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g^*-CLOSED SETS AND A NEW SEPARATION AXIOM IN ALEXANDROFF SPACES

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Abstract. In this paper we introduce the concept of g^*-closed sets and investigate some of its properties in the spaces considered by A. D. Alexandroff [1] where only countable unions of open sets are required to be open. We also introduce a new separation axiom called T_w-axiom in the Alexandroff spaces with the help of g^*-closed sets and investigate some of its consequences.

1. Introduction

The notion of a topological space can be generalized by requiring only countable unions of open sets to be open (A. D. Alexandroff [1]).

In this paper, starting with an equivalent form of generalised closed sets of Levine [10] as the definition, we obtain, a generalization of closed sets in the Alexandroff spaces [1] which we call g^*-closed sets. We investigate various properties of these sets in Section 3 which shows that g^*-closed sets do not always behave like generalized closed sets. In some of these cases we try to find out the conditions under which their behaviour appear to be same.

Finally in section 5, we use g^*-closed sets to obtain a new separation axiom in the Alexandroff spaces, namely T_w-axiom, which is defined in the same way as Levine defined $T_{1/2}$-axiom in topological spaces [10] and compare it with $T_{1/2}$-axiom (see Note 5). Where needed, results are always substantiated by examples.

2. Preliminaries

Definition 1 ([1]). An Alexandroff space (or σ-space, briefly space) is a set X together with a system $\mathcal{F}$ of subsets satisfying the following axioms

(i) the intersection of a countable number of sets from $\mathcal{F}$ is a set in $\mathcal{F}$.
(ii) The union of a finite number of sets from $\mathcal{F}$ is a set in $\mathcal{F}$.
(iii) ϕ and X are in $\mathcal{F}$.

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Members of $\mathcal{F}$ are called closed sets. Their complementary sets are called open sets. It is clear that instead of closed sets in the definition of an Alexandroff space, one may put open sets with subject to the conditions of countable summability, finite intersectability and the condition that X and ϕ should be open. The collection of all such open sets will sometimes be denoted by τ and the Alexandroff space by (X, τ). When there is no confusion, (X, τ) will simply be denoted by X.

Note 1. In general τ is not a topology as can be easily seen by taking $X = R$ and τ as the collection of all F_σ sets in R.

Throughout the paper by a space we shall always mean an Alexandroff space.

Definition 2 ([1])**.** With every M ⊂ X we associate its closure $\overline{M}$, the intersection of all closed sets containing M.

Note that $\overline{M}$ is not necessarily closed.

Definition 3 ([1])**.** A space (X, τ) is said to be bicompact if every open cover of it has a finite subcover.

Definition 4 ([8])**.** Two sets A, B in X are said to be weakly separated if there are two open sets U, V such that $A \subset U$, $B \subset V$ and $A \cap V = B \cap U = \phi$.

Definition 5 ([1])**.** (X, τ) is said to be a T_0 space if for any two distinct points $\overline{x}$, $\overline{y}$ in X, there exists an open set U which contains one of them but not the other.

Definition 6 ([1])**.** (X, τ) is said to be a T_1 space if for any two distinct points x, y in X, there exist open sets U, V such that $x \in U$, $y \notin U$, $y \in V$ and $x \notin V$.

Definition 7 ([5])**.** (X, τ) is called a regular space if for any $x \in X$ and any closed set F such that $x \notin F$, there exist U, $V \in \tau$ such that $x \in U$, $F \subset V$ and $U \cap V = \phi$.

Definition 8 ([10])**.** A set A in a topological space is said to be generalized closed (g-closed for short) if and only if $\overline{A} \subset U$ whenever $A \subset U$ and U is open.

We shall also make use of the following theorems

Theorem 1 ([5])**.** *(X, τ) is regular if and only if for any $x \in X$ and any open set U containing x, there is an open set V and a closed set F such that*

$$x \in V \subset F \subset U\,.$$

Theorem 2. *(X, τ) is T_0 if and only if $x \neq y$ in X implies $\{\overline{x}\} \neq \{\overline{y}\}$.*

Throughout X stands for a space and unless otherwise stated, sets are always subsets of X. The letters R and Q stand respectively for the set of real numbers and the set of rational numbers.

