WEITZENBÖCK FORMULAS ON POISSON PROBABILITY SPACES

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Abstract. This paper surveys and compares some recent approaches to stochastic infinite-dimensional geometry on the space \( \Gamma \) of configurations (i.e. locally finite subsets) of a Riemannian manifold \( M \) under Poisson measures. In particular, different approaches to Bochner–Weitzenböck formulas are considered. A unitary transform is also introduced by mapping functions of \( n \) configuration points to their multiple stochastic integral.

1. Weitzenböck Formula under a Measure

Let \( M \) be a Riemannian manifold with volume measure \( dx \), covariant derivative \( \nabla \), and exterior derivative \( d \). Let \( \nabla^*_\mu \) and \( d^*_\mu \) denote the adjoints of \( \nabla \) and \( d \) under a measure \( \mu \) on \( M \) of the form \( \mu(dx) = e^{\phi(x)} dx \). The classical Weitzenböck formula under the measure \( \mu \) states that

\[
d^*_\mu d + dd^*_\mu = \nabla^*_\mu \nabla + R - \text{Hess} \phi,
\]

where \( R \) denotes the Ricci tensor on \( M \). In terms of the de Rham Laplacian \( H_R = d^*_\mu d + d d^*_\mu \) and of the Bochner Laplacian \( H_B = \nabla^*_\mu \nabla \) we have

\[
H_R = H_B + R - \text{Hess} \phi.
\]

In particular the term \( \text{Hess} \phi \) plays the role of a curvature under the measure \( \mu \).

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