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ON SECOND ORDER HAMILTONIAN SYSTEMS

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Abstract. The aim of the paper is to announce some recent results concerning Hamiltonian theory. The case of second order Euler–Lagrange form non-affine in the second derivatives is studied. Its related second order Hamiltonian systems and geometrical correspondence between solutions of Hamilton and Euler–Lagrange equations are found.

1. Introduction

The purpose of this paper is to announce some recent result in Hamiltonian field theory. We work within the framework of Krupka's theory of Lagrange stuctures on fibered manifolds [1] and Krupková's Hamiltonian systems (e.g., Lepagean equivalent of Euler–Lagrange form)[3].

In [3] Krupková proposed a concept of a Hamiltonian system, which, contrary to usual approach (c.f. Shadwick [6]), is not related with a single Lagrangian, but rather with an Euler–Lagrange form (i.e., with the class of equivalent Lagrangians, possibly of different orders). Using the concept she formulated a Hamiltonian field theory and studied the corresponding geometric structures [2], [3], [4].

In this paper we are interested in non-affine second order Euler–Lagrange equations which give rise to second order Lepagean equivalents (i.e., Hamiltonian systems). All these Hamiltonian systems have a special stucture of their principal part (i.e., at most 2-contact part). The principal part admits a noninvariant decomposition $\hat{\alpha} = \hat{\alpha}_E + \hat{\alpha}_C$, where $\hat{\alpha}_E$ depends on the Euler–Lagrange form, and $\hat{\alpha}_C$ does not depent on the Euler–Lagrange form. The arising Hamilton equations depend not only on the Euler–Lagrange form, but also on some "free" functions, which correspond to the choice of a concrete Hamiltonian system. A very interesting property of Hamiltonian systems is regularity. In the case studied in this paper Hamiltonian systems cannot be regular. We study a weaker correspondence between solutions of Euler–Lagrange and Hamilton equations. The condition for Hamilton extremals satisfying $\pi_{2,1} \circ \delta = J^1\gamma$ is found. We note that the condition depends on the choice of a Hamiltonian system (i.e, on some "free" functions).

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This consideration is illustrated on an example of "quadratic" Euler–Lagrange equations.

Throughout the paper all manifolds and mappins are smooth and summation convention is used. We consider a fibered manifold (i.e., surjective submersion) $\pi : Y \to X$, dim $X = n$, dim $Y = n + m$, its r-jet prolongation $\pi_r : J^rY \to X$, $r \geq 1$ and canonical jet projections $\pi_{r,k} : J^rY \to J^kY$, $0 \leq k \leq r$ (with an obvious notation $J^0Y = Y$). A fibered chart on Y (resp. associated fibered chart on J^rY) is denoted by (V, ψ), $\psi = (x^i, y^\sigma)$ (resp. (V_r, ψ_r), $\psi_r = (x^i, y^\sigma, y^\sigma_i, \dots, y^\sigma_{i_1\dots i_r})$).

A vector field ξ on J^rY is called π_r*-vertical* if it projects onto the zero vector field on X. A q-form η on J^rY is called π_r*-horizontal* if $i_\xi\eta = 0$ for every π_r-vertical vector field ξ on J^rY.

A q-form η on J^rY is called *contact* if $h\eta = 0$. A contact q-form η on J^rY is called 1*-contact* if for every π_r-vertical vector field ξ on J^rY the $(q-1)$-form $i_\xi\eta$ is horizontal. A contact q-form η on J^rY is called i*-contact* if for every π_r-vertical vector field ξ on J^rY the $(q-1)$-form $i_\xi\eta$ is $(i-1)$-contact.

Recall that every q-form η on J^rY admits a unique (canonical) decomposition into a sum of q-forms on $J^{r+1}Y$ as follows:

$$\pi^*_{r+1,r}\eta = h\eta + \sum_{k=1}^{q} p_k\eta \,,$$

where $h\eta$ is a horizontal form, called the *horizontal part of* η, and $p_k\eta$, $1 \leq k \leq q$, is a k*-contact part of* η (see [1]).