3. g^*-closed sets in a space

Definition 9 (cf. Definition 2.1 [10])**.** A set A is said to be a g^*-closed set if and only if there is a closed set F containing A such that $F \subset U$ whenever $A \subset U$ and U is open.

Remark 1. Every closed set is g^*-closed but the converse is not true as shown by the following example.

Example 1. Let $X = R - Q$ and $\tau = \{X, \phi, G_i\}$ where G_i runs over all countable subsets of $R - Q$. Then (X, τ) is a space but not a topological space. Let B be the set of all irrational numbers in $(0, \infty)$. Then B is not closed but B is g^*-closed, since X is the only open and closed set containing B.

Theorem 3 (cf. Theorem 2.2 [10])**.** *A set A is g^*-closed if and only if there is a closed set F containing A such that $F - A$ does not contain any non-void closed set.*

Proof. Let A be g^*-closed. Then there is a closed set F containing A such that $F \subset U$ whenever $A \subset U$ and U is open. Assume $F_1 \subset F - A$ and F_1 is closed. Since F_1^c is open and $A \subset F_1^c$ where c denotes the complement operator, it follows that $F \subset F_1^c$ i.e. $F_1 \subset F^c$ and so $F_1 \subset F \cap F^c = \phi$. Hence the condition is necessary.

Conversely suppose that the given condition is satisfied. Let $A \subset U$ and U be open. If $F \not\subset U$, then $F \cap U^c$ is a non-void closed set contained in $F - A$, a contradiction. So A is g^*-closed. □

Corollary 1. *A g^*-closed set A is closed if and only if both $\overline{A}$ and $\overline{A} - A$ are closed.*

Proof. If A is both closed and g^*-closed then evidently $\overline{A} = A$ and $\overline{A} - A = \phi$ are closed.

Conversely let A be a g^*-closed set such that both $\overline{A}$ and $\overline{A} - A$ are closed. Since A is g^*-closed, by Theorem 3 there is a closed set F containing A such that $F - A$ does not contain any non-void closed set. Now since $\overline{A} - A$ is closed and $\overline{A} - A \subset F - A$, $\overline{A} - A = \phi$ i.e. $A = \overline{A}$ and so A is closed. □

Theorem 4. *A set A is g^*-closed if and only if there is a closed set F containing A such that $F \subset \ker(A) = \bigcap\{U; U$ is open and $U \supset A\}$.*

The proof is omitted.

Theorem 5. *Union of two g^*-closed sets is g^*-closed.*

The proof is omitted.

Note 2. Intersection of two g^*-closed sets is not necessarily g^*-closed as can be seen from Example 2.5 [10].

Theorem 6. *If A is g^*-closed and $A \subset B \subset \overline{A}$, then B is g^*-closed.*

Proof. Let $B \subset U$ and U is open. Then $A \subset U$. Since A is g^*-closed, there is a closed set F containing A such that $F \subset U$. Now $F \supset \overline{A} \supset B$ and this shows that B is also g^*-closed. □

The following theorem is an improvement of Theorem 2.6 [10].

Theorem 7. *Let $B \subset A$ where A is open and g^*-closed. Then B is g^*-closed relative to A if and only if B is g^*-closed.*

Proof. Since A is g^*-closed, there is a closed set F containing A such that $F \subset U$ whenever $A \subset U$. Now since $A \subset A$ and A is open, $F \subset A$, i.e. $A = F$ and so A is closed.

Now let B be g^*-closed. Then there is a closed set F_1 witnessing the g^*-closeness of B. Now since A is open and $B \subset A$, $F_1 \subset A$. Also if $B \subset U'$, U' is open in A, then U' is open in X and so $F_1 \subset U'$. This shows that B is g^*-closed in A.

