A FIRST ORDER PROBABILITY LOGIC - \(LP_Q\)

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Communicated by Žarko Mijajlović

Abstract. A conservative extension of the classical first order logic which allows making statements about probability is introduced. Some classes of probability models are described. An infinitary axiomatic system which is sound and complete with respect to these classes of models is given.

1. Introduction

The first order probability logic \(LP\) is given in [3,4] Its language is obtained by adding probability operators of the form \(P_{\geq s}\) to the classical first order language, where \(s\) belongs to a set \(\text{Index}\) which is a finite subset of \([0,1]\). Formulas in the scope of a probability operator are classical first order formulas. \(LP\) allows making formulas such as \(P_{\geq s}\alpha\), with the intended meaning “the probability of \(\alpha\) is greater then or equal to \(s\)”. In [3,4] a finitary axiomatic system is provided, and the corresponding extended completeness theorem is proved.

In this paper we investigate another first order probability logic, denoted \(LP_Q\), whose language contains a list of probability operators of the form mentioned above, but the set \(\text{Index}\) is the set of all rational numbers from \([0,1]\). It turns out that such an assumption makes \(LP_Q\) different from \(LP\). Namely, the compactness theorem does not hold for \(LP_Q\), while it holds for \(LP\): consider an arbitrary classical sentence \(\alpha\) and the set \(T = \{\neg P_{\geq s}\alpha\} \cup \{P_{\geq 1/n}\alpha : n\text{ is a positive integer}\}\); although every finite subset of \(T\) is satisfiable, the set \(T\) is not. A consequence is that, if we want the extended completeness theorem, we cannot obtain a finitary axiomatization. In this paper, we describe some classes of probability models, give an axiomatization with an infinitary rule, and prove the corresponding extended completeness theorems. We also discuss (un)decidability of \(LP_Q\).

AMS Subject Classification (1991): Primary 03B48
Supported by Ministry of Science and Technology of Serbia, grant 04M02/C
2. Syntax

Let Index be the set of all rational numbers from \([0,1]\). The language of \(LP_\Omega\) is a countable classical first-order language extended by a list of probability operators \(P_{\geq s}\), for every \(s \in \text{Index}\). Let us denote the set of all classical first-order formulas by \(\text{For}_C\). The formulas from the set \(\text{For}_C\) will be denoted by \(\alpha, \beta, \ldots\). If \(\alpha \in \text{For}_C\), and \(s \in \text{Index}\), then \(P_{\geq s}\alpha\) is a basic probability formula. The set of all probability formulas is the least set \(\text{For}_P\) containing all basic probability formulas, and closed under formation rules: if \(A, B \in \text{For}_P\), then \(\neg A\), \(A \land B \in \text{For}_P\). The formulas from the set \(\text{For}_P\) will be denoted by \(A, B, \ldots\) Let \(\text{For}_C \cup \text{For}_P\) be denoted by \(\text{For}\), and the set of all sentences from \(\text{For}\) by \(\text{Sentences}\). The formulas from the set \(\text{For}\) will be denoted by \(\Phi, \Psi, \ldots\) We use the usual abbreviations for the other classical connectives, and also denote \(\neg P_{\geq s}(\alpha)\) by \(P_{<s}(\alpha)\), \(P_{\geq 1-s}(\alpha)\) by \(P_{\leq s}(\alpha)\), \(\neg P_{\leq s}(\alpha)\) by \(P_{>s}(\alpha)\), and \(\Phi \land \neg \Phi\) for an arbitrary \(\Phi \in \text{For}\) by \(\bot\).

3. Semantics

We use the possible-worlds approach to give semantics to formulas and interpret formulas such that they remain either true or false. An \(LP_\Omega\)-model is a structure \(M = \langle W, D, I, A, \mu \rangle\) where:

- \(W\) is a non empty set of objects called worlds,
- \(D\) is a function which assigns to every \(w \in W\) a domain \(D(w)\),
- \(I\) is a function which assigns to every \(w \in W\) a classical interpretation \(I(w)\),
- \(A\) is an algebra of subsets of \(W\), and
- \(\mu\) is a finitely additive probability measure, \(\mu : A \rightarrow [0,1]\).

