ARCHIVUM MATHEMATICUM (BRNO)
Tomus 56 (2020), 277–286

ON VARIANTS OF ARNOLD CONJECTURE

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Abstract. In this note we discuss the collection of statements known as Arnold conjecture for Hamiltonian diffeomorphisms of closed symplectic manifolds. We provide an overview of the homological, stable and strong versions of Arnold conjecture for non-degenerate Hamiltonian systems, a few versions of Arnold conjecture for possibly degenerate Hamiltonian systems, the degenerate version of Arnold conjecture for Hamiltonian homeomorphisms and Sandon's version of Arnold conjecture for contactomorphisms.

1. Versions of Arnold conjecture

One of the origins of Arnold conjecture is Poincarés last geometric theorem (or the Poincaré-Birkhoff fixed point theorem). Let ϕ be a self-diffeomorphism of an annulus $S^1 \times [0,1]$ which is area and orientation preserving. If ϕ maps $S^1 \times \{0\}$ and $S^1 \times \{1\}$ to themselves and "rotates" them in opposite directions, then the theorem states that there are at least two fixed points of ϕ, we refer the reader to [2] for more details. Arnold conjecture for closed symplectic manifolds can be seen in a way as a natural generalization of the Poincaré-Birkhoff theorem and concerns the existence of fixed points of a time-dependent Hamiltonian vector field on a closed symplectic manifold. It has been formulated by Arnold in 1960s and since then it has become such a landmark result that most of the symplectic community thinks that the modern symplectic topology emerged from Arnold conjecture.

Let (M,ω) be a closed connected symplectic manifold and $H\colon S^1 \times M \to \mathbb{R}$ be a periodic smooth function. Then there is a time-dependent Hamiltonian vector field $X_H\colon S^1 \times M \to TM$ given by $\omega(X_H(t,x),\cdot) = dH(t,\cdot)$. From now on we write $H_t(x) := H(t,x)$. We consider the set of contractible periodic orbits of X_H denoted by

$$P(H) := \{\gamma : S^1 \to M \mid \gamma \text{ is contractible}, \gamma'(t) = X_H(t,\gamma(t))\}\,. \tag{1.1}$$

Observe that $P(H)$ can be identified with the set of fixed points of the time 1-flow of X_H, $\phi\colon M \to M$, that we denote by $\mathrm{Fix}(\phi)$.

If $\phi^1_H \times M$ is transverse to the diagonal Δ, such a Hamiltonian system is called nondegenerate; for such Hamiltonian systems the set of fixed points is finite.

2020 *Mathematics Subject Classification*: primary 53D12; secondary 53D42.
Key words and phrases: Arnold conjecture, fixed points, Hamiltonian symplectomorphisms.
Received April 20, 2020. Editor M. Čadek.
DOI: 10.5817/AM2020-5-277

There are many different forms of Arnold conjecture. The literature on this subject is quite large with many partial solutions. In this short note it would be difficult to state all the results, we will mention the most general one and will refer the reader to [1, 26, 35] for more details.

1.1. **Strong Arnold conjecture.** We start with the strongest form of Arnold conjecture for non-degenerate Hamiltonian systems. The bound for the number of fixed points in this case is given by the Morse number defined by

$$\mathrm{Morse}(M) = \min\{\#\text{ critical points of } f \mid f \text{ is a Morse function on } M\}\,.$$

Conjecture 1.1. *For a non-degenerate Hamiltonian system*

$$|\operatorname{Fix}(\phi)| \geq \mathrm{Morse}(M)\,.$$

This statement is usually called the strong Arnold conjecture. Conjecture 1.1 is still wide open.

Arnold noted that Conjecture 1.1 holds in the case when Hamiltonian H is a C^2-small function. In such a situation Conjecture 1.1 follows from the elementary differential topology. For closed symplectic 2-manifolds it was proved by Eliashberg [9] and for the 2n-torus it was proved by Conley-Zehnder in [6]. Then Floer's original result [15] proves Conjecture 1.1 under the assumptions that $\dim M \geq 6$ and $\pi_1(M) = 0$ (under these assumptions $\mathrm{Morse}(M)$ is determined by the integer graded homology groups with $\mathbb{Z}$-coefficients).

1.2. **Stable Arnold conjecture.** There is a version called stable Arnold conjecture. It is closely related to the strong Arnold conjecture, but is slightly weaker than Conjecture 1.1. In this version Morse number is substituted with the stable Morse number.

