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WEAK PSEUDO-COMPLEMENTATIONS ON ADL'S

R. Vasu Babu, Ch. Santhi Sundar Raj, and B. Venkateswarlu

Abstract. The notion of an Almost Distributive Lattice (abbreviated as ADL) was introduced by U. M. Swamy and G. C. Rao [6] as a common abstraction of several lattice theoretic and ring theoretic generalization of Boolean algebras and Boolean rings. In this paper, we introduce the concept of weak pseudo-complementation on ADL's and discuss several properties of this.

1. Introduction

O. Frink [2] has proved that, in any pseudo-complemented semi lattice S, the set $S^* = \{a^* \mid a \in S\}$ becomes a Boolean algebra which is a sub semi lattice of S. K. B. Lee [3] has proved that the class of distributive pseudo-complemented lattice is equationally definable and hence a variety (a class which is closed under the formation of subalgebras, homomorphic images and products). Further, U. M. Swamy, G. C. Rao and G. N. Rao [7] have introduced the notion of pseudo-complementation on an Almost Distributive Lattice (ADL) and proved that the class of pseudo-complemented ADL's is also equationally definable. Here, we introduce the concept of weak pseudo-complementation on an ADL and discuss several properties of ADL's with weak pseudo-complementation. In particular, we prove that an ADL is pseudo-complemented if and only if it is weakly pseudo-complemented, even though a weak pseudo-complementation need not be a pseudo-complementation in general.

2. Preliminaries

We first recall certain elementary definitions and results concerning Almost Distributive Lattices. These are collected from [6] and [7].

Definition 2.1. An algebra $A = (A, \wedge, \vee, 0)$ of type $(2, 2, 0)$ is called an Almost Distributive Lattice (abbreviated as ADL) if it satisfies the following identities

(1) $0 \wedge a \approx 0$;
(2) $a \vee 0 \approx a$;
(3) $a \wedge (b \vee c) \approx (a \wedge b) \vee (a \wedge c)$;

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(4) $(a \vee b) \wedge c \approx (a \wedge c) \vee (b \wedge c)$;
(5) $a \vee (b \wedge c) \approx (a \vee b) \wedge (a \vee c)$;
(6) $(a \vee b) \wedge b \approx b$.

Any distributive lattice bounded below is an ADL, where 0 is the smallest element. Also, a commutative regular ring $(R, +, \cdot, 0, 1)$ with unity can be made into an ADL by defining the operations $\wedge$ and $\vee$ on R by

$$a \wedge b = a_0 b \quad \text{and} \quad a \vee b = a + b - a_0 b\,,$$

where, for any $a \in R$, a_0 is the unique idempotent in R such that $aR = a_0 R$ and 0 is the additive identity in R. Further any non empty set X can be made into an ADL by fixing an arbitrarily choosen element 0 in X and by defining the operations $\wedge$ and $\vee$ on by X by

$$a \wedge b = \begin{cases} 0, & \text{if } a = 0 \\ b, & \text{if } a \neq 0 \end{cases} \quad \text{and} \quad a \vee b = \begin{cases} b, & \text{if } a = 0 \\ a, & \text{if } a \neq 0. \end{cases}$$

This ADL $(X, \wedge, \vee, 0)$ is called a discrete ADL. An ADL A is said to be associate ADL if the operation $\vee$ on A is associate. Through out this paper, by an ADL we mean an associate ADL only.

Definition 2.2. Let $A = (A, \wedge, \vee, 0)$ be an ADL. For any a and $b \in A$, define

$$a \leq b \quad \text{if and only if} \quad a = a \wedge b\,, \quad \text{this is equivalent to} \quad a \vee b = b\,.$$

Then $\leq$ is a partial order on A.