Let \(M = \langle W, D, I, A, \mu \rangle\) be an \(LP_\Omega\)-model. A variable valuation \(v\) assigns some element of the domain \(D(w)\) to every world \(w\) and every variable \(x\), i.e., \(v(w)(x) \in D(w)\). If \(D(w)\) is a domain, \(d \in D(w)\), and \(v\) is a valuation, then \(v_w[d/x]\) is a valuation like \(v\) except that \(v_w[d/x](w)(x) = d\). The values of terms and classical formulas in a world is defined as usual. For example, the value of a classical formula \((\forall x)\alpha\) in \(w \in W\) for a given valuation \(v\) (denoted by \(I(w)((\forall x)\alpha)_v\)) is true if and only if for every \(d \in D(w)\), \(I(w)(\alpha)_{v[d/x]}\) is true. A classical formula holds in a world \(w\) of an \(LP_\Omega\) model \(M\) (denoted by \((M,w) \models \alpha\)) if for every valuation \(v\), \(I(w)(\alpha)_v\) is true.

Let \(M\) be an \(LP_\Omega\) model and \(\alpha\) a classical sentence. The set \(\{w \in W : (M,w) \models \alpha\}\) is denoted by \([\alpha]_M\). We will omit the subscript \(M\) from \([\alpha]_M\) and write \([\alpha]\), if \(M\) is clear from the context. An \(LP_\Omega\)-model \(M\) is measurable if \([\alpha]\) is measurable for every classical sentence \(\alpha\). In this paper we will focus on the class \(LP_{\Omega,\text{Meas}}\) of all measurable \(LP_\Omega\)-models, as well as on its subclasses: \(LP_{\Omega,\text{AH}}\), the class of all \(LP_{\Omega,\text{Meas}}\)-models such that a model \(M = \langle W, D, I, A, \mu \rangle\) belongs to \(LP_{\Omega,\text{AH}}\) if \(A\) is the power set of \(W\), and \(LP_{\Omega,\sigma}\), the class of all \(LP_{\Omega,\text{Meas}}\)-models with \(\sigma\)-additive measure.

Let \(L\) be one of the above class of models. The satisfiability relation \(\models\subset L \times \text{Sentences}\) fulfills the following conditions:
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- if \( \alpha \in \text{For}_C \), \( M \models \alpha \) if \( (\forall w \in W)(M, w) \models \alpha \),
- \( M \models P_{\geq s} \alpha \) if \( \mu([\alpha]) \geq s \),
- if \( A \in \text{For}_P \), \( M \models \neg A \) if \( M \not\models A \), and
- if \( A, B \in \text{For}_P \), \( M \models A \land B \) if \( M \models A \) and \( M \models B \).

A set \( T \) of sentences is \( L \)-satisfiable if there is an \( L \)-model \( M \) such that every sentence from \( T \) is satisfied in \( M \). A sentence \( \Phi \in \text{For} \) is \( L \)-valid if it is satisfied in every \( L \)-model.

### 4. Complete Axiomatization

The axiom schemata for \( LP_Q \) are:

1. axiom schemata of the classical first order logic
2. \( P_{\geq 0} \alpha \)
3. \( P_{\leq r} \alpha \rightarrow P_{< s} \alpha, \ s > r \)
4. \( P_{< s} \alpha \rightarrow P_{< s} \alpha \)
5. \( (P_{\geq r} \alpha \land P_{\geq s} \beta \land P_{\geq 1}(-\alpha \land \beta)) \rightarrow P_{\geq \min(1, r+s)}(\alpha \lor \beta) \)
6. \( (P_{\leq r} \alpha \land P_{< s} \beta) \rightarrow P_{< r+s}(\alpha \lor \beta), r + s \leq 1 \)

while the inference rules are:

1. From \( \Phi \) and \( \Phi \rightarrow \Psi \) infer \( \Psi \).
2. From \( \alpha \) infer \( (\forall x)\alpha \)
3. From \( \alpha \) infer \( P_{s} \alpha \).
4. From \( A \rightarrow P_{\geq 1/s} \alpha \), for every integer \( k \geq 1/s \), and \( s > 0 \) infer \( A \rightarrow P_{s} \alpha \).

The main difference between the axiomatic system for the logic \( LP \) and the one given above is that the inference rule 4 does not appear in the former system. Note that formulas obtained by applications of the inference rules must obey the formation rules, i.e., in the inference rules 2 and 3, \( \alpha \) must be a classical formula.

