



COMMON COUPLED FIXED POINT THEOREMS IN d -COMPLETE TOPOLOGICAL SPACES

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ABSTRACT. In this paper, we give two unique common coupled fixed point theorems for mappings satisfying a generalized condition in d -complete topological spaces.

1. INTRODUCTION AND PRELIMINARIES

In 1992, Hicks [5] introduced the notion of d -complete topological spaces as a generalization of complete metric spaces.

Definition 1.1. (See [5]) Let (X, τ) be a topological space and $d : X \times X \rightarrow [0, \infty)$ satisfy

- (i) $d(x, y) = 0$ if and only if $x = y$,
- (ii) for any sequence $\{x_n\}$ in X ,

$$\sum_{n=1}^{\infty} d(x_n, x_{n+1}) < \infty \Rightarrow \{x_n\} \text{ is convergent in } (X, \tau).$$

Then the triplet (X, τ, d) is called a d -complete topological space.

For details on d -complete topological spaces, we refer to Iseki [9] and Kasahara [10, 11, 12]. Hicks [5] and Hicks and Rhoades [6, 7] proved several fixed point theorems in d -complete topological spaces. Hicks and Saliga [8] and Saliga [18] obtained fixed point theorems for non-self maps in d -complete topological spaces.

In 2006, Bhaskar and Lakshmikantham [2] introduced the notion of a coupled fixed point in partially ordered metric spaces, also discussed some problems of

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the uniqueness of a coupled fixed point and applied their results to the problems of the existence and uniqueness of a solution for the periodic boundary value problems.

Later several authors proved coupled fixed and common coupled fixed point theorems in partial ordered metric spaces, partially ordered cone metric spaces and cone metric spaces for two maps (see e.g. [1, 3, 4],[13]-[17],[19]-[22].)

In this paper, we prove a common coupled fixed point theorem for four mappings in d -complete topological spaces.

Definition 1.2. (See [2]) Let X be a nonempty set. An element $(x, y) \in X \times X$ is called a coupled fixed point of the mapping $F : X \times X \rightarrow X$ if $x = F(x, y)$ and $y = F(y, x)$.

Definition 1.3. (See [1]) Let X be a nonempty set. An element $(x, y) \in X \times X$ is called

- (i) a coupled coincidence point of $F : X \times X \rightarrow X$ and $g : X \rightarrow X$ if $gx = F(x, y)$ and $gy = F(y, x)$.
- (ii) a common coupled fixed point of $F : X \times X \rightarrow X$ and $g : X \rightarrow X$ if $x = gx = F(x, y)$ and $y = gy = F(y, x)$.

Definition 1.4. (See [1]) Let X be a nonempty set. The mappings $F : X \times X \rightarrow X$ and $g : X \rightarrow X$ are called weakly-compatible if $g(F(x, y)) = F(gx, gy)$ whenever $gx = F(x, y)$ and $gy = F(y, x)$ for some $(x, y) \in X \times X$.

2. AN IMPLICIT RELATION

Let Φ_7 be the family of all continuous mappings $\phi : \mathbb{R}_+^7 \rightarrow \mathbb{R}$ satisfying the following conditions :

- (ϕ_1) there exists $0 \leq h_1 < 1$ such that $u, v, w, p \geq 0$ with
 - (ϕ_a) $\phi(u, v, w, u, v, p, 0) \leq 0$ or,
 - (ϕ_b) $\phi(u, v, w, v, u, 0, p) \leq 0$
 implies $u \leq h_1 \max\{v, w\}$,
- (ϕ_2) there exists $0 \leq h_2 < 1$ such that $u, v \geq 0$ with

$$\phi(u, u, v, 0, 0, u, u) \leq 0 \Rightarrow u \leq h_2 v.$$

Example 2.1.

$$\phi(t_1, t_2, t_3, t_4, t_5, t_6, t_7) = t_1 - \frac{\alpha}{2}(t_2 + t_3) - \beta(t_4 + t_5),$$

where $\alpha, \beta \geq 0$ with $\alpha + 2\beta < 1$.

