INEQUALITIES AMONG THE NUMBER OF THE GENERATORS AND RELATIONS OF A KÄHLER GROUP

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Abstract. The present note announces some inequalities on the number of the generators and relations of a Kähler group $\pi_1(X)$, involving the irregularity $q(X)$, the Albanese dimension $a(X)$ and the Albanese genera $g_k(X)$, $1 \leq k \leq a(X)$, of the corresponding compact Kähler manifold $X$. The principal ideas for their derivation are outlined and the proofs are postponed to be published elsewhere.

Let $X$ be an irregular compact Kähler manifold, i.e., with an irregularity $q = q(X) := \dim \mathbb{C}H^{1,0}(X) > 0$. The Albanese variety $\text{Alb}(X) = H^{1,0}(X)^*/H_1(X,\mathbb{Z})_{\text{free}}$ admits a holomorphic Albanese map $\text{alb}_X : X \to \text{Alb}(X)$, given by integration $\text{alb}_X(x)(\omega) := \int_{x_0}^x \omega$ of holomorphic $(1,0)$-forms $\omega \in H^{1,0}(X)$ from a base point $x_0 \in X$ to $x \in X$. The complex rank of the Albanese map $\text{alb}_X$ is called an Albanese dimension $a = a(X)$ of $X$. A compact Kähler manifold $Y$ is said to be Albanese general if $\dim \mathbb{C}Y = a(Y) < q(Y)$. The surjective holomorphic maps $f_k : X \to Y_k$ of a compact Kähler manifold $X$ onto Albanese general $Y_k$ are referred to as Albanese general $k$-fibrations of $X$. The maximum irregularity $q(Y_k)$ of a base $Y_k$ of an Albanese general $k$-fibration $f_k : X \to Y_k$ is called $k$-th Albanese genus of $X$ and denoted by $g_k(Y) = g_k(X)$. The present note states lower bounds on the Betti numbers $b_i(\pi_1(X)) := rk_{\mathbb{Z}}H^i(\pi_1(X),\mathbb{Z})$ of the fundamental group $\pi_1(X)$, in terms of the irregularity $q(X)$, the Albanese dimension $a(X)$ and the Albanese genera $g_k(X)$, $1 \leq k \leq a(X)$.

On the other hand, $b_i(\pi_1(X))$ are estimated above by the number of the generators $s$ and the number of the relations $r$ of $\pi_1(X)$ and, eventually, by the irregularity $q(X)$, exploiting to this end few abstract results on the group cohomologies.