CONFERENCES AT
UNIVERSITY COLLEGE DUBLIN
September 1994

7th Annual Meeting
of the Irish Mathematical Society
5–6 September 1994

Speakers: J. M. Anderson (London), P. M. Gauthier (Montreal),
B. Goldsmith (DIT), A. J. O’Farrell (Maynooth), J. V. Pulé
(UCD), R. Ryan (UCG).

Requests for accommodation should be submitted by 1 July, 1994.
Further information: S. Dineen, S. Gardiner (addresses below).

Polynomials and Holomorphic Functions
on Infinite Dimensional Spaces
7–9 September, 1994


Tel: +353 1 706 8242
+353 1 706 8265
Fax: +353 1 706 1196
email: sdineen@irllearn.bitnet
gardiner@irllearn.bitnet

TRACE-ZERO MATRICES AND
POLYNOMIAL COMMUTATORS

T. J. Laffey and T. T. West

Let $F$ denote a field and $M_n(F)$ the algebra of $n \times n$ matrices
over the field $F$. If $X \in M_n(F)$, $\text{tr}(X)$ will denote the trace of
the matrix $X$. A well known result of Albert and Muckenhoupt [1]
states that if $\text{tr}(X) = 0$ then there exist matrices $A, B \in M_n(F)$
such that $X$ is the commutator of $A$ and $B$,

$$X = [A, B] = AB - BA.$$

Let $p$ denote a polynomial in $F[x]$ of degree greater than or equal
to one. The Polynomial Commutator of $A$ and $B$ relative to $p$ is
defined to be

$$p[A, B] = p(AB) - p(BA).$$

It is easy to check, by examining the eigenvalues, that $\text{tr}(p[A, B])$
is always zero. The Albert-Muckenhoupt result states that if $X \in
M_n(F)$ with $\text{tr}(X) = 0$ then, for $p(x) = x$,

$$X = p[A, B],$$

for some $A, B \in M_n(F)$. We show that, if the field $F$ has character-
istic zero the Albert-Muckenhoupt result may be extended to
general polynomials of degree greater than, or equal to, one.

**Theorem.** Let $F$ be a field of characteristic zero and let $p \in F[x]$
have degree greater than or equal to one. If $X \in M_n(F)$ is of trace
zero then there exist matrices $A, B \in M_n(F)$ such that

$$X = p[A, B].$$

First we prove the following elementary