Theorem 2.3. *The following hold for any a, b and c in an ADL $A = (A, \wedge, \vee, 0)$.*
(1) $a \wedge 0 = 0 = 0 \wedge a$ *and* $a \vee 0 = a = 0 \vee a$;
(2) $a \wedge a = a = a \vee a$;
(3) $a \wedge b \leq b \leq b \vee a$;
(4) $a \wedge b = a \Leftrightarrow a \vee b = b$;
(5) $a \vee b = a \Leftrightarrow a \wedge b = b$;
(6) $(a \wedge b) \wedge c = a \wedge (b \wedge c)$;
(7) $a \vee (b \vee a) = a \vee b$;
(8) $a \leq b \Rightarrow a \wedge b = a = b \wedge a \ \Leftrightarrow a \vee b = b = b \vee a$;
(9) $(a \wedge b) \wedge c = (b \wedge a) \wedge c$;
(10) $(a \vee b) \wedge c = (b \vee a) \wedge c$;
(11) $a \wedge b = b \wedge a \Leftrightarrow a \vee b = b \vee a$;
(12) $a \wedge b = \inf\{a, b\} \Leftrightarrow a \wedge b = b \wedge a \Leftrightarrow a \vee b = \sup\{a, b\}$.

Definition 2.4. A non empty subset I of an ADL A is said to be an ideal of A if $a \vee b \in I$ for all $a \in I$ and $b \in I$ and $x \wedge a \in I$ for all $x \in I$ and $a \in A$.

It follows as a consequence that $a \wedge x \in I$ for all $x \in I$ and $a \in A$. For any $X \subseteq A$, the smallest ideal of A containing X is called the ideal generated by X and is denoted by $\langle X]$. If $X = \{x\}$, we simply write $\langle x]$ for $\langle \{x\}]$. We have the following for any $X \subseteq A$ and $x \in A$.

$$\langle X] = \Big\{ \big(\bigvee_{i=1}^{n} x_i \big) \wedge a \mid n \geq 0,\ x_i \in X \text{ and } a \in A \Big\}$$

$$\text{and} \qquad \langle\ x] = \langle\ \{x\}] = \{x \wedge a \mid a \in A\} = \{y \in A \mid x \wedge y = y\}\,,$$

$\langle\ x]$ is called the principal ideal generated by x.

3. Weak pseudo-complementations on ADL's

The concept of pseudo-complementation on an ADL was first introduced by U. M. Swamy, G. C. Rao and G. N. Rao [7] and they have proved that the class of pseudo-complemented ADL's is an equationally definable class. Also, for any ADL A in this class, they have exhibited a one-to-one correspondence between maximal elements in A and pseudo-complementations on A. We prove certain important properties of pseudo-complemented ADL's by making a slight modifications of the definition of pseudo-complementations given in [7].

First, let us recall that, for any elements a and b in an ADL A, $a \wedge b = 0 \Leftrightarrow b \wedge a = 0$ (since $a \wedge b \wedge a = b \wedge a$). For any subset S of A, let

$$S^* = \{a \in A \mid a \wedge s = 0 \quad \text{for all} \quad s \in S\}\,.$$

Then S^* is always an ideal of A for all $S \subseteq A$. It can be easily proved that $S^* = \langle\ S]^*$. For any $a \in A$, we have

$$\langle\ a]^* = \{a\}^* = \{x \in A \mid a \wedge x = 0\} = \{x \in A \mid x \wedge a = 0\}\,.$$

Definition 3.1. Let $A = (A, \wedge, \vee, 0)$ be an ADL. A mapping $a \mapsto a^*$ of A into itself is called a weak pseudo-complementation on A if

$$a \wedge b = 0 \Leftrightarrow a^* \wedge b = b$$

for any a and $b \in A$.

The following is a straight forward verification.

Theorem 3.2. *The following are equivalent to each other for any mapping $a \mapsto a^*$ of an ADL A into itself.*

(1) *$a \mapsto a^*$ is a weak pseudo-complementation on A;*
(2) *$\{a\}^* = \langle\ a^*]$ for any $a \in A$;*
(3) *For any $a \in A$, $a \wedge a^* = 0$; and $a \wedge b = 0 \;\Rightarrow\; a^* \wedge b = b$ for any $b \in A$.*

Definition 3.3. An ADL A is said to be weakly pseudo-complemented if there is a weak pseudo-complementation $a \mapsto a^*$ on A.

The following is an immediate consequence of Theorem 3.2 and the axiom of choice.

Corollary 3.4. *An ADL A is weakly pseudo-complemented if and only if $\{a\}^*$ is a principal ideal for any $a \in A$.*

Note that a principal ideal in an ADL may have more than one generators, unlike the case of a lattice in which any principal ideal has a unique generator. However, for any a and b in an ADL, we have

$$\begin{aligned} \langle\, a] = \langle\, b] &\Leftrightarrow a \wedge b = b \quad \text{and} \quad b \wedge a = a \\ &\Leftrightarrow a \vee b = a \quad \text{and} \quad b \vee a = b \end{aligned}$$

and we denote this situation by writing $a \sim b$ and calling a and b as associates to each other. In this context, we have the following.

