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INFINITESIMAL CR AUTOMORPHISMS OF HYPERSURFACES OF FINITE TYPE IN $\mathbb{C}^2$

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Abstract. We study the Chern-Moser operator for hypersurfaces of finite type in $\mathbb{C}^2$. Analysing its kernel, we derive explicit results on jet determination for the stability group, and give a description of infinitesimal CR automorphisms of such manifolds.

1. Introduction

Let $M \subset \mathbb{C}^2$ be a smooth real hypersurface and $p \in M$ a point of *finite type* $m \geq 2$ in the sense of Kohn ([1],[4]).

It is shown in [4] that one may choose local coordinates (z, w) vanishing at p such that M is described near p by

$$\text{Im } w = P(z,\bar z) + o(Re\ w, |z|^m)\,, \tag{1.1}$$

where $P(z,\bar z)$ is a nonzero homogeneous polynomial of degree m with *no harmonic* terms.

The model hypersurface M_H associated to M at p is given by

$$M_H = \{(z,w) \in \mathbb{C}^2 \ \mid \text{Im } w = P(z,\bar z)\}\,. \tag{1.2}$$

In [3], P. Ebenfelt, B. Lamel and D. Zaitsev have shown that the local automorphisms fixing p of such hypersurfaces are uniquely determined by their $2-$jets at p.

The following stronger result, characterizing precisely which derivatives are needed when M is given by (1.1), is implicitly contained in the normal form construction of the first author ([5]).

Theorem 1.1. *Let $M \subset \mathbb{C}^2$ be a smooth hypersurface of finite type given by* (1.1) *and let $h\colon M \longrightarrow M$ be a germ at p of a biholomorphism mapping M into itself and fixing p. Then h is uniquely determined by its first order derivatives and its second derivative in the direction w, that is h_{w^2}, at p. Moreover h is uniquely determined by its first order derivatives if $P(z,\bar z) \neq Cz^k\bar z^k$.*

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In this paper we give an explicit direct proof of this fact, based on a new technique recently developed by the authors for hypersurfaces of finite Catlin multitype (cf. [7]). In the same time, we describe all possible infinitesimal CR vector fields which occur as symmetries of Levi degenerate hypersurfaces of finite type.

Note that while the kernel of the generalized Chern-Moser operator is a CR invariant of the manifold, arguably the most important such invariant, there is some deal of arbitrariness in choosing the cokernel and the normal form (cf. [6]). We also note that the technique used below applies automatically to smooth manifolds, while the normal form construction a priori requires the hypersurface to be real analytic.

In the nondegenerate case, when $m = 2$, the above jet determination result follows already from the work of Chern and Moser. Hence we will restrict ourselves to the case $m > 2$.

The paper is organized as follows. In Section 2, we study the generalized Chern-Moser operator, and show how to reduce analysis of the jet determination problem for $\mathrm{Aut}\,(M,p)$, the stability group of M, to the study of $\mathrm{hol}(M_H,p)$, the set of real-analytic infinitesimal CR automorphisms of M_H at p. In Section 3, we introduce the notion of rigid vector fields and prove results regarding the jet determination problem for $\mathrm{hol}\,(M_H,p)$. In Section 4, we use these techniques to give a complete description of $\mathrm{hol}\,(M_H,p)$.

2. The generalized Chern-Moser operator

Let $M \subset \mathbb{C}^2$ be a smooth hypersurface of finite type m given by

$$\mathrm{Im}\, w = \psi(z,\bar z,\mathrm{Re}\, w) = P(z,\bar z) + o(\mathrm{Re}\, w, |z|^m)\,, \tag{2.1}$$

where $P(z,\bar z)$ is a nonzero homogeneous polynomial of degree m with *no harmonic* terms. Recall the following definitions.

Definition 2.1. We denote by $\mathrm{Aut}\,(M,p)$ the stability group of M, that is, those germs at p of biholomorphisms mapping M into itself and fixing p.

Definition 2.2. We denote by $\mathrm{hol}\,(M,p)$ the set of germs of real-analytic infinitesimal CR automorphisms of M at p.

