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POLYNOMIAL IDENTIFICATION IN UNIFORM AND OPERATOR ALGEBRAS

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ABSTRACT. Let \mathcal{A} be a unital Banach algebra, and denote the spectral radius of $f \in \mathcal{A}$ by $\rho(f)$. If \mathcal{A} is a uniform algebra and $\rho(fh + 1) = \rho(gh + 1)$ for all $h \in \mathcal{A}$, then it can be shown that $f = g$, a result that also carries in algebras of bounded linear operators on Banach spaces. On the other hand $\rho(fh) = \rho(gh)$ does not imply $f = g$ in any unital algebra, marking a distinction between the polynomials $p(z, w) = zw + 1$ and $p(z, w) = zw$. Such results are known as *spectral identification lemmas*, and in this work we demonstrate first- and second-degree polynomials of two variables that lead to identification via the spectral radius, peripheral spectrum, or full spectrum in uniform algebras and in algebras of bounded linear operators on Banach spaces. The primary usefulness of identification lemmas is to determine the injectivity of a class of mappings that preserve portions of the spectrum, and results corresponding to the given identifications are also presented.

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