CONFORMAL SCHWARZIAN DERIVATIVES
AND DIFFERENTIAL EQUATIONS

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Abstract. We investigate the fundamental system of equations in the
theory of conformal geometry, whose coefficients are considered as
the conformal Schwarzian derivative. The integrability condition of
the system is obtained in a simple method, which allow us to find a
natural geometric structure on the solution space. From the solution
spaces, using this geometric structure, we get a transformation whose
Schwarzian derivative is equal to the given coefficients of the equation.

1. Introduction

Some years ago Sasaki and Yoshida [10] gave the fundamental system of linear
equations, which is the key system connecting the theory of conformal connec-
tions and the uniformizing differential equations in the geometry of symmetric
domains of type IV. It is a system of equations with \( n \) variables such that
the maximal dimension of the solution space is \( n + 2 \). The solutions natu-
 rally provide a map into the projective space whose image is contained in the
hyperquadric, and accepts the conformal transformation group as its symmetry.
Sasaki and Yoshida considered the equations as a higher dimensional analogue
of Gauss–Schwarz equation. In projective geometry of higher dimension, they
defined Schwarzian derivatives as a difference of normal Cartan connections
moved by a diffeomorphism. Using the Schwarzian derivatives, they got the
system of linear equations such that the maximal dimension of the solution
space is \( n + 1 \) on \( n \) variables [9, 12]. In conformal geometry of higher dimen-
sion, the problem is much harder. As the Schwarzian derivatives we need more