FINDING THE ZEROS OF THE FUNCTIONS TREATED BY $q$-CALCULUS

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Abstract. We construct $q$-Taylor formula for the functions of several variables and develop some new methods for solving equations and systems of equations. They are much easier for application than well known ones and very useful when the continuous function does not have fine smooth properties. Especially, we will demonstrate their power in solving the equations where the function is defined by some $q$-integral. We will discuss the convergence and accuracy of those methods and compare them with well known methods. The conclusions are illustrated by examples.

1. Introduction

We will start with basic notions from $q$-calculus which can be found, for example, in [3] and [5].

Let $q \in (0, 1)$. A $q$-natural number $[n]_q$ is defined by

$$[n]_q := 1 + q + \cdots + q^{n-1}, \quad n \in \mathbb{N}.$$  

Generally, a $q$-complex number $[a]_q$ is

$$[a]_q := \frac{1 - q^a}{1 - q}, \quad a \in \mathbb{C}. $$

We define the factorial of the number $[n]_q$ and the $q$-binomial coefficient by

$$[0]_q! := 1, \quad [n]_q! := [n]_q[n-1]_q \cdots [1]_q, \quad \binom{n}{k}_q := \frac{[n]_q!}{[k]_q! [n-k]_q!}.$$ 

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