Conversely let B be g^*-closed in A. Then there is a closed set F_2 in A witnessing the g^*-closeness of B in A. Since A is closed, F_2 is closed in X. Further if $B \subset U_1$, U_1 open in X, then $B \subset U_1 \cap A$ where $U_1 \cap A$ is open in A and so $F_2 \subset U_1 \cap A \subset U_1$. This completes the proof of the theorem. □

Note 3. An open g^*-closed set is closed.

Corollary 2. *Let A be g^*-closed and open. Then $A \cap B$ is g^*-closed if B is g^*-closed.*

Theorem 8. *If every subset of X is g^*-closed then $\tau = c(\tau)$ where $c(\tau)$ is the collection of all closed sets in (X, τ).*

The proof follows from Note 3.

Remark 2. The converse of Theorem 8 is true for g-closed sets (recall Definition 8) in a topological space as shown by Theorem 2.10 in [10]. But this may not be true for g^*-closed set as shown by:

Example 2. Let $X = R - Q$ and $\tau = \{X, \phi, G_i, A_i\}$ where G_i and A_i runs over all countable and co-countable subsets of X respectively. Then (X, τ) is a space but not a topological space where $\tau = c(\tau)$. But the set B (say) of all irrational numbers in $(0, \infty)$ is clearly not closed and so is not g^*-closed since $\ker(B) = B$.

However we have the following theorem.

Theorem 9. *In a space (X, τ) with $\tau = c(\tau)$, every subset of X is g^*-closed if and only if X is a topological space.*

Proof. If X is a topological space, then the proof is evident in view of Remark 2.

Conversely let every subset of X be g^*-closed. Let $\{F_i\}$ be an arbitrary collection of closed sets and $F = \cap F_i$. Now by the given condition F is g^*-closed. Then there is a closed set F' containing F such that $F' \subset \ker(F)$ (by Theorem 4). Now since each $F_i \in c(\tau) = \tau$ and $F \subset F_i$, $F' \subset \ker(F) \subset \bigcap F_i = F$, i.e. $F = F'$ and so F is closed. This shows that X is a topological space. □

Remark 3. Levine showed that in a compact topological space, g-closed sets are compact (Theorem 3.1 [10]). But this is not true for g^*-closed sets in a space as shown by:

Example 3. Let $X = R - Q$ and $\tau = \{X, \phi, G_i\}$ where G_i runs over all countable subsets of $X - \{\sqrt{2}\}$. Then (X, τ) is a space but not a topological space. X is clearly bicompact. Take $B =$ the set of all irrational numbers in $(0, 1)$. Then B is g^*-closed, since X is the only open set containing B. But B is not bicompact.

Remark 4. Levine also showed that in a regular topological space compact sets are g-closed (Theorem 3.5 [10]). But in a regular space bicompact sets are not necessarily g^*-closed as shown by:

Example 4. Let $X = R - Q$ and $\tau = \{\phi, X, G_i, A_i\}$ where G_i runs over all countable subsets of $X - \{\sqrt{2}\}$ and A_i runs over all cofinite subsets of X containing $\sqrt{2}$. Then (X, τ) is a space but not a topological space. We first show that X is regular. Let $\alpha \in X$ and F be a closed set such that $\alpha \notin F$. If $\alpha \neq \sqrt{2}$, then $\{\alpha\}$ and $X - \{\alpha\}$ are the two disjoint open sets containing α and F respectively. If $\alpha = \sqrt{2}$, then F must be of the form $X - A_i = \{\alpha_1, \alpha_2, \ldots, \alpha_n\}$ (say), $\alpha_k \neq \sqrt{2}$, for $k = 1$ to n. Clearly then F and $V = X - F$ are the two disjoint open sets containing F and α respectively. This shows that X is regular.

Now let B be the set of all irrational numbers in $(1, 2)$. Since $\sqrt{2} \in B$, B is bicompact. Clearly B is not closed. Then B is not g^*-closed, since here $\ker(B) \subset \bigcap\{X - \{\nu\}\nu \notin B\} = B$.