Theorem 3.5. *Let $a \mapsto a^*$ and $a \mapsto a^+$ be two weak pseudo-complementations on an ADL A. Then the following hold for any a and $b \in A$.*

(1) $a^* \sim a^+$;
(2) $a^{*+} \sim a^{++}$;
(3) $a^* \sim b^* \Leftrightarrow a^+ \sim b^+$;
(4) $a^* = 0 \Leftrightarrow a^+ = 0$;
(5) $a^* \wedge 0^+ \sim a^+$;
(6) $a^* \vee a^{**} \sim 0^* \Leftrightarrow a^+ \vee a^{++} \sim 0^+$.

Proof.

(1) We have $\langle\, a^*] = \{a\}^* = \langle\, a^+]$ (by Theorem 3.2) and therefore $a^* \sim a^+$.

(2) We have $\langle\, a^{*+}] = \{a^*\}^* = \langle\, a^*]^* = \langle\, a^+]^* = \{a^+\}^* = \langle\, a^{++}]$ and therefore $a^{*+} \sim a^{++}$.

(3) $a^* \sim b^* \Leftrightarrow \langle\, a^*] = \langle\, b^*]$
$\Leftrightarrow \{a\}^* = \{b\}^* \Leftrightarrow \langle\, a^+] = \langle\, b^+] \Leftrightarrow a^+ \sim b^+$.

(4) $a^* = 0 \Leftrightarrow \langle\, a^*] = \{0\}$
$\Leftrightarrow \{a\}^* = \{0\} \Leftrightarrow \langle\, a^+] = \{0\} \Leftrightarrow a^+ = 0$.

(5) $\langle\, a^+] = \{a\}^* \cap A = \langle\, a^*] \cap \{0\}^* = \langle\, a^*] \cap \langle\, 0^+] = \langle\, a^* \wedge 0^+]$
and therefore $a^* \wedge 0^+ \sim a^+$.

(6) $a^* \vee a^{**} \sim 0^* \Rightarrow a^+ \vee a^{++} \sim a^+ \vee a^{*+} \sim (a^* \wedge 0^+) \vee (a^{**} \wedge 0^+)$
$= (a^* \vee a^{**}) \wedge 0^+ = 0^* \wedge 0^+ \sim 0^+$.

□

Since $a \sim b$ implies $a = b$ for any elements a and b in a lattice, we have the following.

Corollary 3.6. *Any distributive lattice with* 0 *has at most one weak pseudo-complementation.*

Let us recall that an element m in an ADL A is maximal in $(A, \leq)$ if and only if $m \wedge a = a (\Leftrightarrow m = m \vee a)$ for all $a \in A$, which is equivalent to saying that $\langle\, m] = A$.

Theorem 3.7. *Let $a \mapsto a^*$ be a weak pseudo-complementation on an ADL A. Then the following hold for any $a \in A$ and $b \in A$.*

(1) 0^* *is a maximal element in* A;
(2) m *is maximal in* $A \Rightarrow m^* = 0$;
(3) $0^{**} = 0$;

(4) $a^* \wedge a = 0$;
(5) $a^{**} \wedge a = a$;
(6) $a \wedge b = 0 \Leftrightarrow a^{**} \wedge b = 0 \Leftrightarrow a \wedge b^{**} = 0 \Leftrightarrow a^{**} \wedge b^{**} = 0$;
(7) $a^* \sim a^{***}$;
(8) $a^* = 0 \Leftrightarrow a^{**}$ *is maximal;*
(9) $a = 0 \Leftrightarrow a^{**} = 0$;
(10) $(a \vee b)^* \sim a^* \wedge b^*$.

Proof.

(1) $\langle\, 0^*] = \{0\}^* = A$ and hence 0^* is maximal.