If $u = 0$, then (ϕ_1) and (ϕ_2) are satisfied. Assume that $u > 0$.

$$\begin{aligned} \phi(u, v, w, u, v, p, 0) \leq 0 &\Rightarrow u - \frac{\alpha}{2}(v + w) - \beta(v + u) \leq 0 \\ &\Rightarrow u \leq \frac{\alpha}{2}(v + w) + \beta(v + u) \\ &\Rightarrow (1 - \beta)u \leq \alpha \max\{v, w\} + \beta \max\{v, w\} \\ &\Rightarrow u \leq h_1 \max\{v, w\}, \end{aligned}$$

where $h_1 = \frac{\alpha + \beta}{\alpha - \beta} < 1$.

Similarly $\phi(u, v, w, v, u, 0, p) \leq 0 \Rightarrow u \leq h_1 \max\{v, w\}$. $\phi(u, u, v, 0, 0, u, u) \leq 0 \Rightarrow u \leq h_2 v$, where $h_2 = \frac{\alpha/2}{1 - \alpha/2} < 1$.

Example 2.2.

$$\phi(t_1, t_2, t_3, t_4, t_5, t_6, t_7) = t_1 - \alpha \max\{t_2, t_3, t_4, t_5\} - \beta \min\{t_6, t_7\},$$

where $\alpha, \beta \geq 0$ with $\alpha + \beta < 1$.

Example 2.3.

$$\phi(t_1, t_2, t_3, t_4, t_5, t_6, t_7) = t_1 - \frac{\alpha}{2}(t_2 + t_3) - L \min\{t_4, t_5, t_6, t_7\},$$

where $0 \leq \alpha < 1$ and $L \geq 0$.

Example 2.4.

$$\phi(t_1, t_2, t_3, t_4, t_5, t_6, t_7) = t_1 - \alpha t_2 - \beta t_3 - L \min\{t_4, t_5, t_6, t_7\},$$

where $\alpha, \beta \geq 0$ such that $\alpha + \beta < 1$ and $L \geq 0$.

Example 2.5.

$$\phi(t_1, t_2, t_3, t_4, t_5, t_6, t_7) = t_1^2 - \frac{\alpha}{2}(t_2^2 + t_3^2) - \beta t_4 t_5 - \gamma t_6 t_7,$$

where $\alpha, \beta, \gamma \geq 0$ with $\alpha + \beta < 1$ and $\alpha + \gamma < 1$.

Example 2.6.

$$\phi(t_1, t_2, t_3, t_4, t_5, t_6, t_7) = t_1^2 - \frac{\alpha}{2}(t_2^2 + t_3^2) - \beta \frac{t_4^2 + t_5^2}{t_6 + t_7 + 1},$$

where $\alpha, \beta \geq 0$ with $\alpha + 2\beta < 1$.

3. MAIN RESULTS

Theorem 3.1. . Let (X, τ) be a Hausdorff topological space. Let $F, G : X \times X \rightarrow X$ and $f, g : X \rightarrow X$ be mappings satisfying

$$\phi \left(\begin{array}{c} d(F(x, y), G(u, v)), d(fx, gu), d(gv, fy), d(F(x, y), fx), \\ d(G(u, v), gu), d(F(x, y), gu), d(G(u, v), fx) \end{array} \right) \leq 0 \quad (3.1)$$

for all $x, y, u, v \in X$, where $\phi \in \Phi_7$,

- (a) $F(X \times X) \subseteq g(X), G(X \times X) \subseteq f(X)$,
- (b) one of $(f(X), \tau, d)$ and $(g(X), \tau, d)$ is d -complete,
- (c) the pairs (F, f) and (G, g) are weakly compatible,
- (d) $d(x, y) = d(y, x)$ for all $x, y \in X$ and
- (e) for each $y \in X, d(x_n, y) \rightarrow d(x, y)$, whenever $\{x_n\} \subseteq X, x \in X$ such that $x_n \rightarrow x$.

Then there exists a unique $(\alpha, \beta) \in X \times X$ such that $f\alpha = g\alpha = \alpha = F(\alpha, \beta) = G(\alpha, \beta)$ and $f\beta = g\beta = \beta = F(\beta, \alpha) = G(\beta, \alpha)$.