A formula \( \Phi \in \text{For} \) is deducible from a set \( T \) of sentences \( T \vdash \phi \) if there is an at most countable sequence of formulas \( \Phi_0, \Phi_1, \ldots, \Phi_n \) such that every formula in the sequence is an axiom or a formula from the set \( T \), or it is derived from the preceding formulas by an application of an inference rule. A set \( T \) of sentences is inconsistent if \( T \vdash \bot \), otherwise it is consistent.

In the proof of the completeness theorem the Henkin procedure will be used. We begin with some auxiliary statements.

**Theorem 4.1** (Deduction theorem) *If \( T \subset \text{Sentences}, \Phi \in \text{Sentences}, \) and \( T \cup \{\Phi\} \vdash \Psi \), then \( T \vdash \Phi \rightarrow \Psi \), where \( \Phi \) and \( \Psi \) are either both classical or both probability formulas.*

*Proof.* We use the transfinite induction on the length of the proof of \( \Psi \) from \( T \cup \{\Phi\} \). For example, we consider the case where \( B = C \rightarrow P_{\geq s} \delta \) is obtained from \( T \cup \{A\} \) by an application of the inference rule 4, and \( A \) is a probability sentence. Then:

\( T, A \vdash C \rightarrow P_{\geq 1/s} \delta \), for every integer \( k \geq 1/s \)
Theorem 4.2. Let \( \alpha \) and \( \beta \) be classical sentences. Then:

1. \( \vdash P_{\geq s}(\alpha \rightarrow \beta) \) \( \rightarrow (P_{\geq s}\alpha \rightarrow P_{\geq s}\beta) \)
2. \( \vdash P_{\geq r}\alpha \rightarrow P_{\geq s}\alpha, \ r > s \)

Proof. 1. If \( s = 0 \), the statement obviously holds. So, let \( s \) be a rational number from \((0, 1]\). First note that by an application of the inference rule 3, we obtain

(1) \[ \vdash P_{\geq 1}(\neg \alpha \lor \bot) \]

from \( \vdash \neg \alpha \lor \bot \). Similarly, from \( \vdash (\neg \alpha \land \bot) \lor \neg \alpha \) we have

(2) \[ \vdash P_{\geq 1}((\neg \alpha \land \bot) \lor \neg \alpha) \]

By the axiom 5, we have \( \vdash (P_{\geq s}\alpha \land P_{\geq 0}\neg \bot \land P_{\geq 1}(\neg \alpha \lor \neg \bot)) \rightarrow P_{\geq s}(\alpha \lor \bot) \). Since \( \vdash P_{\geq 0}\neg \bot \) by the axiom 2, from (1) it follows that

(3) \[ \vdash P_{\geq s}(\alpha \lor \bot) \rightarrow P_{\geq s}(-\alpha \lor \bot) \]

The expressions \( P_{\geq s}(\alpha \lor \bot) \) and \( -P_{\geq s}(-\alpha \land \bot) \) denote \( P_{\leq 1-s}(\neg \alpha \land \bot) \), and \( P_{\leq s}(-\alpha \land \bot) \), respectively. By the axiom 6, we have \( \vdash (P_{\leq 1-s}(\neg \alpha \land \bot) \land P_{\leq s}(-\alpha) \rightarrow P_{\leq 1}(\neg \alpha \land \bot) \lor \neg \alpha) \). From (2) we obtain that \( \vdash (P_{\leq 1-s}(\neg \alpha \land \bot) \land P_{\leq s}(-\alpha) \rightarrow (P_{\leq 1}(\neg \alpha \land \bot) \lor \neg \alpha)) \land P_{\leq 1}(\neg \alpha \land \bot) \lor \neg \alpha \). It follows that \( \vdash P_{\leq 1-s}(\neg \alpha \land \bot) \rightarrow P_{\leq 1-s}(-\alpha) \), i.e.