Remark 2.3 ([1])**.** Recall that $X \in \mathrm{hol}\,(M,p)$ if and only if there exists a germ Z at p of a holomorphic vector field in $\mathbb{C}^2$ such that $\mathrm{Re}\, Z$ is tangent to M and $X = \mathrm{Re}\, Z|_M$. By abuse of notation, we also say that $Z \in \mathrm{hol}\,(M,p)$.

Let $h = (z', w') \in \mathrm{Aut}\,(M,p)$. Using (2.1), one can check that h is of the form

$$\begin{aligned} z' &= z + f(z,w) \\ w' &= w + g(z,w)\,, \end{aligned} \tag{2.2}$$

where

$$f(0) = 0\,, \quad g(0) = 0\,, \quad \frac{\partial^k g}{\partial z^k}(0) = 0\,, \quad k = 1,\dots,m\,. \tag{2.3}$$

Assigning the weight $\frac{1}{m}$ to the variable z, and the weight 1 to the variable $w = u+iv$, we define the weighted degree κ of a monomial $q(z,\bar{z},u) = c_{\alpha\beta l} z^\alpha \bar{z}^\beta u^l$, $l \in \mathbb{N}$, to be

$$\kappa := l + \frac{1}{m}(\alpha+\beta)\,.$$

Also we say that a polynomial $Q(z,\bar{z},u)$ is weighted homogeneous of weighted degree κ if it is a sum of monomials of weighted degree κ.

Using (2.3), we observe that f consists of terms of weighted degree greater or equal to $\frac{1}{m}$ while g consists of terms of weighted degree greater or equal to 1.

Decomposing the power series f and g into weighted homogeneous polynomials f_μ and g_μ of weighted degree μ, we may write

$$f = \sum_\mu f_\mu\,, \qquad g = \sum_\mu g_\mu\,.$$

Since $h \in \mathrm{Aut}\,(M,p)$, substituting (2.2) into (2.1), we obtain the transformation formula

$$\begin{aligned}(2.4) \qquad &\psi(z + f(z,u+i\psi(z,\bar{z},u)), \overline{z + f(z,u+i\psi(z,\bar{z},u))}, u \\ &\qquad + \mathrm{Re}\; g(z,u+i\psi(z,\bar{z},u)) = \psi(z,\bar{z},u) + \mathrm{Im}\; g\big(z,u+i\psi(z,\bar{z},u)\big)\,.\end{aligned}$$

By the above, we obtain for $\mu \geq 1$ in (2.4)

$$(2.5) \qquad 2\,\mathrm{Re}\; P_z(z,\bar{z}) f_{\mu-1+\frac{1}{m}}(z,u+iP(z,\bar{z})) = \mathrm{Im}\; g_\mu(z,u+iP(z,\bar{z})) + \dots$$

where dots denote terms depending on $f_{\nu-1+\frac{1}{m}}, g_\nu, \psi_\nu$, for $\nu < \mu$, and $P_z = \frac{\partial P}{\partial z}$.

We now define the generalized Chern-Moser operator.

Definition 2.4. Let F and G be holomorphic functions. The generalized Chern-Moser operator, denoted by L, is defined by

$$(2.6) \qquad L(F,G) = \mathrm{Re}\;\left\{ iG(z,u+iP(z,\bar{z})) + 2\frac{\partial P}{\partial z} F(z,u+iP(z,\bar{z}))\right\}.$$

The following lemma shows the relation between the kernel of L and the infinitesimal CR automorphisms of the model hypersurface given by (1.2). (See also [2] for the analogue in the Levi non degenerate case).

Lemma 2.5. *Let L be given by* (2.6) *and let* (F,G) *be holomorphic functions. Then* (F,G) *lies in the kernel of L if and only if the vector field*

$$Y = F(z,w)\frac{\partial}{\partial z} + G(z,w)\frac{\partial}{\partial w}$$

lies in $\mathrm{hol}\,(M_H,p)$, *where M_H is given by* (1.2).

Proof. Applying Y to $v - P$, we obtain

$$\begin{aligned}(2.7) \quad &\mathrm{Re}\; Y(v-P)_{|M_H} = \\ &-\frac{1}{2}\,\mathrm{Re}\;\Big\{ iG\,(z,u+iP(z,\bar{z})) + 2\frac{\partial P}{\partial z} F\,(z,u+iP(z,\bar{z}))\Big\} = -\frac{1}{2} L(F,G)\,.\end{aligned}$$

The conclusion follows, using the characterization of the set of germs of real-analytic infinitesimal CR automorphisms. $\square$

We have the following theorem which shows how to reduce the weighted jet determination problem from $\operatorname{Aut}(M,p)$ to $\operatorname{hol}(M_H,p)$.