(2) $$\begin{aligned} m \text{ is a maximal in } A \;&\Rightarrow\; \langle\, m] = A \\ &\Rightarrow\; \langle\, m^*] = \langle\, m]^* = A^* = \{0\} \\ &\Rightarrow\; m^* = 0\,. \end{aligned}$$

(3) $\langle\, 0^{**}] = \{0^*\}^* = A^* = \{0\}$ and therefore $0^{**} = 0$.

(4) Since $a \wedge a^* = 0$, we have $a^* \wedge a = a^* \wedge a \wedge a = a \wedge a^* \wedge a = 0 \wedge a = 0$.

(5) Since $a^* \wedge a = 0$, we have $a \in \{a^*\}^* = \langle\, a^{**}]$ and hence $a^{**} \wedge a = a$.

(6) $$\begin{aligned} a \wedge b = 0 \;&\Rightarrow\; a^* \wedge b = b \\ &\Rightarrow\; a^{**} \wedge b = a^{**} \wedge (a^* \wedge b) = 0 \wedge b = 0 \\ &\Rightarrow\; b \wedge a^{**} = 0 \\ &\Rightarrow\; b^{**} \wedge a^{**} = 0 \\ &\Rightarrow\; a^{**} \wedge b^{**} = 0 \\ &\Rightarrow\; a \wedge b = a^{**} \wedge a \wedge b^{**} \wedge b \\ &\qquad = a^{**} \wedge b^{**} \wedge a \wedge b = 0 \wedge a \wedge b = 0\,. \end{aligned}$$

(7) By (6), we have $\{a\}^* = \{a^{**}\}^*$ and therefore $\langle\, a^*] = \langle\, a^{***}]$ which implies that $a^* \sim a^{***}$.

(8) This follows from (1), (2) and (7) (Note that $x \sim 0 \Rightarrow x = 0$).

(9) Follows from (1), (2) and (5).

(10) We have $$\begin{aligned} \langle\, a^* \wedge b^*] &= \langle\, a^*] \cap \langle\, b^*] \\ &= \{a\}^* \cap \{b\}^* \\ &= \{a \vee b\}^* \quad \text{(by the distributivity of } \wedge \text{ over } \vee) \\ &= \langle\, (a \vee b)^*] \end{aligned}$$
and therefore $(a \vee b)^* \sim a^* \wedge b^*$.

$\square$

Theorem 3.8. *Let A be an ADL and $a \mapsto a^*$ be a weak pseudo-complementation on A. Then the following hold for any a and $b \in A$.*

(1) $a \sim b \Rightarrow a^* \sim b^*$;
(2) $(a \wedge b)^* \sim (b \wedge a)^*$;
(3) $(a \vee b)^* \sim (b \vee a)^*$;
(4) $(a \wedge b)^* \wedge a^* = a^*$;
(5) $(a \wedge b)^* \wedge b^* = b^*$;

(6) $(a \wedge b)^{**} \sim a^{**} \wedge b^{**}$.

Proof. First, let us recall that $S^* = \langle\, S]^*$ for any $S \subseteq A$ and, in particular, $\{a\}^* = \langle\, a]^*$ for any $a \in A$.

(1) $a \sim b \Rightarrow \langle\, a] = \langle\, b] \Rightarrow \langle\, a]^* = \langle\, b]^* \Rightarrow \{a\}^* = \{b\}^*$
$\Rightarrow \langle\, a^*] = \langle\, b^*] \Rightarrow a^* \sim b^*$.

(2) For any $c \in A$, we have $a \wedge b \wedge c = 0 \Leftrightarrow b \wedge a \wedge c = 0$ and therefore $\langle\, a \wedge b]^* = \langle\, b \wedge a]^*$. This implies that $\langle\, (a \wedge b)^*] = \langle\, (b \wedge a)^*]$ and hence $(a \wedge b)^* \sim (b \wedge a)^*$.

(3) This is similar to (2), since $(a \vee b) \wedge c = (b \vee a) \wedge c$.

(4) Since $(a \wedge b) \wedge a^* = b \wedge a \wedge a^* = b \wedge 0 = 0$, we get that $(a \wedge b)^* \wedge a^* = a^*$.

(5) Since $(a \wedge b) \wedge b^* = 0$, we have $(a \wedge b)^* \wedge b^* = b^*$.