Proof. Let x_0 and y_0 be in X . Since $F(X \times X) \subseteq g(X)$, we can choose $x_1, y_1 \in X$ such that

$$gx_1 = F(x_0, y_0) \text{ and } gy_1 = F(y_0, x_0).$$

Since $G(X \times X) \subseteq f(X)$, we can choose $x_2, y_2 \in X$ such that

$$fx_2 = G(x_1, y_1) \text{ and } fy_2 = G(y_1, x_1).$$

Continuing this process, we can construct the sequences $\{x_n\}$ and $\{y_n\}$ in X such that

$$\begin{aligned} gx_{2n+1} &= F(x_{2n}, y_{2n}) = z_{2n+1}, \\ gy_{2n+1} &= F(y_{2n}, x_{2n}) = p_{2n+1}, \\ fx_{2n+2} &= G(x_{2n+1}, y_{2n+1}) = z_{2n+2}, \\ fy_{2n+2} &= G(y_{2n+1}, x_{2n+1}) = p_{2n+2}. \end{aligned}$$

Putting $x = x_{2n}, y = y_{2n}, u = x_{2n+1}, v = y_{2n+1}$ in (3.1), we get

$$\phi \left(\begin{array}{c} d(z_{2n+1}, z_{2n+2}), d(z_{2n}, z_{2n+1}), d(p_{2n+1}, p_{2n}), d(z_{2n+1}, z_{2n}), \\ d(z_{2n+2}, z_{2n+1}), 0, d(z_{2n+2}, z_{2n}) \end{array} \right) \leq 0$$

From (ϕ_b) , we have

$$d(z_{2n+1}, z_{2n+2}) \leq h_1 \max\{d(z_{2n}, z_{2n+1}), d(p_{2n+1}, p_{2n})\}$$

where $0 \leq h_1 < 1$. Also putting $x = y_{2n}, y = x_{2n}, u = y_{2n+1}, v = x_{2n+1}$ in (3.1) and using (ϕ_b) , we get

$$d(p_{2n+1}, p_{2n+2}) \leq h_1 \max\{d(z_{2n}, z_{2n+1}), d(p_{2n+1}, p_{2n})\}.$$

Thus

$$\max\{d(z_{2n+1}, z_{2n+2}), d(p_{2n+1}, p_{2n+2})\} \leq h_1 \max\{d(z_{2n}, z_{2n+1}), d(p_{2n+1}, p_{2n})\}.$$

Similarly, using (3.1) and (ϕ_a) , we can show that

$$\max\{d(z_{2n}, z_{2n+1}), d(p_{2n+1}, p_{2n})\} \leq h_1 \max\{d(z_{2n}, z_{2n-1}), d(p_{2n-1}, p_{2n})\}.$$

Hence

$$\max\{d(z_n, z_{n+1}), d(p_n, p_{n+1})\} \leq h_1 \max\{d(z_{n-1}, z_n), d(p_{n-1}, p_n)\}.$$

Inductively we have

$$\max\{d(z_n, z_{n+1}), d(p_n, p_{n+1})\} \leq h_1^n \max\{d(z_0, z_1), d(p_0, p_1)\}.$$

Since $\sum_{n=1}^{\infty} h_1^n$ is convergent, it follows that $\sum_{n=1}^{\infty} d(z_n, z_{n+1})$ and $\sum_{n=1}^{\infty} d(p_n, p_{n+1})$ are convergent. Hence $d(z_n, z_{n+1}) \rightarrow 0$ and $d(p_n, p_{n+1}) \rightarrow 0$ as $n \rightarrow \infty$.

Suppose $(f(X), \tau, d)$ is d -complete. Then $\{z_{2n+2}\} = \{fx_{2n+2}\} \subseteq f(X)$ and $\{p_{2n+2}\} = \{fy_{2n+2}\} \subseteq f(X)$ converge to α and β respectively for some $\alpha, \beta \in f(X)$. Hence there exist x and y in X such that $\alpha = fx$ and $\beta = fy$. Also the subsequences $\{z_{2n+1}\}$ and $\{p_{2n+1}\}$ converge to α and β , respectively.