Proposition 2.6. *Let* $h=(z+f,w+g)\in\operatorname{Aut}(M,p)$ *be given by* (2.2). *Let*

$$(f,g)=\sum(f,g)_\mu\,,$$

where

$$(f,g)_\mu=(f_{\mu-1+\frac{1}{m}},g_\mu)\,.$$

Let $\mu_0\ge 1$ *be minimal such that* $(f,g)_{\mu_0}\ne 0$. *Then the (non trivial vector) field*

$$Y=f_{\mu_0-1+\frac{1}{m}}\frac{\partial}{\partial z}+g_{\mu_0}\frac{\partial}{\partial w} \tag{2.8}$$

lies in $\operatorname{hol}(M_H,p)$, *where* M_H *is given by* (1.2).

Proof. Using (2.5) and the definition of μ_0, we obtain that

$$L\big((f,g)_{\mu_0}\big)=0\,.$$

Therefore, using Lemma 2.5,

$$Y=f_{\mu_0-1+\frac{1}{m}}\frac{\partial}{\partial z}+g_{\mu_0}\frac{\partial}{\partial w}$$

belongs to $\operatorname{hol}(M_H,p)$. This achieves the proof of the proposition. □

Definition 2.7. We say that the vector field

$$Y=F(z,w)\frac{\partial}{\partial z}+G(z,w)\frac{\partial}{\partial w}$$

has homogeneous weight μ (≥ -1) if F is a weighted homogeneous polynomial of weighted degree $\mu+\frac{1}{m}$, and G is a homogeneous polynomial of weighted degree $\mu+1$,

Remark 2.8. The weights introduce a natural grading on $\operatorname{hol}(M_H,p)$ in the following sense. Writing $\operatorname{hol}(M_H,p)$ as

$$\operatorname{hol}(M_H,p)=\oplus_\mu G_\mu\,,$$

where G_μ consists of weighted homogeneous vector fields of weight μ, we observe that each weighted homogeneous component $X_\mu\in G_\mu$ of $X\in\operatorname{hol}(M_H,p)$ lies also in $\operatorname{hol}(M_H,p)$. The reason is that $v-P$ is weighted homogeneous.

Gathering all the previous results, we obtain the following theorem.

Proposition 2.9. *Let* $M\subset\mathbb{C}^2$ *be a smooth hypersurface of finite type* m, *given by* (1.1). *Let* M_H *be the model hypersurface given by* (1.2). *Assume that there exists* μ_0 *such that*

$$\operatorname{hol}(M_H,p)=\oplus_{-1\le\mu<\mu_0}G_\mu\,. \tag{2.9}$$

Then any $h=(z+f,w+g)\in\operatorname{Aut}(M,p)$ *given by* (2.2) *such that such that* $(f,g)_\mu=0$ *for all* $\mu<\mu_0$ *is the identity map.*

Proof. Apply Proposition 2.6. □

Remark 2.10. In the light of Proposition 2.9, we see that in order to study the jet determination problem for $\operatorname{Aut}(M,p)$, it is enough to describe $\operatorname{hol}(M_H,p)$.

3. Rigid vector fields

In this section, we describe an important class of vector fields $X \in \mathrm{hol}\,(M_H, p)$, called rigid vector fields. As we will see, they play a crucial role in the study of the jet determination problem for $\mathrm{hol}\,(M_H, p)$.

As before, let $M \subset \mathbb{C}^2$ be a smooth hypersurface of finite type m, given by (1.1). We have the following definition.

Definition 3.1. Let X be a vector field in $\mathrm{hol}\,(M_H, p)$ of the form

$$X = f(z,w)\frac{\partial}{\partial z} + g(z,w)\frac{\partial}{\partial w}\,.$$

We say that X is rigid if f and g are independent of the variable w.

Remark 3.2. Note that if $X \in \mathrm{hol}\,(M_H, p)$ is rigid and of nonnegative homogeneous weight, then $g = 0$.