We use the following notations:

$$\omega_0 = dx^1 \wedge dx^2 \wedge \dots \wedge dx^n \,, \quad \omega_i = i_{\partial/\partial x^i}\omega_0 \,, \quad \omega_{ij} = i_{\partial/\partial x^j}\omega_i, \ \dots$$

and

$$\omega^\sigma = dy^\sigma - y^\sigma_j dx^j, \ \dots, \quad \omega^\sigma_{i_1 i_2\dots i_k} = dy^\sigma_{i_1 i_2\dots i_k} - y^\sigma_{i_1 i_2\dots i_k j}dx^j$$

For more details on fibered manifolds and the corresponding geometric stuctures we refer e.g. to [5].

In this section we briefly recall basic concepts on Lepagean equivalents of of Euler–Lagrange forms and generalized Hamiltonian field theory, due to Krupková [2], [3], [4].

By an *r-th order Lagrangian* we shal mean a horizontal n-form λ on J^rY.

A *closed* $(n+1)$-form α is called a *Lepagean equivalent of an Euler–Lagrange form* $E = E_\sigma\omega^\sigma \wedge \omega_0$ if $p_1\alpha = E$.

Recall that the Euler–Lagrange form corresponding to an r-th order Lagrangian $\lambda = L\omega_0$ is the following $(n+1)$-form of order $\leq 2r$

$$E = \Big(\frac{\partial L}{\partial y^\sigma} - \sum_{l=1}^{r}(-1)^l d_{p_1} d_{p_2} \dots d_{p_l} \frac{\partial L}{\partial y^\sigma_{p_1\dots p_l}}\Big)\omega^\sigma \wedge \omega_0 \,. \tag{1}$$

The family of Lepagean equivalents of E is also called a *Lagrangian system*, and denoted by $[\alpha]$. The corresponding Euler–Lagrange equations now take the form

$$J^s\gamma^* i_{J^s\xi}\alpha = 0 \quad \text{for every } \pi - \text{vertical vector field } \xi \text{ on } Y \,, \tag{2}$$

where α is any representative of order s of the class $[\alpha]$. A (single) Lepagean equivalent α of E on J^sY is also called a *Hamiltonian system of order* s and the equations

$$\delta^* i_\xi \alpha = 0 \quad \text{for every } \pi_s-\text{vertical vector field } \xi \text{ on } J^sY \tag{3}$$

are called *Hamilton equations*. They represent equations for integral sections δ (called *Hamilton extremals*) of the *Hamiltonian ideal*, generated by the system $\mathcal{D}^s_\alpha$ of n-forms $i_\xi\alpha$, where ξ runs over π_s-vertical vector fields on J^sY. Also, considering π_{s+1}-vertical vector fields on $J^{s+1}Y$, one has the ideal $\mathcal{D}^{s+1}_{\hat\alpha}$ of n-forms $i_\xi\hat\alpha$ on $J^{s+1}Y$, where $\hat\alpha$ (called *principal part* of α) denotes the at most 2-contact part of α. Its integral sections which moreover annihilate all at least 2-contact forms, are called *Dedecker–Hamilton extremals*. It holds that if γ is an extremal then its s-prolongation (resp. $(s+1)$-prolongation) is a Hamilton (resp. Dedecker–Hamilton) extremal, and (up to a projection) every Dedecker-Hamilton extremal is a Hamilton extremal.

2. Second Order Hamiltonian Systems

We shall consider a second order Euler–Lagrange form which is not affine in the second derivatives, i.e.,

$$\frac{\partial^2 E_\nu}{\partial y^\sigma_{kl}\partial y^\kappa_{pq}} \neq 0\,.$$

As pointed out in [2] the Euler–Lagrange form affine in the second derivatives has first order Hamiltonian systems. In what follows, we shall study second order Hamiltonian systems corresponding to a Lepagean equivalent of such Euler–Lagrange form. The Hamiltonian systems admits a decomposition

$$\pi^*_{3,2}\alpha = \hat\alpha + \mu\,, \tag{4}$$

where $\hat\alpha = p_1\alpha + p_2\alpha$ is the principal part of α, μ is at least 2-contact part of α.