We say that a function $f\colon M \times \mathbb{R}^l \to \mathbb{R}$ is almost quadratic at infinity if there is a non-degenerate quadratic form G on $\mathbb{R}^l$ with the property that

$$\|df - dG\|_{T^*(M\times\mathbb{R}^l)}$$

is bounded. Here $\|\cdot\|_{T^*(M\times\mathbb{R}^l)}$ is a norm on $T^*(M\times\mathbb{R}^l)$ which is given by a product metric on $M \times \mathbb{R}^l$. The stable Morse number of a closed smooth manifold M is defined by

$$\mathrm{StableMorse}(M) = \min_{\substack{f\in C^\infty(M\times\mathbb{R}^l)\\ l\in\mathbb{N}\cup\{0\}}}\{\#\text{ critical points of } f \mid f \text{ is Morse and quadratic at } \infty\}\,.$$

Conjecture 1.2. *For a non-degenerate Hamiltonian system*

$$|\operatorname{Fix}(\phi)| \geq \mathrm{StableMorse}(M)\,.$$

We call symplectic manifold M symplectically aspherical if the symplectic area class and the first Chern class vanish on the elements of $\pi_2(M)$. According to [8], Conjecture 1.2 holds for closed symplectic manifolds which are symplectically aspherical.

Observe that from the definition of $\mathrm{StableMorse}(M)$ and $\mathrm{Morse}(M)$ it follows that

$$\mathrm{StableMorse}(M) \leq \mathrm{Morse}(M)\,.$$

It is known that Morse number and stable Morse number are the same for many manifolds. In particular, they coincide for surfaces, simply connected closed three-manifolds (3-sphere by Perelman's proof of the Poincaré conjecture [31]), and for simply connected closed k-manifolds, where $k \geq 6$, see [40].

On the other hand, Damian in [7] proved that for closed symplectic 4-manifolds with a 'reasonably complicated' fundamental group $\mathrm{Morse}(M) > \mathrm{StableMorse}(M)$. Examples of the same sort exist for general symplectic manifolds of dimension $2n$, $n \geq 2$, this follows from the combination of the result of Damian and the result of Gompf [22] saying that every finitely presentable group can be realised as a fundamental group of a closed symplectic $2n$-manifold, $n \geq 2$.

1.3. **Homological Arnold conjecture.**

There is a weaker, but more feasible and well-known version of Arnold conjecture which is completely proven these days.

It is accessible by Floer theory, introduced by Floer in the 1980s, see [13, 12, 14, 16, 15]. It constructs a chain complex generated by $\mathrm{Fix}(\phi)$, this complex can be compared with the Morse complex generated by the critical points of a Morse function. When well-defined, Floer homology is independent of the Hamiltonian and can be identified with Morse homology. This approach was used by Floer in the absence of pseudoholomorphic spheres to prove a version of the following homological Arnold conjecture (we will provide more details in Section 2):

Theorem 1.3. *Let (M, ω) be a closed symplectic manifold and $H\colon S^1 \times M \to \mathbb{R}$ a nondegenerate periodic Hamiltonian function. Then*

$$\#|\,\mathrm{Fix}(\phi)| \;\geq\; \sum_{i=0}^{\dim M} \dim H_i(M; \mathbb{Q})\,. \tag{1.2}$$

Floer's proof was later extended to general closed symplectic manifolds.

Theorem 1.3 was proven for a general closed symplectic manifold by Fukaya-Ono [20, 21], Liu-Tian [28], and Ruan [33], based on the work of Floer. There is also a proof of Piunikhin-Salamon-Schwarz [32]. Using an abstract perturbation scheme provided by the polyfold theory of Fish-Hofer-Wysocki-Zehnder [11, 25], following an approach by Piunikhin-Salamon-Schwarz, and building on polyfold descriptions of Gromov-Witten moduli spaces as well as their degenerations in symplectic field theory, Filippenko and Wehrheim in [10] also provided a proof of Theorem 1.3.

In addition, we would like to mention that there are improvements of the bound 1.2 involving the fundamental group of a symplectic manifold (we refer to the works of Barraud [3] and Ono-Pajitnov [30].

From Morse theory it follows that $\mathrm{Morse}(M)$ and $\mathrm{StableMorse}(M)$ are bounded from below by the sum of Betti numbers of M and hence the statement of Theorem 1.3 is weaker than statements of Conjectures 1.2 and 1.1.

1.4. **Degenerate version of Arnold conjecture.**

There is a version of Arnold conjecture for the situation when one omits the restriction of non-degeneracy of Hamiltonian system. Sometimes this version of Arnold conjecture is called the degenerate Arnold conjecture.