Putting $x = x, y = y, u = x_{2n+1}, v = y_{2n+1}$ in (3.1), we get

$$\phi \left(\begin{array}{c} d(F(x, y), z_{2n+2}), d(fx, z_{2n+1}), d(p_{2n+1}, fy), d(F(x, y), fx), \\ d(z_{2n+2}, z_{2n+1}), d(F(x, y), z_{2n+1}), d(z_{2n+2}, fx) \end{array} \right) \leq 0$$

Letting $n \rightarrow \infty$, we get

$$\phi(d(F(x, y), fx), 0, 0, d(F(x, y), fx), 0, d(F(x, y), fx), 0) \leq 0.$$

From (ϕ_a) , we have $F(x, y) = fx = \alpha$. Replacing x, y, u, v with y, x, y_{2n+1}, x_{2n+1} respectively in (3.1) and letting $n \rightarrow \infty$ and by using (ϕ_a) , we get

$$F(y, x) = fy = \beta.$$

Since the pair (F, f) is weakly compatible, we have

$$\begin{aligned} f\alpha &= f(F(x, y)) = F(fx, fy) = F(\alpha, \beta) \text{ and} \\ f\beta &= f(F(y, x)) = F(fy, fx) = F(\beta, \alpha). \end{aligned}$$

Putting $x = \alpha, y = \beta, u = x_{2n+1}, v = y_{2n+1}$ in (3.1), we get

$$\phi \left(\begin{array}{l} d(f\alpha, z_{2n+2}), d(f\alpha, z_{2n+1}), d(p_{2n+1}, f\beta), 0, \\ d(z_{2n+2}, z_{2n+1}), d(f\alpha, z_{2n+1}), d(z_{2n+2}, f\alpha) \end{array} \right) \leq 0$$

Letting $n \rightarrow \infty$, we get

$$\phi(d(f\alpha, \alpha), d(f\alpha, \alpha), d(f\beta, \beta), 0, 0, d(f\alpha, \alpha), d(f\alpha, \alpha)) \leq 0.$$

From (ϕ_2) , we have $d(f\alpha, \alpha) \leq h_2 d(f\beta, \beta)$, where $0 \leq h_2 < 1$.

Putting $x = \beta, y = \alpha, u = y_{2n+1}, v = x_{2n+1}$ in (3.1) and letting $n \rightarrow \infty$ and then using (ϕ_2) , we get

$$d(f\beta, \beta) \leq h_2 d(f\alpha, \alpha).$$

Hence $f\alpha = \alpha$ and $f\beta = \beta$. Thus

$$\alpha = f\alpha = F(\alpha, \beta) \text{ and } \beta = f\beta = F(\beta, \alpha). \quad (3.2)$$

Since $F(X \times X) \subseteq g(X)$, there exist $\gamma, \delta \in X$ such that

$$g\gamma = F(\alpha, \beta) = f\alpha = \alpha \text{ and } g\delta = F(\beta, \alpha) = f\beta = \beta.$$

Putting $x = \alpha, y = \beta, u = \gamma, v = \delta$ in (3.1), we get

$$\phi(d(g\gamma, G(\gamma, \delta)), 0, 0, 0, d(G(\gamma, \delta), g\gamma), 0, d(G(\gamma, \delta), g\gamma)) \leq 0.$$

From (ϕ_b) , we have $g\gamma = G(\gamma, \delta)$.

Putting $x = \beta, y = \alpha, u = \delta, v = \gamma$ in (3.1) and using (3.2) and (ϕ_b) , we have $g\delta = G(\delta, \gamma)$. Since the pair (G, g) is weakly compatible, we have

$$\begin{aligned} g\alpha &= g(g\gamma) = g(G(\gamma, \delta)) = G(g\gamma, g\delta) = G(\alpha, \beta) \text{ and} \\ g\beta &= g(g\delta) = g(G(\delta, \gamma)) = G(g\delta, g\gamma) = G(\beta, \alpha). \end{aligned}$$