Remark 3.3. Note that the rigid vector field W of homogeneous weight -1, given by

$$W = \frac{\partial}{\partial w} \tag{3.1}$$

lies in $\mathrm{hol}\,(M_H, p)$.

Definition 3.4. Let E be the vector field of homogeneous weight 0 defined by

$$E = \frac{z}{m}\frac{\partial}{\partial z} + w\frac{\partial}{\partial w}\,. \tag{3.2}$$

E is called the weighted Euler field.

Remark 3.5. Note that E is a non rigid vector field lying in $\mathrm{hol}\,(M_H, p)$.

We have the following result.

Lemma 3.6. *Let $M \subset \mathbb{C}^2$ be a hypersurface of finite type. Then the rigid vector fields $X \in \mathrm{hol}\,(M_H, p)$ of nonnegative homogeneous weight given by*

$$X = f(z)\frac{\partial}{\partial z}$$

are of the form

$$X = cz\frac{\partial}{\partial z}\,, \tag{3.3}$$

where c is a constant.

Proof. We write

$$P(z,\bar z) = \sum_{j=l}^{m-l} A_j z^j \bar z^{m-j}\,, \tag{3.4}$$

where l is the smallest integer such that $A_j \neq 0$.
Assume that we may write X as

$$X = cz^u \frac{\partial}{\partial z}\,, \quad u > 1\,, \tag{3.5}$$

where c is a constant.

Applying Re X to $v - P(z, \bar z)$, and using the assumption, we obtain

$$\sum_{j=l}^{m-l} jcA_j z^{j+u-1} \bar z^{m-j} + \sum_{j=l}^{m-l} j\overline{cA_j} z^{m-j} \bar z^{j+u-1} = 0\,. \tag{3.6}$$

For the coefficient of $z^{m-l+u-1}\bar z^l$ in this equation we get $A_{m-l} = \bar A_l = 0$. This gives a contradiction. Hence $u = 1$, which proves the lemma. □

We have the following lemma.

Lemma 3.7. *Let $X \in \mathrm{hol}\,(M_H, p)$ be a weighted homogeneous vector field, and let $W \in \mathrm{hol}\,(M_H, p)$ be given by* (3.1)*. There exists an integer $l \geq 1$, and a rigid vector field $Y \in \mathrm{hol}\,(M_H, p)$ such that*

$$[\dots[[X; W]; W]; \dots]; W] = Y\,,$$

where the string of brackets is of length l.

Proof. Observe that the effect of taking the bracket of X with W is simply differentiation of the coefficients with respect to w. Also note that

$$(\mathrm{Re}\,[X; W])\big(v - P(z, \bar z)\big) = [\mathrm{Re}\ X, \mathrm{Re}\ W]\big(v - P(z, \bar z)\big)\,.$$

□

Definition 3.8. We say that $X \in \mathrm{hol}\,(M_H, p)$ is an l-integration of a rigid vector $Y \in \mathrm{hol}\,(M_H, p)$ if the string of brackets described in the above lemma is of length l.

Remark 3.9. By the above lemma, the general case will be reduced to the rigid case by taking sufficiently many commutators with the vector field W. The problem reduces then to

(i) describing rigid vector fields

(ii) analysing to what extent rigid fields can be "integrated".

The results of this section provide the first step. We consider the second step in the following section.

4. A complete description of hol (M_H, p)

In this section we use the results obtained in Section 3 for describing completely hol (M_H, p), where $M \subset \mathbb{C}^2$ is of finite type $m > 2$, given by (1.1), that is

$$M = \{(z, w) \in \mathbb{C}^2 \mid \mathrm{Im}\ w = P(z, \bar z) + o(\mathrm{Re}\ w, |z|^m)\}\,. \tag{4.1}$$

Proposition 4.1. *Let $X \in \mathrm{hol}\,(M_H, p)$ be a nonzero rigid vector field of nonnegative weight of the form*

$$X = cz\,\frac{\partial}{\partial z}\,. \tag{4.2}$$

Then there exists no nonzero vector field $Y \in \mathrm{hol}\,(M_H, p)$ *such that*

$$[Y, W] = X\,.$$

Proof. Integrating the coefficients of X given by (4.2) with respect to w, we obtain that Y would have to be of the form

$$Y = wz\frac{\partial}{\partial z} + \phi(z)\frac{\partial}{\partial z} + \psi(z)\frac{\partial}{\partial w}\,.$$