(6) We have $a \wedge b \wedge (a \wedge b)^* = 0 = b \wedge a \wedge (a \wedge b)^*$. By repeated use of 3.7(6), we get that $a^{**} \wedge b^{**} \wedge (a \wedge b)^* = 0$.

$$\therefore\ (a \wedge b)^* \wedge a^{**} \wedge b^{**} = 0$$

$$\therefore\ (a \wedge b)^{**} \wedge a^{**} \wedge b^{**} = a^{**} \wedge b^{**}\,. \tag{3.1}$$

On the other hand, we have $(a \wedge b) \wedge b^* = 0$ and hence (again by 3.7(6)), $(a \wedge b)^{**} \wedge b^* = 0$.

$$\therefore\ b^* \wedge (a \wedge b)^{**} = 0$$

$$\therefore\ b^{**} \wedge (a \wedge b)^{**} = (a \wedge b)^{**}$$

$$\text{Similarly} \qquad a^{**} \wedge (a \wedge b)^{**} = (a \wedge b)^{**}$$

$$\therefore\ a^{**} \wedge b^{**} \wedge (a \wedge b)^{**} = (a \wedge b)^{**} \tag{3.2}$$

By (3.1) and (3.2), we get that $(a \wedge b)^{**} \sim a^{**} \wedge b^{**}$. □

4. Pseudo-complementations on ADL'S

For any weak pseudo-complementation $*$ on an ADL A, Theorem 3.7(10) gives us that $(a \vee b)^*$ and $a^* \wedge b^*$ are associates to each other, for any a and b in A. In this context, let us recall the following from [7].

Definition 4.1. A weak pseudo-complementation $*$ on an ADL A is called a pseudo-complementation if

$$(a \vee b)^* = a^* \wedge b^* \quad \text{for all} \quad a \quad \text{and} \quad b \in A\,.$$

A is said to be pseudo-complemented if there is a pseudo-complementation on A.

For any elements a and b in a lattice, we have $a \wedge b = b \wedge a$ and hence $a \sim b \Rightarrow a = b$. This together with 3.7(10) implies the following.

Theorem 4.2. *Let $L = (L, \wedge, \vee, 0)$ be a distributive lattice with smallest element 0. Then any weak pseudo-complementation on L is a pseudo-complementation.*

The above theorem is not valid for a general ADL. For, consider the example given in the following.

Example 4.3. Let $A = \{0, 1, 2\}$ be the 3-element discrete ADL with 0 as the zero element and $A^3 = A \times A \times A$ be the product ADL whose operations are defined coordinate-wise. For any $a \in A^3$, let $|a|$ be the number of non zero coordinates of a. If $0 \neq a = (a_1, a_2, a_3) \in A^3$, define $a^* = (a_1^*, a_2^*, a_3^*)$, where

$$a_i^* = \begin{cases} 0\,, & \text{if} \quad a_i \neq 0 \\ 1\,, & \text{if} \quad a_i = 0 \quad \text{and} \quad |a| = 1 \\ 2\,, & \text{if} \quad a_i = 0 \quad \text{and} \quad |a| > 1 \end{cases}$$

and define $0^* = (2, 2, 2)$. For example, $(1, 0, 0)^* = (0, 1, 1)$, $(1, 2, 0)^* = (0, 0, 2)$ and $(2, 0, 1)^* = (0, 2, 0)$. It can be easily checked that $a \mapsto a^*$ is a weak pseudo-complementation on A^3. But this is not a pseudo-complementation; for, let

$$\begin{aligned} & & a = (1,0,0) \quad &\text{and} \quad b = (0,1,0)\,. \\ &\text{Then} & a \vee b = (1,1,0) \quad &\text{and} \quad (a \vee b)^* = (0,0,2)\,. \\ &\text{But} & a^* = (0,1,1) \quad &\text{and} \quad b^* = (1,0,1) \\ &\text{and hence} & a^* \wedge b^* = (0,0,1) &\neq (a \vee b)^*\,. \end{aligned}$$

Even though a particular weak pseudo-complementation need not be a pseudo-complementation, it induces one such. This is proved in the following.