Putting $x = x_{2n}, y = y_{2n}, u = \alpha, v = \beta$ in (3.1) and letting $n \rightarrow \infty$ and then using (ϕ_2) , we get

$$d(\alpha, g\alpha) \leq h_2 d(g\beta, \beta).$$

Similarly, by putting $x = y_{2n}, y = x_{2n}, u = \beta, v = \alpha$ in (3.1) and letting $n \rightarrow \infty$ and then using (ϕ_2) , we get

$$d(g\beta, \beta) \leq h_2 d(\alpha, g\alpha).$$

Hence $g\alpha = \alpha$ and $g\beta = \beta$.

$$\alpha = g\alpha = G(\alpha, \beta) \text{ and } \beta = g\beta = G(\beta, \alpha) \quad (3.3)$$

From (3.2) and (3.3), we have

$$\begin{aligned} f\alpha &= g\alpha = \alpha = F(\alpha, \beta) = G(\alpha, \beta) \text{ and} \\ f\beta &= g\beta = \beta = F(\beta, \alpha) = G(\beta, \alpha). \end{aligned} \quad (3.4)$$

Suppose there exists $(\alpha_1, \beta_1) \in X \times X$ such that

$$\begin{aligned} f\alpha_1 &= g\alpha_1 = \alpha_1 = F(\alpha_1, \beta_1) = G(\alpha_1, \beta_1) \text{ and} \\ f\beta_1 &= g\beta_1 = \beta_1 = F(\beta_1, \alpha_1) = G(\beta_1, \alpha_1). \end{aligned}$$

By putting $x = \alpha_1, y = \beta_1, u = \alpha, v = \beta$ and $x = \beta_1, y = \alpha_1, u = \beta, v = \alpha$ in (3.1) and using (ϕ_2) , one can show that $\alpha_1 = \alpha$ and $\beta_1 = \beta$. Thus (α, β) is the unique pair satisfying (3.4). \square

The following example illustrates Theorem 3.1.

Example 3.2. Let $X = [0, 1]$ and $d(x, y) = |x^2 - y^2|$ for all $x, y \in X$. Define $F(x, y) = \frac{1}{2} = G(x, y)$ and

$$fx = \begin{cases} \frac{1}{2}, & \text{if } x \in [0, \frac{1}{2}], \\ x, & \text{if } x \in (\frac{1}{2}, 1], \end{cases}, \quad gx = \begin{cases} \frac{1}{2}, & \text{if } x \in [0, \frac{1}{2}], \\ 1, & \text{if } x \in (\frac{1}{2}, 1]. \end{cases}$$

Now, we prove a unique common coupled fixed point theorem for a pair of Jungck type maps without using symmetry of d .

Theorem 3.3. . Let (X, τ) be a Hausdorff topological space. Let $F : X \times X \rightarrow X$ and $f : X \rightarrow X$ be mappings satisfying

$$\phi \left(\begin{array}{c} d(F(x, y), F(u, v)), d(fx, fu), d(fy, fv), d(fu, F(u, v)), \\ d(fx, F(x, y)), d(fx, F(u, v)), d(F(x, y), fu) \end{array} \right) \leq 0 \quad (3.5)$$

for all $x, y, u, v \in X$, where $\phi \in \Phi_7$ without (ϕ_b) ,

- (a) $F(X \times X) \subseteq f(X)$,
- (b) $(f(X), \tau, d)$ is d -complete,
- (c) the pair (F, f) is weakly compatible,
- (d) for each $y \in X, d(x_n, y) \rightarrow d(x, y)$, whenever $\{x_n\} \subseteq X, x \in X$ such that $x_n \rightarrow x$.

Then the mappings F and f have a unique common coupled fixed point.

Proof. Let x_0 and y_0 be in X .

Since $F(X \times X) \subseteq f(X)$, we can choose $x_1, y_1 \in X$ such that

$$fx_1 = F(x_0, y_0) \text{ and } fy_1 = F(y_0, x_0).$$

Continuing this process, we can construct two sequences $\{x_n\}$ and $\{y_n\}$ in X such that (for $n = 0, 1, 2, \dots$)

$$fx_{n+1} = F(x_n, y_n) = z_n \text{ and } fy_{n+1} = F(y_n, x_n) = p_n.$$

Putting $x = x_n, y = y_n, u = x_{n+1}, v = y_{n+1}$ in (3.5), we get

$$\phi \left(\begin{array}{c} d(z_n, z_{n+1}), d(z_{n-1}, z_n), d(p_{n-1}, p_n), d(z_n, z_{n+1}), \\ d(z_{n-1}, z_n), d(z_{n-1}, z_{n+1}), 0 \end{array} \right) \leq 0$$