Suppose that there exists such a $Y \in \mathrm{hol}\,(M_H, p)$. We then have

$$\begin{aligned}\mathrm{Re}\; Y(P - v) &= \mathrm{Re}\,\Big(2\frac{\partial P}{\partial z} z(u + iP(z, \bar z)) + 2\frac{\partial P}{\partial z}\phi(z) + i\psi(z)\Big)\\ &= \mathrm{Re}\,\Big(2\frac{\partial P}{\partial z} ziP(z, \bar z) + 2\frac{\partial P}{\partial z}\phi(z)\Big) - \mathrm{Im}\;\psi(z) = 0\,,\end{aligned}$$

where we have used Re $X(P - v) = 0$. The first summand contains only mixed terms, while the second summand is harmonic. It implies that $\psi(z) = 0$. By homogeneity, ϕ has weight $1 + \frac{1}{m}$, hence $\phi(z) = \alpha z^{m+1}$ for some $\alpha \in \mathbb{C}$. Hence

$$-P(z, \bar z)\,\mathrm{Im}\,\Big(\frac{\partial P}{\partial z} z\Big) + \mathrm{Re}\,\Big(\frac{\partial P}{\partial z}\alpha z^{m+1}\Big) = 0\,. \tag{4.3}$$

Since P contains no harmonic terms, the coefficient of $z^{2m+1-l}\bar z^{l-1}$ on the left hand side is $\frac{\alpha}{2}$. Hence $\alpha = 0$ and $X(P) = 0$, which gives a contradiction. □

Proposition 4.2. *Let* $W \in \mathrm{hol}\,(M_H, p)$ *be given by* (3.1)*. There exists no vector field lying in* $\mathrm{hol}\,(M_H, p)$ *that is a* 3*-integration of* W*.*

Proof. By integrating W we obtain a field of the form

$$cz\frac{\partial}{\partial z} + (w + \phi(z))\,\frac{\partial}{\partial w}\,. \tag{4.4}$$

Applying (4.4) to $P - v$ we obtain

$$\mathrm{Re}\,(2cz\frac{\partial P}{\partial z}) - P(z, \bar z) + \mathrm{Im}\;\phi(z) = 0\,. \tag{4.5}$$

Since the first two terms are mixed, we obtain $\phi(z) = 0$.
Therefore the 1-integration of W satisfies

$$\mathrm{Re}\,(2cz\frac{\partial P}{\partial z}) = P(z, \bar z)\,. \tag{4.6}$$

Integrating (4.4), we obtain a field of the form

$$(czw + \psi(z))\,\frac{\partial}{\partial z} + \Big(\frac{1}{2}w^2 + \phi(z)\Big)\frac{\partial}{\partial w}\,. \tag{4.7}$$

Applying (4.7) to $P - v$, we obtain

$$-P(z, \bar z)\,\mathrm{Im}\,\Big(2cz\frac{\partial P}{\partial z}\Big) + \mathrm{Re}\;\frac{\partial P}{\partial z}\psi(z) + \mathrm{Re}\;i\phi(z) = 0\,. \tag{4.8}$$

Since the first two summands contain only mixed terms, we obtain $\phi(z) = 0$. Hence

$$-P(z, \bar z)\,\mathrm{Im}\,\Big(2\frac{\partial P}{\partial z}cz\Big) + 2\,\mathrm{Re}\,\Big(\frac{\partial P}{\partial z}\psi(z)\Big) = 0\,. \tag{4.9}$$

By homogeneity, $\psi(z) = \alpha z^{m+1}$ for some $\alpha \in \mathbb{C}$. Again, from the coefficient of $z^l \bar{z}^{2m-l}$ we obtain $\alpha = 0$, and therefore

$$\operatorname{Im}\, \Big(2\frac{\partial P}{\partial z}\phi(z)\Big) = 0\,, \quad \psi = 0\,, \quad \phi = 0\,. \tag{4.10}$$

Hence the 2-integration of W satisfies

$$2cz\frac{\partial P}{\partial z} = P(z,\bar{z})\,. \tag{4.11}$$

Integrating (4.7), and using (4.11), we obtain a field of the form

$$Y = \Big(\frac{1}{2}w^2cz + \psi(z)\Big)\frac{\partial}{\partial z} + \Big(\frac{1}{6}w^3 + \phi(z)\Big)\frac{\partial}{\partial w}\,. \tag{4.12}$$