Conjecture 1.4. *Given a Hamiltonian diffeomorphism ϕ of a closed and connected symplectic manifold M. Then*

$$|\operatorname{Fix}(\phi)| \geq \operatorname{Crit}(M)\,.$$

Here $\operatorname{Crit}(M)$ *denotes the minimal number of critical points of a smooth function on M.*

There are a few other versions of this bound in the degenerate case.

There is a version where one substitutes $\operatorname{Crit}(M)$ with the Lusternik-Schnirelmann category of M.

Lusternik-Schnirelmann category (or LS category) of a topological space X is the smallest integer n such that there is an open covering $U_1, \ldots, U_n$ of X such that every inclusion map $U_i \hookrightarrow X$ is nullhomotopic. Then the conjecture can be written as:

Conjecture 1.5. *Given a Hamiltonian diffeomorphism ϕ of a closed and connected symplectic manifold M, then*

$$|\operatorname{Fix}(\phi)| \geq \operatorname{LS}(M)\,.$$

Here $\operatorname{LS}(M)$ *denotes the Lusternik-Schnirelmann category of M.*

There is another version of Arnold conjecture involving the cup-length of M. The cup-length of M is defined to be

$$\begin{aligned}\operatorname{CL}(M, \mathbb{F}) := \max\{k+1 \mid \text{there are } \alpha_i \in H^{d_i}(M, \mathbb{F}), d_i \geq 1, i = 1, \ldots, k \\ \text{such that } \alpha_1 \cup \cdots \cup \alpha_k \neq 0\}\,.\end{aligned}$$

In this case, the conjecture has a form

Conjecture 1.6. *Given a Hamiltonian diffeomorphism ϕ of a closed and connected symplectic manifold M, then*

$$|\operatorname{Fix}(\phi)| \geq \operatorname{CL}(M)\,.$$

Note that the following inequality holds

$$\operatorname{Crit}(M) \geq \operatorname{LS}(M) \geq \operatorname{CL}(M)\,,$$

which implies that Conjecture 1.4 is stronger than Conjecture 1.5 which is stronger than Conjecture 1.6.

The most general answer for such settings appears in the work of of Rudyak-Oprea [34], which shows that Conjecture 1.4 (and hence Conjectures 1.5 and 1.6) hold for closed symplectically aspherical manifolds.

1.5. **Degenerate Arnold conjecture for Hamiltonian homeomorphisms.** Given a symplectic manifold M with a Riemannian distance denoted by d, for maps ϕ, $\psi\colon M \to M$ we define

$$d_{C^0}(\phi, \psi) = \max_{p \in M} d\big(\phi(p), \psi(p)\big)\,.$$

A sequence of compactly supported maps $\phi_k \colon M \to M$ is said to be C^0-convergent to ϕ, if there exists a compact set $X \subset M$ such that support$(\phi_k) \subset X$ for all k and

$$\lim_{k\to\infty} d_{C^0}(\phi_k, \phi) = 0\,.$$

A homeomorphism $\phi : M \to M$ is called Hamiltonian if it is the C^0-limit of a sequence of Hamiltonian diffeomorphisms.

There is a version of Arnold conjecture for Hamiltonian homeomorphisms.

Conjecture 1.7. *Given a Hamiltonian homeomorphism ϕ of a closed and connected symplectic manifold M, then*

$$|\operatorname{Fix}(\phi)| \geq \operatorname{Crit}(M)\,.$$

Using the earlier work of Franks [17], Matsumoto in [29] has proven that Hamiltonian homeomorphisms of surfaces satisfy Conjecture 1.7. In addition, there is a proof of Le Calvez [27].

In contrast, Buhovsky-Humiliere-Seyfaddini in [5] proved that Hamiltonian homeomorphisms do not satisfy Conjecture 1.7 in high dimensions:

Theorem 1.8 (Buhovsky-Humiliere-Seyfaddini)**.** *Let M be a closed connected symplectic manifold of dimension $2n$, $n \geq 2$, then there is a Hamiltonian homeomorphism $\phi\colon M \to M$ such that*

$$|\operatorname{Fix}(\phi)| = 1\,.$$

Despite Theorem 1.8, Buhovsky-Humiliere-Seyfaddini in a recent work [4] showed that certain Arnold-type statement (involving spectral invariants) survives in C^0 settings.

