = b_2 s_2^{s_2} + (b_0 + b_1) s_0^{s_1} s_1^{s_0} s_2^{s_2} + b_0 s_0^{s_1} s_1^{s_0} s_2^{s_0} + b_1 s_0^{s_1} s_1^{s_0} s_2^{s_0}
\]
\[
A_1 = b_2 s_2^{s_2} + (b_0 + b_1) s_0^{s_1} s_1^{s_0} s_2^{s_2} + b_0 s_0^{s_1} s_1^{s_0} s_2^{s_0} + b_1 s_0^{s_1} s_1^{s_0} s_2^{s_0}
\]
\[
A_2 = b_2 s_2^{s_2} + (b_0 + b_1) s_0^{s_1} s_1^{s_0} s_2^{s_2} + b_0 s_0^{s_1} s_1^{s_0} s_2^{s_0} + b_1 s_0^{s_1} s_1^{s_0} s_2^{s_0}
\]

**Remark 1.** Let \( p = 2^n - 1 \) (\( n \) is a natural number), \( \{0, 1\}^n = \{D_0, \ldots, D_p\} \), \( f : \{0, 1\}^n \to \{0, 1\} \) be a Boolean function and

\[
f(x_1, \ldots, x_n) = 0
\]

be a consistent Boolean equation, i.e. \( \prod_{i=0}^{p} f(D_i) = 0 \).

In accordance with Theorem 1 one can effectively find all general solutions of (4) in the form

\[
X = A_0 (f(D_0), \ldots, f(D_p)) T^{D_0} \cup \cdots \cup A_p (f(D_0), \ldots, f(D_p)) T^{D_p}
\]

where \( X = (x_1, \ldots, x_n) \) and \( T = (t_1, \ldots, t_n) \).

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