However the following theorem shows that the result of Remark 4 holds in a space under some additional supposition.

The following definition will be needed in Theorem 10.

Definition 10. A set A is called g^*-open if and only if A^c is g^*-closed.

Theorem 10. *In a regular space X, bicompact subsets are g^*-closed if and only if arbitrary union of open sets whose complement is the closure of a bicompact set is g^*-open.*

Proof. First let every bicompact subset of X be g^*-closed. Let $\{U_i\}$ be an arbitrary collection of open sets and $U = \bigcup U_i$ be such that $X - U = \overline{A}$ (say) where A is a bicompact set. Now by our hypothesis A is g^*-closed in X and so is then $\overline{A}$ (by Theorem 6). Hence U is g^*-open and the condition is necessary.

Conversely let the given condition hold. Let A be bicompact. Let $A \subset U$ where U is open. Let $x \in A$. Then $x \in U$. Since X is regular, by Theorem 1, there is an open set V_x and a closed set F_x such that

$$x \in V_x \subset F_x \subset U\,.$$

Now $\{V_x; x \in A\}$ form an open cover of A. Since A is bicompact, there are finite number of points $x_1, \ldots, x_n \in A$ such that

$$A \subset \bigcup_{i=1}^{n} V_{x_i} \subset \bigcup_{i=1}^{n} F_{x_i} \subset U$$

and so $A \subset \overline{A} \subset \bigcup_{i=1}^{n} F_{x_i} \subset U$. Now $X - \overline{A} = \bigcup\{X - F;$ F is a closed set containing $A\}$ whose complement $\overline{A}$ is the closure of a bicompact set A. Then by the given condition $X - \overline{A}$ is g^*-open and so $\overline{A}$ is g^*-closed. Hence there is a closed set F witnessing the g^*-closeness of $\overline{A}$. Now clearly $A \subset \overline{A} \subset F \subset U$. Since this is true for all open sets U containing A, A is g^*-closed. $\square$

Remark 5. The following example shows that the condition of Theorem 10 is strictly weaker than the requirement of (X, τ) to be a topological space.

Example 5. Let $X = R - Q$ and $\tau = \{\phi, X, G_i, A_i\}$ where G_i and A_i respectively runs over all countable and cofinite subsets of X. Then X is a space but not a topological space where for any set $A, \overline{A} = A$ and a set is bicompact only if it is finite. Hence the union of an arbitrary number of open sets whose complement is the closure of a bicompact set must be a cofinite set which is open and so g^*-open.

We shall make use of the following theorem in Section 5.

Theorem 11. *For each $x \in X, \{x\}$ is closed or its complement $\{x\}^c$ is g^*-closed.*

The proof is straight forward and so is omitted.

4. g^*-open sets in a space

Theorem 12 (cf. Theorem 4.2 [10])**.** *A set A is g^*-open if and only if there is an open set U contained in A such that $F \subset U$ whenever F is closed and $F \subset A$.*

The proof is omitted.

Theorem 13 (cf. Theorem 4.5 [10])**.** *A is g^*-open if and only if there is an open set $V \subset A$ such that $V \cup A^c \subset U$ and U is open implies $U = X$.*

Proof. First suppose that A is g^*-open. Then there is an open set $V \subset A$ satisfying the properties of Theorem 12. Now let U be open and $V \cup A^c \subset U$. Then $U^c \subset V^c \cap A$. Since U^c is closed and $U^c \subset A$, $U^c \subset V$. Hence $U^c \subset V \cap V^c = \phi$, i.e. $U = X$.

Conversely let there be an open set $V \subset A$ such that $V \cup A^c \subset U$ and U is open implies $U = X$. Let F be a closed set contained in A. Now $V \cup A^c \subset V \cup F^c$ which is clearly open and so by the given condition $V \cup F^c = X$ which implies $F \subset V$. This proves the theorem. □

Theorem 14 (cf. Theorem 4.9 [10])**.** *If $\overline{A}$ is closed then A is g^*-closed if and only if $\overline{A} - A$ is g^*-open.*

The proof of the theorem is parallel to Theorem 4.9 [10] and so is omitted.