Theorem 4.4. *Let $A = (A, \wedge, \vee, 0)$ be an ADL. Then A is weakly pseudo-complemented if and only if it is pseudo-complemented.*

Proof. Suppose that $*$ is a weak pseudo-complementation on A. Choose a maximal element m in A (A has one such; for example, 0^* is maximal). For any $a \in A$, define $a^+ = a^* \wedge m$. Then $a \wedge a^+ = a \wedge a^* \wedge m = 0 \wedge m = 0$ and, for any $b \in A$,

$$a \wedge b = 0 \;\Rightarrow\; a^* \wedge b = b \;\Rightarrow\; a^+ \wedge b = a^* \wedge m \wedge b = a^* \wedge b = b\,.$$

Thus $a \mapsto a^+$ is a weak pseudo-complementation on A. Also, for any a and $b \in A$, we have $(a \vee b)^+ \sim a^+ \wedge b^+$ (by Theorem 3.7(10)). Since $x^+ \leq m$ for all $x \in A$, we have that m is an upper bound of $(a \vee b)^+$ and $a^+ \wedge b^+$. This implies that

$$(a \vee b)^+ = (a^+ \wedge b^+) \wedge (a \vee b)^+ = (a \vee b)^+ \wedge (a^+ \wedge b^+) = a^+ \wedge b^+\,.$$

Thus $a \mapsto a^+$ is a pseudo-complementation on A and hence A is pseudo-complemented. The converse is trivial. □

Definition 4.5. Let A be an ADL and $PC(A)$ and $WPC(A)$ be respectively the sets of pseudo-complementations and weak pseudo-complementations on A. Any $*$ and $+$ in $WPC(A)$ are said to be equivalent (and denote this by $* \approx +$) if $0^* = 0^+$. Then clearly $\approx$ is an equivalence relation on $WPC(A)$.

The proof of Theorem 4.4 suggests the following, whose proof is a straight forward verification.

Theorem 4.6. *Let $*$ be weak pseudo-complementation on an ADL A. For any $a \in A$, define $a^{\overline{*}} = a^* \wedge 0^*$. Then $\overline{*}$ is a pseudo-complementation on A.*

Theorem 4.7. *For any ADL A, the correspondence $* \mapsto \overline{*}$ induces a bijection of $WPC(A)/_{\approx}$ onto $PC(A)$.*

Proof. First we observe that, for any $*$ in $PC(A)$,

$$a^{\overline{*}} = a^* \wedge 0^* = (a \vee 0)^* = a^* \quad \text{for all} \quad a \in A$$

and hence $\overline{*} = *$. This implies that $* \mapsto \overline{*}$ is a surjection correspondence. Also, for any $*$ and $+$ in $PC(A)$,

$$\begin{aligned} * \approx + &\Rightarrow 0^* = 0^+ \\ &\Rightarrow a^{\overline{*}} = a^* \wedge 0^* = 0^+ \wedge a^* \wedge 0^+ \qquad (\text{since } \ 0^+ \ \text{ is maximal}) \\ &\qquad = a^* \wedge 0^* = a^* \wedge 0^+ \wedge 0^+ = a^+ \wedge 0^+ \qquad \big(\text{by } 3.5(5)\big) \\ &\qquad = a^{\overline{+}} \quad \text{for all} \quad a \in A \\ &\Rightarrow \ \overline{*} = \overline{+}. \end{aligned}$$

Also, $\overline{*} = \overline{+} \ \Rightarrow \ 0^{\overline{*}} = 0^{\overline{+}} \ \Rightarrow \ 0^* \wedge 0^* = 0^+ \wedge 0^+ \ \Rightarrow \ 0^* = 0^+ \ \Rightarrow * \approx +$. Thus $* \mapsto \overline{*}$ induces a bijection of $WPC(A)$ onto $PC(A)$. $\square$

Corollary 4.8. *Let A be a pseudo-complemented ADL. Then $* \mapsto 0^*$ induces a bijection of $WPC(A)/_{\approx}$ onto the set $M(A)$ of all maximal elements of A and therefore $PC(A)$ is bijective with $M(A)$.*

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Department of Mathematics,
Shri Vishnu Engineering college for Women,
Bhimavaram - 534201, W.G.Dist., A.P., India
E-mail: vasukgr@gmail.com

Department of Engineering Mathematics,
Andhra University,
Visakhapatnam - 530 003, A.P., India
E-mail: santhisundarraj@yahoo.com

Corresponding author: B. Venkateswarlu,
Department of Mathematics,
GIT, GITAM University,
Visakhapatnam - 530 045, A.P., India
E-mail: bvlmaths@gmail.com