(4) \[ \vdash P_{\geq s}(\alpha \lor \bot) \rightarrow P_{\geq s}(-\alpha) \]

From (3) and (4) we obtain:

(5) \[ \vdash P_{\geq s}\alpha \rightarrow P_{\geq s}(-\alpha) \]

The negation of the formula \( P_{\geq 1}(\alpha \rightarrow \beta) \rightarrow (P_{\geq s}\alpha \rightarrow P_{\geq s}\beta) \) is equivalent to \( P_{\geq 1}(\neg \alpha \lor \beta) \land P_{\leq s}\alpha \land P_{\leq s}\beta \). By (5) this formula implies \( P_{\geq 1}(\neg \alpha \lor \beta) \land P_{\leq s}(-\alpha) \land P_{\leq s}\beta \) which can be rewritten as \( P_{\geq 1}(\neg \alpha \lor \beta) \land P_{\leq 1-s}\neg \alpha \land P_{\leq s}\beta \). From the axiom 6, \( P_{\leq 1-s}\neg \alpha \land P_{\leq s}\beta \rightarrow P_{\leq 1}(\neg \alpha \lor \beta) \), and \( P_{\leq 1}\alpha = -P_{\leq 1}\alpha \), we have \( \vdash \neg (P_{\geq 1}(\alpha \rightarrow \beta) \rightarrow (P_{\geq s}\alpha \rightarrow P_{\geq s}\beta)) \rightarrow P_{\geq 1}(\neg \alpha \lor \beta) \land \neg P_{\geq 1}(\neg \alpha \lor \beta) \), a contradiction. It follows that \( \vdash P_{\geq 1}(\alpha \rightarrow \beta) \rightarrow (P_{\geq s}\alpha \rightarrow P_{\geq s}\beta) \).
2. By the axioms 3 and 4, we have \( \vdash P_{2r} \alpha \rightarrow P_{>s} \alpha \), for \( r > s \), and \( \vdash P_{>s} \alpha \rightarrow P_{>s} \alpha \). Thus, \( \vdash P_{2r} \alpha \rightarrow P_{>s} \alpha \), for \( r > s \).

**Theorem 4.3. (Completeness theorem for LPQ,\text{\textit{Meas}}) Let** \( T \subset \text{Sentences} \). Then, \( T \) is consistent if and only if \( T \) has an LPQ,\text{\textit{Meas}}-model.

**Proof.** The \((\Rightarrow)-\)direction follows from the soundness of the above axiomatic system. In order to prove the \((\Rightarrow)-\)direction let us suppose that \( T \) is a consistent set of sentences, that conseq\((T)\) is the set of all classical sentences that are consequences of \( T \) and that \( A_0, A_1, \ldots \) is an enumeration of all probability sentences. We define a sequence of sets \( T_i, i = 0, 1, 2, \ldots \) such that:

1. \( T_0 = T \cup \text{conseq}(T) \cup \{P_{\geq} \alpha : \alpha \in \text{conseq}(T)\} \)
2. for every \( i \geq 0 \), if \( T_i \cup \{A_i\} \) is consistent, then \( T_{i+1} = T_i \cup \{A_i\}, \) otherwise, \( T_{i+1} = T_i \cup \{\neg A_i\}, \)
3. if the set \( T_{i+1} \) is obtained by adding a formula of the form \( \neg(B \rightarrow P_{>s} \gamma) \)
   then for some positive integer \( n \), \( B \rightarrow \neg P_{>s-1/n} \gamma \), is also added to \( T_{i+1} \), so that \( T_{i+1} \) is consistent.

Every \( T_i \) is a consistent set. \( T_0 \) is consistent because it is a set of consequences of a consistent set. Suppose that \( T_i \) is obtained by the step 2 of the above construction and that neither \( T_i \cup \{A_i\} \), nor \( T_i \cup \{\neg A_i\} \) are consistent. It follows by the deduction theorem that \( T_i \vdash A_i \land \neg A_i \), which is a contradiction. Consider the step 3 of the construction. If \( T_i \cup \{B \rightarrow P_{>s} \gamma\} \) is not consistent, then the set \( T_i \) can be consistently extended as above. Suppose that it is not the case. Then:

1. \( T_i \vdash \neg(B \rightarrow P_{>s} \gamma), B \rightarrow \neg P_{>s-1/k} \gamma \vdash \bot \), for every \( k > 1/s \), by the hypothesis
2. \( T_i \vdash \neg(B \rightarrow P_{>s} \gamma) \rightarrow \neg(B \rightarrow \neg P_{>s-1/k} \gamma) \) for every \( k > 1/s \), by the deduction theorem
3. \( T_i \vdash \neg(B \rightarrow P_{>s} \gamma) \rightarrow B \rightarrow P_{>s-1/k} \gamma \) for every \( k > 1/s \), from 2, by the classical tautology \( \neg(\alpha \rightarrow \gamma) \rightarrow (\alpha \rightarrow \neg \gamma) \)
4. \( T_i \vdash \neg(B \rightarrow P_{>s} \gamma) \rightarrow B \rightarrow P_{>s} \gamma \), from 3, by the inference rule 4
5. \( T_i \vdash \neg(B \rightarrow P_{>s} \gamma) \rightarrow B \rightarrow P_{>s} \gamma \), from 4, by the deduction theorem
6. \( T_i \vdash B \rightarrow P_{>s} \gamma \)

Since \( T_i \cup \{B \rightarrow P_{>s} \gamma\} \) is not consistent, from \( T_i \vdash B \rightarrow P_{>s} \gamma \) it follows that \( T_i \) is not consistent, a contradiction.

Let \( T^* = \bigcup_i T_i \). The set \( T^* \) is a deductively closed set that does not contain all sentences. First note that for every \( \Phi \in \text{Sentences} \), if \( T_i \vdash \Phi \), then it must be \( \Phi \in T^* \). If \( \Phi \) is a classical sentence, then \( T \vdash \Phi \), and \( \Phi \in T_0 \). If \( \Phi = A_k \) is a probability sentence, and \( \Phi \not\in T^* \), then \( T_{\max\{i,k\}+1} \vdash \Phi \) and \( T_{\max\{i,k\}+1} \vdash \neg \Phi \), a contradiction. Since \( T \) is a consistent set, there is at least a classical sentence \( \alpha \) such that \( T \not\vdash \alpha \). If \( A \) is a probability sentence, it cannot be \( A = A_k \in T^* \), and \( \neg A = A_m \in T^* \), because \( T_{\max\{k,m\}+1} \) is consistent. Finally, we can prove that if \( A \) is a probability sentence, and \( T \vdash A \), then \( A \in T^* \). Suppose that the sequence \( \Phi_1, \Phi_2, \ldots, A \) of formulas which forms the proof of \( A \) from \( T^* \) is countably infinite (otherwise there must be some \( k \) such that \( T_k \vdash A \), and it must be \( A \in T^* \)). We can show that for every \( i \), if \( \Phi_i \) is obtained by an application of an inference rule,
and all the premises of $\Phi_i$ belong to $T^*$, then $\Phi_i \in T^*$. Suppose $\Phi_i$ is obtained by the inference rule 1 (modus ponens) and its premises $\Phi_i^1$ and $\Phi_i^2$ belong to $T^*$. There must be some $k$ such that $\Phi_i^1, \Phi_i^2 \in T_k$. From $T_k \vdash \Phi_i$, it follows $\Phi_i \in T^*$. If $\Phi_i$ is obtained by the inference rules 2 and 3, then $T_0 \vdash \Phi_i$, and $\Phi_i \in T^*$. Suppose that $\Phi_i = B \rightarrow P_{2^s} \gamma$ is obtained by the infinitary inference rule 4, and that the premises $\Phi_i^1 = B \rightarrow P_{2^s/k} \gamma, \Phi_i^2 = B \rightarrow P_{2^{s-1}} \gamma, \ldots$ belong to $T^*$. If $\Phi_i \notin T^*$, by the step 3 of the construction of $T^*$, there is some $j > 1/s$, such that $B \rightarrow -P_{2^s-1/j} \gamma \in T^*$. Let $l = \max\{k, j\}$. By the axioms 3 and 4, $B \rightarrow P_{2^{s-l} \gamma} \in T^*$, and $B \rightarrow -P_{2^{s-l} \gamma} \in T^*$. There must be a set $T_m$ which also contains these formulas. It follows that $T_m \cup \{B\}$ is not consistent. Thus, $B \notin T^*$, and there is some $j$ such that $-B \in T_j$, $T_j \vdash B \rightarrow \bot$, $T_j \vdash B \rightarrow P_{2^s} \gamma$, and $B \rightarrow P_{2^s} \gamma \in T^*$, which is a contradiction. Hence, from $T^* \vdash A$, it follows $A \in T^*$.