Note 4. The requirement that $\overline{A}$ be closed in the preceeding theorem is essential as can be seen by taking $A = \{\sqrt{2}\}$ in the space of Example 1.

Theorem 15. *Union of two weakly separated g^*-open sets is g^*-open.*

Proof. Let A_1 and A_2 be two weakly separated g^*-open sets. Since A_1 and A_2 are weakly separated, there are open sets U_1, U_2 such that

$$A_1 \subset U_1\,, \quad A_2 \subset U_2\,, \quad U_1 \cap A_2 = U_2 \cap A_1 = \phi\,.$$

Let $F_i = U_i^c$ for $i = 1, 2$. Then F_i are closed and $A_1 \subset F_2, A_2 \subset F_1$. Again since A_1, A_2 are g^*-open there are open sets V_1, V_2 witnessing the g^*-openness of A_1,

A_2 respectively. Clearly $V_1 \cup V_2$ is open and $V_1 \cup V_2 \subset A_1 \cup A_2$. Let $F \subset A_1 \cup A_2$ and F be closed. Now

$$\begin{aligned} F &= F \cap (A_1 \cup A_2) = (F \cap A_1) \cup (F \cap A_2) \\ &\subset (F \cap F_2) \cup (F \cap F_1) \end{aligned}$$

where $F \cap F_i$ is closed for $i = 1, 2$. Further $F \cap F_1 \subset (A_1 \cup A_2) \cap F_1 \subset A_2$ and so $F \cap F_1 \subset V_2$. Similarly $F \cap F_2 \subset V_1$ and hence $F \subset V_1 \cup V_2$ and so by Theorem 12, $A_1 \cup A_2$ is g^*-open. □

5. T_w-SPACES

Definition 11. A space X is called a T_w-space if and only if every g^*-closed set is closed in X.

Note 5. Levine [10] defined a topological space to be a $T_{1/2}$ space if and only if every g-closed set is closed and showed that it is properly placed between T_0 and T_1 axioms (Corollary 5.6 [10]). But T_w-axiom in a space does not have this property (see Example 6).

Theorem 16. *Every T_w-space is T_0.*

Proof. If possible let (X, τ) be a T_w-space which is not T_0. Then by Theorem 2, there exist x, $y \in X$ such that $x \neq y$ but $\overline{\{x\}} = \overline{\{y\}}$. Let $A = \{x\}^c$. We first note that $\{x\}$ is not closed for otherwise $\overline{\{x\}} = \{x\} \neq \overline{\{y\}}$. Then by Theorem 11, A is g^*-closed. But A is not closed for otherwise $y \in \{x\}^c = A$ implies $\overline{\{y\}} \subset \{x\}^c$ and so $\overline{\{x\}} \neq \overline{\{y\}}$, which is a contradiction. This shows that (X, τ) is T_0. □

Example 6. The space of Example 1 is T_2 and so T_1 and also T_0 space. But it is not T_w.

We now recall the following example.

Example 7 ([10])**.** Let $X = \{a, b\}$ and let $\tau = \{\phi, X, \{a\}\}$. Then (X, τ) is a $T_{1/2}$ topological space and so a T_w-space which is not T_1.

Remark 6. Examples 6 and 7 show that the T_w and T_1 axioms in a space are independent of each other.

Remark 7. Dunham [6] showed that for a topological space Y the following are equivalent.

a) Y is $T_{1/2}$.
b) For each $x \in Y$, $\{x\}$ is either open or closed.
c) Every subset of Y is the intersection of all open sets and all closed sets containing it.

But for a space to be T_w, though the conditions (b), (c) are necessary, they are not sufficient as can be seen from Example 1.

Next we find some necessary sufficient conditions for a space to be T$_w$. For this we introduce the following definition.