1.6. **Arnold conjecture for contactomorphisms.** Given a cooriented contact manifold $(M, \xi = ker(\alpha))$ and a contactomorphism ϕ contact isotopic to the identity. In general, ϕ does not have a fixed point. For example, we can take flow of the Reeb vector field R_α, i.e. vector field R_α determined by the system $\alpha(R_\alpha) = 1$, $d\alpha(R_\alpha) = 0$. R_α never vanishes, and hence for a small $t > 0$ time-t map of the Reeb flow does not have fixed points.

In contact geometric settings Arnold conjecture makes sense only for translated points of ϕ.

Given a cooriented contact manifold $(M, \xi = \ker(\alpha))$, its contactomorphism $\phi\colon M \to M$ and a smooth function $f\colon M \to \mathbb{R}$ such that $\phi^*\alpha = e^f\alpha$. We say that a point $p \in M$ is a translated point or ϕ with respect to the contact form α if both p and $\phi(p)$ belong to the same Reeb orbit (i.e. integral curve of the Reeb flow) and $f(p) = 0$. The set of translated points of ϕ is denoted by Translated(ϕ)

The following version of Arnold conjecture for translated points has been formulated by Sandon [37, 38]:

Conjecture 1.9. *Assume that $(M, \xi = \ker(\alpha))$ is a compact contact manifold and that $\phi\colon M \to M$ is a contactomorphism contact isotopic to the identity. Then*

$$|\operatorname{Translated}(\phi)| \geq \operatorname{Crit}(M)\,.$$

Conjecture 1.9 was previously established for the standard contact sphere S^{2n+1} and real projective space $\mathbb{R}P^{2n+1}$ in [38], for all lens spaces in [23] and for the case of prequantization bundle over a closed monotone toric manifold in [41].

2. Floer homology and its relation to Arnold conjecture

Most of the previously known results about Arnold conjecture in non-degenerate settings have been established using Floer homology which is based upon Gromov's theory of pseudoholomorphic curves [24]. We finish this note by explaining the original approach of Floer to the homological Arnold conjecture for certain basic symplectic manifolds.

We will show the construction of Floer homology with $\mathbb{Z}_2$-coefficients for a symplectic manifold (M, ω) with vanishing $\pi_2(M)$ and the way to prove a variant of homological Arnold conjecture in these settings. Observe that in particular $\pi_2(M) = 0$ implies that M is symplectically aspherical. For some extra details about Floer theory we refer the reader to [12, 13, 14, 15, 16]. In addition, we refer the reader to the works of Fukaya-Oh-Ohta-Ono [18, 19] for the construction of Floer homology in a much more general setting.

Take a space of smooth contractible loops on M, denote it by $\mathcal{L}M$, and consider a functional $\mathcal{A}_H \colon \mathcal{L}M \to \mathbb{R}$ defined by

$$\mathcal{A}_H(x) = -\int_{D^2} u^*\omega - \int_0^1 H_t(x(t))dt\,, \tag{2.1}$$

where $x \in \mathcal{L}M$ and $u \colon D^2 \to M$ is an extension of x. Note that u exists because of contractibility of x, and the right hand side of (2.1) is independent of the choice of u by the symplectic asphericity.

The periodic solutions of Hamilton's equation appear as the critical points of the symplectic action functional $\mathcal{A}_H$. Note that before Floer no one tried to develop a Morse theory for the functional $\mathcal{A}_H$ since there is no well-defined L^2 gradient flow, and also every critical point has infinite Morse index and coindex. Floer observed that the L^2 gradient flow of $\mathcal{A}_H$ is an elliptic equation, and this allowed him to make Morse theory work in this settings.

To define the gradient of $\mathcal{A}_H$, one needs an L^2 metric on $\mathcal{L}M$ compatible with the symplectic structure. This metric is determined by a periodic smooth family of compatible almost complex structures J_t, $t \in S^1$ on (M, ω) (Recall that compatible almost complex structure J on (M, ω) is an almost complex structure such that the bilinear form $g_J(u, v) := \omega(u, Jv)$ is a Riemannian metric.) J_t induces an L^2-inner product on the tangent space of $\mathcal{L}M$, and with respect to it

$$grad\ \mathcal{A}_H(x) = J_t(x)\big(x' - X_{H_t}(x)\big)\,.$$

Then we take a negative gradient trajectory of $\mathcal{A}_H$, which is a path $\psi \colon \mathbb{R} \to \mathcal{L}M$ such that

$$\psi' = -grad\ \mathcal{A}_H(\psi)\,,$$

it can be interpreted as a contractible smooth map $u\colon \mathbb{R}\times S^1 \to M$ satisfying

$$\partial_s u + J_t(u)\big(\partial_t u - X_{H_t}(u)\big) = 0\,. \tag{2.2}$$

Here s is a coordinate on the $\mathbb{R}$-factor and t is a coordinate on the S^1-factor.