From (ϕ_a) , there exists $h_1 \in [0, 1)$ such that

$$d(z_n, z_{n+1}) \leq h_1 \max\{d(z_{n-1}, z_n), d(p_{n-1}, p_n)\}.$$

Similarly, by putting $y = x_n, x = y_n, v = x_{n+1}, u = y_{n+1}$ in (3.5) and using (ϕ_a) , we get

$$d(p_n, p_{n+1}) \leq h_1 \max\{d(z_{n-1}, z_n), d(p_{n-1}, p_n)\}.$$

Thus

$$\begin{aligned} \max\{d(z_n, z_{n+1}), d(p_n, p_{n+1})\} &\leq h_1 \max\{d(z_{n-1}, z_n), d(p_{n-1}, p_n)\} \\ &\leq h_1^2 \max\{d(z_{n-2}, z_{n-1}), d(p_{n-2}, p_{n-1})\} \\ &\vdots \\ &\vdots \\ &\leq h_1^n \max\{d(z_0, z_1), d(p_0, p_1)\}. \end{aligned}$$

Since $\sum_{n=1}^{\infty} h_1^n$ is convergent, it follows that $\sum_{n=1}^{\infty} d(z_n, z_{n+1})$ and $\sum_{n=1}^{\infty} d(p_n, p_{n+1})$ are convergent. Hence $d(z_n, z_{n+1}) \rightarrow 0$ and $d(p_n, p_{n+1}) \rightarrow 0$ as $n \rightarrow \infty$.

Suppose $(f(X), \tau, d)$ is d -complete. Then there exist α and β in $f(X)$ such that $\{z_n\}$ and $\{p_n\}$ converge to α and β , respectively. Hence there exist $x, y \in X$ such that $\alpha = fx$ and $\beta = fy$.

Replacing x, y, u, v with x_n, y_n, x, y respectively in (3.5), we get

$$\phi \left(\begin{array}{l} d(z_n, F(x, y)), d(z_{n-1}, fx), d(p_{n-1}, fy), d(fx, F(x, y)), \\ d(z_{n-1}, z_n), d(z_{n-1}, F(x, y)), d(z_n, fx) \end{array} \right) \leq 0$$

Letting $n \rightarrow \infty$ and using (d), we get

$$\phi(d(fx, F(x, y)), 0, 0, d(fx, F(x, y)), 0, d(fx, F(x, y)), 0) \leq 0.$$

Now, from (ϕ_a) , we have $F(x, y) = fx$.

Similarly, replacing x, y, u, v with y_n, x_n, y, x in (3.5) and letting $n \rightarrow \infty$ and using (d), (ϕ_a) , we can show that $F(y, x) = fy$.

Since (F, f) is a weakly compatible pair, we have

$$f\alpha = ffx = f(F(x, y)) = F(fx, fy) = F(\alpha, \beta) \text{ and}$$

$$f\beta = ffy = f(F(y, x)) = F(fy, fx) = F(\beta, \alpha).$$

Putting $x = x_n, y = y_n, u = \alpha, v = \beta$ and $x = y_n, y = x_n, u = \beta, v = \alpha$ in (3.5) and letting $n \rightarrow \infty$ and using (ϕ_2) , we can show that $f\alpha = \alpha$ and $f\beta = \beta$.

Thus

$$\alpha = f\alpha = F(\alpha, \beta) \text{ and } \beta = f\beta = F(\beta, \alpha). \quad (3.6)$$

Using (3.5) and (ϕ_2) , one can show that (α, β) is the unique pair in $X \times X$ satisfying (3.6). \square

The following example illustrates Theorem 3.3.

Example 3.4. Let $X = \{0, 1\}$ and $d(x, y) = |x^2 - y|$ for all $x, y \in X$. Define $F(x, y) = 1$ and $f0 = 0$ and $f1 = 1$.

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