Definition 12 (cf. Definition 3.2 [7])**.** For any $E \subset X$, let $\overline{E}^* = \cap\{A; E \subset A$ and A is g^*-closed in $X\}$. $\overline{E}^*$ is called the g^*-closure of the set E.

Now we have the following theorems.

Theorem 17. *A space X is T_w if and only if*

a) *for each $x \in X$, $\{x\}$ is either open or closed and*

b) *$C = C^*$ where*
$C = \{A; \overline{(X - A)}$ is closed$\}$ and
$C^* = \{A; \overline{(X - A)} $ is g^*-closed$\}$.*

Proof. First let X be a T_w-space.

a) Let $x \in X$. If $\{x\}$ is not closed then by Theorem 11, $\{x\}^c$ is g^*-closed and so is closed. Therefore $\{x\}$ is open.

b) $A \in C \Rightarrow \overline{(X - A)}$ is closed $\Rightarrow \overline{(X - A)}$ is g^*-closed $\Rightarrow \overline{(X - A)}* = \overline{(X - A)}$ is g^*-closed (since X is T_w) $\Rightarrow A \in C^*$.

Similarly we can show that $A \in C^* \Rightarrow A \in C$. Hence $C = C^*$.

Conversely let the conditions (a) and (b) hold. Let A be any g^*-closed set in X. Then $\overline{A}^* = A$ is g^*-closed and so $A^c \in C^*$. Since by (b) $C^* = C$, $A^c \in C$ i.e. $\overline{A}$ is closed. We shall show that $\overline{A} = A$. If not, then there exists a $x \in \overline{A} - A$. Since A is g^*-closed, by Theorem 3, $\{x\}$ can not be closed. Then by (a), $\{x\}$ is open and so $\{x\}^c$ is closed. But $x \notin A$ implies $A \subset \{x\}^c$ which again implies $\overline{A} \subset \{x\}^c$, a contradiction. Therefore $\overline{A} = A$ i.e. A is closed and this proves the theorem. □

Theorem 18. *A space X is T_w if and only if*

a) *every subset of X is the intersection of all open sets and all closed sets containing it.*

b) *$C = C^*$ where C and C^* are as in Theorem 17.*

The proof follows from Theorem 17 in view of the fact that the conditions (a) in Theorems 17 and 18 are equivalent (Corollary 2.6 [6]).

Definition 13. A space X is called a door space if and only if each subset of X is either open or closed.

Theorem 19. *A door space X is T_w.*

The proof is omitted.

Definition 14 (cf. Definition 8.1 [10])**.** A space X is called symmetric if and only if for any $x, y \in X$, $x \in \overline{\{y\}} \Rightarrow y \in \overline{\{x\}}$.

Remark 8. As in [10] it can be shown that a T_1-space is symmetric though the converse is not true. But while in a symmetric topological space, T_0, T_1 and $T_{1/2}$ axioms are equivalent, in a symmetric space though T_0 and T_1 axioms are equivalent but a symmetric T_1 space may not be T_w as can be seen from Example 1.

Remark 9. Levine showed that a topological space is symmetric if and only if $\{x\}$ is g-closed for each x in X (Theorem 8.2 [10]). But Example 1 shows that a space may be symmetric without any of $\{x\}$ being g^*-closed.

So we introduce the following definition.

Definition 15. A space X is called strongly symmetric if $\{x\}$ is g^*-closed for each x in X.

Theorem 20. *A strongly symmetric space is symmetric.*

Proof. Let $x \in \overline{\{y\}}$ but $y \notin \overline{\{x\}}$. Then there is a closed set F containing x such that $y \notin F$. Now since $\{y\}$ is g^*-closed and $\{y\} \subset F^c$ which is open, then there is a closed set F' such that $y \in F' \subset F^c$ which implies

$$x \in \overline{\{y\}} \subset F' \subset F^c$$

which is a contradiction. This proves the theorem. □

Corollary 3. *In a strongly symmetric space, T_w axiom implies T_1 axiom.*

The result follows from Theorems 16 and 20 and Remark 8.

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