EXTENDED HAMILTONIAN FORMALISM OF FIELD THEORIES: VARIATIONAL ASPECTS AND OTHER TOPICS

ARTURO ECHEVERRÍA-ENRÍQUEZ, MANUEL DE LEÓN†
MIGUEL C. MUÑOZ-LECANDA and NARCISO ROMÁN-ROY

Dept. Matemática Aplicada IV. Ed. C-3, Campus Norte UPC
Jordi Girona 1, E-08034 Barcelona, Spain
†Instituto de Matemáticas y Física Fundamental, CSIC
Serrano 123, E-28006 Madrid, Spain

Abstract. We consider Hamiltonian systems in first-order multisymplectic field theories. In particular, we introduce Hamiltonian systems in the extended multimomentum bundle. The resulting extended Hamiltonian formalism is the generalization to field theories of the extended (symplectic) formalism for non-autonomous mechanical systems. In order to derive the corresponding field equations, a variational principle is stated for these extended Hamiltonian systems and, after studying the geometric properties of these systems, we establish the relation between this extended formalism and the standard one.

1. Introduction

It is well known that the structure of autonomous Hamiltonian dynamical systems is especially suitable for analyzing certain kinds of problems concerning these systems, such as: symmetries and related topics (existence of conservation laws and reduction), integrability (including numerical methods), and quantization. Geometrically, many of the characteristics of these systems arise from the existence of a “natural” geometric structure in the phase space: the symplectic form. The dynamic information is carried out by the Hamiltonian function, which is “independent” of the geometry.

We wish to generalize the structure of Hamiltonian systems in autonomous mechanics to first-order multisymplectic field theories. In these models, multisymplectic forms play the same role as symplectic forms in autonomous mechanics [2,4–8,11,12,14].