In the following Proposition the stucture of the principal part of α (4) is found.

Proposition 1. *Let* $\dim X \geq 2$. *Let* $E = E_\sigma\, \omega^\sigma \wedge \omega_0$ *be a second order Euler–Lagrange form* (*nontrivially*) *of order* 2, *and* α *its Lepagean equivalent of the form* (4). *Let the form*

$$\begin{aligned}\hat\alpha = E + F = E_\sigma\omega^\sigma \wedge \omega_0 + A^i_{\sigma\nu}\omega^\sigma \wedge \omega^\nu \wedge \omega_i + B^{ki}_{\sigma\nu}\omega^\sigma \wedge \omega^\nu_k \wedge \omega_i \\ + C^{kli}_{\sigma\nu}\omega^\sigma \wedge \omega^\nu_{kl} \wedge \omega_i + D^{kli}_{\sigma\nu}\omega^\sigma_k \wedge \omega^\nu_l \wedge \omega_i\,,\end{aligned} \tag{5}$$

where

$$A^i_{\sigma\nu} = -\,A^i_{\nu\sigma},\ C^{kli}_{\sigma\nu} = C^{lki}_{\sigma\nu},\ D^{kli}_{\sigma\nu} = -D^{lki}_{\nu\sigma}\,, \tag{6}$$

be the principal part of a Lepagean equivalent α (4) *of the Euler–Lagrange form* E. *Then the following conditions are satisfied*

1) $\left(\frac{\partial E_\sigma}{\partial y^\nu} + d_i A^i_{\nu\sigma}\right)_{\mathrm{Alt}(\sigma\nu)} = 0$,

2) *Coefficient conditions:*

$D^{kli}_{\sigma\nu} = \frac{1}{2}C^{kil}_{\sigma\nu} + d^{kli}_{\sigma\nu}$, *where* $d^{kli}_{\sigma\nu}$ *are arbitrary functions satisfying* $d^{kli}_{\sigma\nu} = -d^{lki}_{\sigma\nu}$,

$A^k_{\sigma\nu} = \frac{1}{2}\big(\frac{\partial E_\nu}{\partial y^\sigma_k} - d_i B^{ki}_{\nu\sigma}\big) - a^k_{\sigma\nu}$, *where* $a^k_{\sigma\nu}$ *are arbitrary functions satisfying* $a^k_{\sigma\nu} = a^k_{\nu\sigma}$,
$B^{kl}_{\sigma\nu} = \frac{\partial E_\sigma}{\partial y^\nu_{kl}} - 2\frac{\partial E_\nu}{\partial y^\sigma_{kl}} - 2d_i\left(C^{kli}_{\sigma\nu} - C^{lki}_{\nu\sigma} + C^{kil}_{\sigma\nu} - d^{kli}_{\sigma\nu}\right) + b^{kl}_{\sigma\nu}$, *where* $b^{kl}_{\sigma\nu}$ *are arbitrary functions satisfying* $b^{kl}_{\sigma\nu} = -b^{lk}_{\sigma\nu}$ *and* $b^{kl}_{\sigma\nu} = -b^{lk}_{\nu\sigma}$,
3) *Projectability conditions:*
$C^{kli}_{\sigma\nu}$, $D^{kli}_{\sigma\nu}$ *do not depend on* y^σ_{kl}, $\left(C^{kli}_{\sigma\nu}\right)_{\mathrm{Sym}(kli)} = 0$,
where Alt$(\sigma\nu)$ *means alternation in the indicated idices and* Sym(kli) *means symmetrization in the indicated indices*

Proof. Proof of Proposition 1 follows from the explicit computation of $d\alpha = 0$. □