Applying (4.12) to $P - v$, and using (4.11), we get (as above $\phi = 0$),

$$\begin{aligned}\operatorname{Re}\, \frac{1}{4}(u^2 - P^2 + 2iuP)P + \operatorname{Re}\, \psi(z)\frac{\partial P}{\partial z}\\ - \frac{1}{12}(3u^2P - P^3) = 0\,.\end{aligned} \tag{4.13}$$

Putting $u = 0$ in (4.13), we obtain

$$-\frac{1}{6}P^3 + \operatorname{Re}\, (\psi(z))\,\frac{\partial P}{\partial z} = 0\,. \tag{4.14}$$

By homogeneity, $\psi(z) = \alpha z^{2m+1}$ for some $\alpha \in \mathbb{C}$. Computing the coefficient of $z^l\bar{z}^{3m-l}$ we obtain $\alpha = 0$, hence $P = 0$, which is a contradiction. Hence, there is no 3-integration of W. This achieves the proof of the proposition. $\square$

Remark 4.3. Using (4.4), (4.6), (4.7) and (4.11) the fields we obtained by integrating W are of the form

$$cz\frac{\partial}{\partial z} + w\frac{\partial}{\partial w} \iff \operatorname{Re}\, (2cz\frac{\partial P}{\partial z}) = P(z,\bar{z}) \tag{4.15}$$

$$czw\frac{\partial}{\partial z} + \frac{1}{2}w^2\frac{\partial}{\partial w} \iff 2cz\frac{\partial P}{\partial z} = P(z,\bar{z})\,. \tag{4.16}$$

Using (3.4), we verify immediately that if $P \neq |z|^m$, 1-integration leads to $c = \frac{1}{2m}$, and if $P = |z|^m$, then it leads to $\operatorname{Re}\, c = \frac{1}{2m}$. Further, 2-integration exists if and only if $P = |z|^m$, with the same value of $c = \frac{1}{2m}$.

Proposition 4.4. *There exists no vector field lying in* $\operatorname{hol}(M_H, p)$ *that is a* $1-$*integration of a nontransversal shift* $\frac{\partial}{\partial z}$.

Proof. If M_H admits such a shift, it is tubular, and in modified coordinates (with harmonic terms allowed) it can be written as $v = x^m$.
Integrating $\frac{\partial}{\partial z}$ we obtain a vector field of the form

$$w\frac{\partial}{\partial z} + bz^m\frac{\partial}{\partial z} + \phi(z)\frac{\partial}{\partial w} \tag{4.17}$$

By homogeneity, applying this to $P - v$, using the hypothesis, we obtain

$$-x^m x^{m-1} + b\operatorname{Re}\, z^{m-1}x^{m-1} + \operatorname{Re}\, \alpha z^{2m-1} = 0\,. \tag{4.18}$$

Applying $\frac{\partial^2}{\partial z \partial \bar{z}}$, we get

$$cx^{2m-3} + d \operatorname{Re} z^{m-1} x^{m-2} = 0 \tag{4.19}$$

for some real constants c and d. Hence Re z^{m-1} is a multiple of x^{m-1}, and therefore $m = 2$. This proves the proposition. $\square$

We may state our main result.

Theorem 4.5. *Let $M \subset \mathbb{C}^2$ be a smooth hypersurface of finite type given by* (1.1) *and let $h \colon M \longrightarrow M$ be a germ at p of a biholomorphism mapping M into itself and fixing p. Then h is uniquely determined by its first order derivatives and its second derivative in the direction w, that is h_{w^2}, at p. Moreover h is uniquely determined by its first order derivatives if $P(z,\bar{z}) \neq Cz^k \bar{z}^k$.*

Proof. For $m = 2$, the result follows from [2]. For $m > 2$, combining Proposition 2.9 with Lemma 3.6, Lemma 3.7, Proposition 4.1, Proposition 4.2, Proposition 4.4 and Remark 4.3, we obtain the first part of the theorem. For the "moreover" part, observe that (4.16) can not occur if $P(z,\bar{z}) \neq Cz^k \bar{z}^k$. This achieves the proof of the theorem. $\square$

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