ON LOCALLY LAGRANGIAN SYMPLECTIC STRUCTURES

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Abstract. Some results on global symplectic forms defined by local Lagrangians of a tangent manifold, studied earlier by the author, are summarized without proofs.

This is a summary of some of our results on locally Lagrange symplectic and Poisson manifolds [3, 4].

The symplectic forms used in Lagrangian dynamics are defined on tangent bundles $TN$, and they are of the type

$$\omega_L = \sum_{i,j=1}^{n} \left( \frac{\partial^2 L}{\partial x^i \partial \xi^j} \, dx^i \wedge dx^j + \frac{\partial^2 L}{\partial \xi^i \partial \xi^j} \, d\xi^i \wedge dx^j \right)$$

(1)

where $(x^i)_{i=1}^n$ $(n = \dim N)$ are local coordinates on $N$, $(\xi^i)$ are the corresponding natural coordinates on the fibers of $TN$, and $L \in C^\infty(TN)$ is a non degenerate Lagrangian.

An almost tangent structure on a differentiable manifold $M^{2n}$ is a tensor field $S \in \Gamma \text{End}(TM)$ (necessarily of rank $n$) such that

$$S^2 = 0, \quad \text{Im} \, S = \text{Ker} \, S.$$  

(2)

If the Nijenhuis tensor vanishes, i.e. $\forall X, Y \in \Gamma TM$,

$$N_S(X, Y) = [SX, SY] - S[SX, Y] - S[X, SY] + S^2[X, Y] = 0,$$

(3)

$S$ is a tangent structure. Then, $V = \text{Im} \, S$, is an integrable subbundle, and we call its tangent foliation the vertical foliation $\mathcal{V}$. Furthermore, $M$ has local