Note that the above Proposition means that the functions $C^{kli}_{\sigma\nu}$, $D^{kli}_{\sigma\nu}$ do not depend on coefficients of the Euler–Lagrange form and $\hat{\alpha}$ admits a noninvariant decomposition

$$\hat{\alpha} = \hat{\alpha}_E + \hat{\alpha}_C\,, \tag{7}$$

where

$$\begin{aligned}\hat{\alpha}_E = {} & E_\sigma \omega^\sigma \wedge \omega_0 + \Big(\frac{1}{2}\frac{\partial E_\nu}{\partial y^\sigma_i} - \frac{1}{2}d_l\frac{\partial E_\sigma}{\partial y^\nu_{il}} - d_l\frac{\partial E_\nu}{\partial y^\sigma_{il}}\Big)\omega^\sigma \wedge \omega^\nu \wedge \omega_i \\ & + \Big(\frac{\partial E_\sigma}{\partial y^\nu_{ki}} - \frac{\partial E_\nu}{\partial y^\sigma_{ki}}\Big)\omega^\sigma \wedge \omega^\nu_k \wedge \omega_i\end{aligned} \tag{8}$$

depends on derivatives of coefficients of the Euler–Lagrange form and

$$\begin{aligned}\hat{\alpha}_C = {} & \big(-a^i_{\sigma\nu} + d_l d_p(C^{kip}_{\nu\sigma} + C^{kip}_{\sigma\nu} - C^{kpi}_{\sigma\nu} + d^{kip}_{\sigma\nu})\big)\omega^\sigma \wedge \omega^\nu \wedge \omega_i \\ & + \big(b^{kl}_{\sigma\nu} + d_p\ (C^{kip}_{\nu\sigma} + C^{kip}_{\sigma\nu} - C^{kpi}_{\sigma\nu} + d^{kip}_{\sigma\nu})\big)\omega^\sigma \wedge \omega^\nu_k \wedge \omega_i \\ & + C^{kli}_{\sigma\nu}\omega^\sigma \wedge \omega^\nu_{kl} \wedge \omega_i + \Big(\frac{1}{2}C^{kil}_{\sigma\nu} + d^{kli}_{\sigma\nu}\Big)\omega^\sigma_k \wedge \omega^\nu_l \wedge \omega_i\,,\end{aligned} \tag{9}$$

does not depend on the Euler–Lagrange form.

A very interesting property of Hamiltonian systems is regularity. A Hamiltonian system of order s is called *regular* if the ideal $\mathcal{D}^{s+1}_{\hat{\alpha}}$ contains all the n-forms

$$\omega^\sigma \wedge \omega_i\,, \quad \omega^\sigma_{(j_1} \wedge \omega_{i)}, \ldots, \quad \omega^\sigma_{(j_1 \ldots j_{r_0-1}} \wedge \omega_{i)}\,,$$

where $(\ldots)$ means symmetrization in the indicated indices and r_0 is the *minimal order* of Lagrangians corresponding to Euler–Lagrange form, [4]. Regularity can be rewritten as the corespondence $\pi_{s,r_0} \circ \delta_D = J^{r_0}\gamma$, $s \geq r_0$ between Dedecker–Hamilton extremals δ_D and extremals γ.

We study the case $s = 2$ and $r_0 = 2$. Unfortunately, these Hamiltonian systems cannot be regular. In this case regularity is a very strong condition. One can, indeed, study regularity of Hamiltonian systems for such second order Euler–Lagrange forms, however, regular Hamiltonian systems have to be considered to be of order ≥ 3. In the following proposition a correspondence between solutions of

Euler–Lagrangange equations (2) (extremals of λ) and solutions of Hamilton equations (3) (Dedecker–Hamilton and Hamilton extremals) is found which is weaker than regularity.

Proposition 2. *Let* $\dim X \geq 2$. *Let* $E = E_\sigma \omega^\sigma \wedge \omega_0$ *the Euler–Lagrange form (nontrivially) of order* 2*, and* α *of the form* (4), (5), (6) *be its Lepagean equivalent.*

Assume that the matrix $C^{kli}_{\sigma\nu}$ *with* mn^2 *rows (resp.* mn *columns) labelled by* νkl *(resp.* σi*) has rank* mn.