The set $T^*$ is used to construct a tuple $M = \langle W, D, I, \{[\alpha] : \alpha$ is a classical sentence$\}, \mu\rangle$, where:

- $W = \{w : w \models \text{clos}_{\text{seq}}(T)\}$ contains all the classical first order interpretations with at most countable domains that satisfy the set $\text{clos}_{\text{seq}}(T)$ of all classical consequences of the set $T$; the corresponding domains are denoted by $D(w)$,
- $D$ maps every $w \in W$ to $D(w)$,
- $I(w)$ is the interpretation $w$,
- $\mu : \{[\alpha] : \alpha$ is a classical sentence$\} \rightarrow [0, 1]$ such that $\mu([\alpha]) = \sup\{s : P_{2^s} \alpha \in T^*\}$.

The axioms guarantee that everything is well defined. For example, by the classical reasoning we can show that $\{[\alpha] : \alpha$ is a classical sentence$\}$ is an algebra of subsets of $W$. The theorem 4.2.1 implies that if $[\alpha] = [\beta]$, then $\mu([\alpha]) = \mu([\beta])$.

From the axioms 2–6 about probability it follows that $\mu$ is a finitely additive probability measure.

By the induction on the complexity of formulas we can prove that for every sentence $\Phi$, $M \models \Phi$ iff $\Phi \in T^*$. For example, let $\Phi$ be a classical sentence. If $\Phi \in \text{clos}_{\text{seq}}(T)$, then by the definition of $M$, $M \models \Phi$. Conversely, let $M \models \Phi$. Then, by the completeness of the classical first order logic, $\Phi \in \text{clos}_{\text{seq}}(T)$. If $\Phi = P_{2^s} \alpha \in T^*$, then $\sup\{r : P_{2^r}(\alpha) \in T^*\} = \mu([\alpha]) > s$, and $M \models \Phi$. For the other direction, suppose that $M \models \Phi$, i.e., that $\sup\{r : P_{2^r}(\alpha) \in T^*\} > s$. If $\mu([\alpha]) > s$, then, by the well known property of supremum and monotonicity of $\mu$ (the theorem 4.2.2), $\Phi \in T^*$. Let $\mu([\alpha]) = s$. If $\Phi \notin T^*$, then by the step 3 of the construction of $T^*$, for some integer $k > 1/s$, $-P_{2^{s-k}} \alpha \notin T^*$. It follows that $s$ cannot be the supremum of the set $\{r : P_{2^r}(\alpha) \in T^*\}$, which is a contradiction. The other cases follow easily.

**Theorem 4.4.** (Completeness theorem for $LP_{Q\&\exists}$) Let $T \subset \text{Sentences}$. Then, $T$ is consistent if and only if $T$ has an $LP_{Q\&\exists}$-model.

**Proof.** The proof can be obtained by applying the extension theorem for additive measures [1] on the measure $\mu$ from the canonical model $M$ described in
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the theorem 4.3. It is proved that there is an additive measure $\mathcal{M}$ defined on the power set of $W$ which is an extension of the measure $\mu$.

**Theorem 4.5.** (Completeness theorem for $LP_{Q,\sigma}$) Let $T \subseteq \text{Sentences}$. Then, $T$ is consistent if and only if $T$ has an $LP_{Q,\sigma}$-model.

**Proof.** By the Loeb process and a bounded elementary embedding [2] we can transform the canonical model $M$ from the theorem 4.3 into a $\sigma$-additive probability model $^*M$ such that for every $\Phi \in \text{Sentences}$, $M \models \Phi$ if $^*M \models \Phi$.

5. Decidability

$LP_Q$-logic is undecidable since it contains the classical first order logic. However, some fragments of $LP_Q$ are decidable. One of these fragments is the monadic first order probability logic (without function symbols except constants) in which the arity of all relation symbols is 1. By the Herbrand theorem, every first order classical sentence $\alpha$ is satisfiable if and only if the set $E(\alpha)$ of formulas that form the Herbrand expansion of $\alpha$ is satisfiable. Formulas from $E(\alpha)$ are without variables and can be understood as formulas in the classical propositional logic. In the monadic case, for every formula $\alpha$ the set $E(\alpha)$ is finite. Thus, the satisfiability of the monadic $LP_Q$-logic can be reduced to the satisfiability of the propositional probability logic. Since the propositional probability logic is decidable [3], the monadic $LP_Q$-logic is decidable.

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(Received 12 05 1998)  
(Revised 17 12 1998)