Then every Hamilton–Dedecker extremal $\delta_D : V \supset \pi(U) \to J^2Y$ *of the Hamiltonian system* α *is of the form* $\pi_{2,1} \circ \delta_D = J^1\gamma$*, where* γ *is an extremal of* λ.

If moreover $\mu = 0$ *in* (4) *then every Hamilton extremal* $\delta : V \supset \pi(U) \to J^2Y$ *of the Hamiltonian system* α *is of the form* $\pi_{2,1} \circ \delta = J^1\gamma$*, where* γ *is an extremal of* λ.

Proof. Expressing the generators of the ideal $\mathcal{D}^{s+1}_{\hat\alpha}$ we get

$$
\begin{aligned}
i_{\frac{\partial}{\partial y^\sigma}} \hat\alpha &= E_\sigma \omega_0 + A^i_{\sigma\nu} \omega^\nu \wedge \omega_i + B^{ki}_{\sigma\nu} \omega^\nu_k \wedge \omega_i + C^{kli}_{\sigma\nu} \omega^\nu_{kl} \wedge \omega_i\,,\\
i_{\frac{\partial}{\partial y^\sigma_k}} \hat\alpha &= B^{ki}_{\nu\sigma} \omega^\nu \wedge \omega_i + D^{kli}_{\sigma\nu} \omega^\nu_k \wedge \omega_i\,,\\
i_{\frac{\partial}{\partial y^\sigma_{kl}}} \hat\alpha &= -C^{kli}_{\sigma\nu} \omega^\sigma \wedge \omega_i\,.
\end{aligned}
$$

Since the rank of the matrix $C^{kli}_{\sigma\nu}$ is equal to mn then the $\omega^\sigma \wedge \omega_i$ are generators of the ideal $\mathcal{D}^{s+1}_{\hat\alpha}$. We obtain $\frac{\partial y^\sigma}{\partial x^i} \circ \delta_D = y^\sigma_i \circ \delta_D$, i.e.

$$\pi_{2,1} \circ \delta_D = J^1\gamma\,, \tag{10}$$

where γ is a section of π. Substituting this into (3) we get

$$\delta^*_D i_{\frac{\partial}{\partial y^\sigma}} \hat\alpha = E_\sigma \circ J^2\gamma = 0\,,$$

showing that γ is an extremal of λ.

If moreover $\mu = 0$, then $\pi^*_{3,2}\alpha = \hat\alpha$, we can easily see that $\pi_{2,1} \circ \delta = J^1\gamma$, where γ is an extremal of λ. This completes the proof. $\square$

Note that in general the condition in Proposition 2 does not depend on the Euler–Lagrange form. In the following we shall study the case than the correspondence between extremals and Hamilton extremals depends on the Euler–Lagrange form.

An interesting case.

If the functions $C^{kli}_{\sigma\nu}$ and $D^{kli}_{\sigma\nu}$ in the principal part (5) vanish then the conditions in Propositon 1 take the form

$$
\begin{aligned}
&\Big(\frac{\partial E_\sigma}{\partial y^\nu} + d_i A^i_{\nu\sigma}\Big)_{\mathrm{Alt}(\sigma\nu)} = 0\,,\\
&\qquad A^i_{\sigma\nu} = \frac{1}{2}\Big(\frac{\partial E_\nu}{\partial y^\sigma_i} - d_l\Big(\frac{\partial E_\sigma}{\partial y^\nu_{il}} - 2\frac{\partial E_\nu}{\partial y^\sigma_{il}} + b^{il}_{\sigma\nu}\Big)\Big) - a^i_{\sigma\nu}\,,\\
&\qquad B^{kl}_{\sigma\nu} = \frac{\partial E_\sigma}{\partial y^\nu_{kl}} - 2\frac{\partial E_\nu}{\partial y^\sigma_{kl}} + b^{kl}_{\sigma\nu}\,,
\end{aligned}
$$

In this case the rank condition in Proposition 2 is not satisfied. In the next Proposition a new condition is found which depends on the Euler–Lagrange form and guarantees the correspondence (10) between extremals and Hamilton extremals.

Proposition 3. *Let* $\dim X \geq 2$. *Let* $E = E_\sigma \omega^\sigma \wedge \omega_0$ *the Euler–Lagrange form (nontrivially) of order* 2, *and* α *of the form* (4), (5), (6) *and with* $C^{kli}_{\sigma\nu}$, $D^{kli}_{\sigma\nu}$ *vanishing, be its Lepagean equivalent.*

Assume that the matrix

$$B^{kl}_{\sigma\nu} = \frac{\partial E_\sigma}{\partial y^\nu_{kl}} - 2\frac{\partial E_\nu}{\partial y^\sigma_{kl}} + b^{kl}_{\sigma\nu} \tag{11}$$

with mn *rows (resp.* mn *columns) labelled by* νk *(resp.* σl*) is regular.*

Then every Hamilton–Dedecker extremal $\delta_D : V \supset \pi(U) \to J^2Y$ *of the Hamiltonian system* α *is of the form* $\pi_{2,1} \circ \delta_D = J^1\gamma$, *where* γ *is an extremal of* λ.

If moreover $\mu = 0$ *in* (4) *then every Hamilton extremal* $\delta : V \supset \pi(U) \to J^2Y$ *of the Hamiltonian system* α *is of the form* $\pi_{2,1} \circ \delta = J^1\gamma$, *where* γ *is an extremal of* λ.

Proof. Expressing the generators of the ideal $\mathcal{D}^{s+1}_{\hat{\alpha}}$ we get

$$\begin{aligned} &i_{\frac{\partial}{\partial y^\sigma}}\hat{\alpha} = E_\sigma\omega_0 + A^i_{\sigma\nu}\omega^\nu \wedge \omega_i + B^{ki}_{\sigma\nu}\omega^\nu_k \wedge \omega_i\,, \\ &i_{\frac{\partial}{\partial y^\sigma_k}}\hat{\alpha} = B^{ki}_{\nu\sigma}\omega^\nu \wedge \omega_i\,, \\ &i_{\frac{\partial}{\partial y^\sigma_{kl}}}\hat{\alpha} = 0\,. \end{aligned}$$

Since the rank of the matrix $B^{kl}_{\sigma\nu}$ is equal to mn then the $\omega^\sigma \wedge \omega_i$ are generators of the ideal $\mathcal{D}^{s+1}_{\hat{\alpha}}$. We obtain $\frac{\partial y^\sigma}{\partial x^i} \circ \delta_D = y^\sigma_i \circ \delta_D$, i.e. $\pi_{2,1} \circ \delta_D = J^1\gamma$, where γ is a section of π. Substituting this into (3) we get

$$\delta_D{}^* i_{\frac{\partial}{\partial y^\sigma}}\hat{\alpha} = E_\sigma \circ J^2\gamma = 0\,,$$

showing that γ is an extremal of λ.

If moreover $\mu = 0$, then $\pi^*_{3,2}\alpha = \hat{\alpha}$, we can easily see that $\pi_{2,1} \circ \delta = J^1\gamma$, where γ is an extremal of λ. This completes the proof. □

The above results can be directly applied to a class of "quadratic" Euler–Lagrange equations. Let us consider the following example as an illustration of the above properties of the second order Hamiltonian systems.

Example. Let us consider an Euler–Lagrange form $E = E_\sigma\ \omega^\sigma \wedge \omega_0$ with the coeficients of the form

$$E_\sigma = P_\sigma + Q^{kl}_{\sigma\nu}\ y^\nu_{kl} + R^{klpq}_{\sigma\nu\kappa}\ y^\nu_{kl}\ y^\kappa_{pq}$$

where $P_\sigma = P_\sigma(x^r, y^\beta, y^\beta_r)$, $Q^{rs}_{\sigma\nu} = Q^{kl}_{\sigma\nu}(x^r, y^\beta, y^\beta_r)$ and $R^{klpq}_{\sigma\nu\kappa} = R^{klpq}_{\sigma\nu\kappa}(x^r, y^\beta, y^\beta_r)$ and

$$Q^{kl}_{\sigma\nu} = Q^{lk}_{\sigma\nu},\quad Q^{kl}_{\sigma\nu} = Q^{kl}_{\nu\sigma},\quad R^{klpq}_{\sigma\nu\kappa} = R^{pqkl}_{\sigma\kappa\nu},\quad R^{klpq}_{\sigma\nu\kappa} = R^{klpq}_{\nu\sigma\kappa}\,.$$

In view of the above considerations we take the principal part (5), (6) in the following form: $C^{kli}_{\sigma\nu} = D^{kli}_{\sigma\nu} = 0$ and

$$\begin{aligned}\hat{\alpha} = &\ \big(P_\sigma + Q^{kl}_{\sigma\nu}\, y^\nu_{kl} + R^{klpq}_{\sigma\nu\kappa}\, y^\nu_{kl}\, y^\kappa_{pq}\big)\, \omega^\sigma \wedge \omega_0 \\ &- \Big(a^i_{\sigma\nu} + \frac{3}{2} d_l (Q^{il}_{\sigma\nu} + 2R^{ilpq}_{\sigma\nu\kappa}\, y^\kappa_{pq})\Big)\omega^\sigma \wedge \omega^\nu \wedge \omega_i \\ &+ \big(b^{ki}_{\sigma\nu} - Q^{ki}_{\sigma\nu} - 2R^{kipq}_{\sigma\nu\kappa}\, y^\kappa_{pq}\big)\, \omega^\sigma \wedge \omega^\nu_k \wedge \omega_i\,,\end{aligned}$$

where $a^i_{\sigma\nu}$, $b^{kl}_{\sigma\nu}$ are arbitrary functions satisfying $a^i_{\sigma\nu} = a^i_{\nu\sigma}$, $b^{kl}_{\sigma\nu} = -b^{lk}_{\sigma\nu}$ and $b^{kl}_{\sigma\nu} = -b^{lk}_{\nu\sigma}$.

We can easily see that the forms in the noninvariant decomposition (7) are

$$\begin{aligned}\hat{\alpha}_E = &\ \big(P_\sigma + Q^{kl}_{\sigma\nu}\, y^\nu_{kl} + R^{klpq}_{\sigma\nu\kappa}\, y^\nu_{kl}\, y^\kappa_{pq}\big)\, \omega^\sigma \wedge \omega_0 \\ &- \Big(\frac{3}{2} d_l\ (Q^{il}_{\sigma\nu} + 2R^{ilpq}_{\sigma\nu\kappa}\, y^\kappa_{pq})\Big)\omega^\sigma \wedge \omega^\nu \wedge \omega_i \\ &- \big(Q^{ki}_{\sigma\nu} + 2R^{kipq}_{\sigma\nu\kappa}\, y^\kappa_{pq}\big)\, \omega^\sigma \wedge \omega^\nu_k \wedge \omega_i\end{aligned}$$

and

$$\hat{\alpha}_C = -\, a^i_{\sigma\nu}\omega^\sigma \wedge \omega^\nu \wedge \omega_i + b^{ki}_{\sigma\nu}\omega^\sigma \wedge \omega^\nu_k \wedge \omega_i\,.$$

The regularity condition for the matrix (11) now takes form

$$\det\big(B^{kl}_{\sigma\nu}\big) = \det\big(b^{kl}_{\sigma\nu} - Q^{kl}_{\sigma\nu} - 2R^{klpq}_{\sigma\nu\kappa}\, y^\kappa_{pq}\big) \neq 0\,.$$

Then every Hamilton–Dedecker extremal $\delta_D : V \supset \pi(U) \to J^2Y$ of the Hamiltonian system α is of the form $\pi_{2,1} \circ \delta_D = J^1\gamma$, where γ is an extremal of λ.

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