Non-degenerate critical points of $\mathcal{A}_H$ are one-periodic solutions such that 1 is not a Floquet multiplier. From now on we assume that all critical points of $\mathcal{A}_H$ are non-degenerate. Then $\mathcal{A}_H$ is a Morse function and the critical points of $\mathcal{A}_H$ form a finite set.

Given a solution u of equation (2.2) (sometimes called Floer equation), the energy of u is given by

$$E(u) := \int_{-\infty}^{\infty}\int_0^1 |\partial_s u|^2 dt\, ds\,.$$

If u is a contractible finite energy solution of Floer's equation, then

$$\lim_{s\to\pm\infty} u(s,t) = x^{\pm}(t) \tag{2.3}$$

exists and is a contractible periodic Hamiltonian trajectory. On the other hand, every solution of (2.2) and (2.3) has finite energy and satisfies

$$E(u) = \mathcal{A}_H(x^-) - \mathcal{A}_H(x^+)\,.$$

Given two critical points $x^{\pm}$ of $\mathcal{A}_H$ we define the moduli space of Floer trajectories connecting x^- and x^+

$$\widetilde{\mathcal{M}}(x^-,x^+) := \{u \mid u \text{ satisfies } (2.2), (2.3)\}\,.$$

The construction of Floer theory is based on three main ingredients: transversality, compactness and gluing.

(i) Transversality is based on the fact that Floer equation is elliptic. There is a Fredholm theory providing that moduli spaces $\widetilde{\mathcal{M}}(x^-,x^+)$ are finite-dimensional transversally cut out manifolds. The dimension of $\widetilde{\mathcal{M}}(x^-,x^+)$ is $\mu_{CZ}(x^-) - \mu_{CZ}(x^+)$, where μ_{CZ} denotes Conley-Zehnder index (see [36]).

(ii) Compactness is based on the Gromov compactness theorem. Solutions of equation (2.2) behave like pseudoholomorphic curves, and hence in general pseudoholomorphic spheres may bubble off, which in this case is impossible because of the property that $\pi_2(M) = 0$. This allows for a generic J to count the elements of the spaces of index-1 Floer gradient trajectories, which is a finite set. This count is used to define the differential ∂ on the Floer chain complex.

(iii) Floer's gluing theorem is needed to show that $\partial^2 = 0$. This theorem is a highly non-trivial result, but in a way it is analogous to the gluing theorem in Morse theory.

We recall that in general the gradient flow of a smooth function f on a manifold M is Morse-Smale if stable manifolds and unstable manifolds intersect transversally. Given a Morse function f such that the gradient flow of f is Morse-Smale. Then the Morse complex of f is a module generated by the critical point of f and Morse differential is defined by counting (with signs) gradient trajectories joining two critical points of index difference 1. The homology of this complex is Morse homology and it is isomorphic to the ordinary homology of M. For details we refer the reader to [39].

The same way as in Morse theory, Floer complex $CF(H,J)$ is defined to be a $\mathbb{Z}_2$-vector space generated by non-degenerate critical points of $\mathcal{A}_H$. Let $CF_i(H,J)$ denote the $\mathbb{Z}_2$-vector space generated by critical points of Conley-Zehnder index $\mu_{CZ} = i$. Hence, $CF(H,J) = \bigoplus_i CF_i(H,J)$. The differential $\partial_{H,J}\colon CF_i(H,J) \to CF_{i-1}(H,J)$ can be written as

$$\partial_{H,J}(x^-) = \sum_{x^+} m(x^-, x^+) x^+ \,.$$

Note that Floer's equation is invariant under translations in the s variable, therefore $\mathbb{R}$ acts on $\widetilde{\mathcal{M}}(x^-, x^+)$. Here

$$m(x^-, x^+) = \#_2 \widetilde{\mathcal{M}}(x^-, x^+)/\mathbb{R}\,,$$

where $\#_2$ denotes the $\mod 2$ count and the quotient by $\mathbb{R}$ is a quotient by the translation in the s variable described above.

From ingredients $(i), (ii), (iii)$ described above it follows that Floer complex is well-defined and Floer homology is given by

$$HF_i(H,J) := H_i(CF(H,J), \partial_{H,J})\,.$$

In the definition of Floer complex we used $\mathbb{Z}_2$-coefficients in order to avoid addressing problems of orientation. However, Floer's construction extends to more sophisticated coefficients.

Floer proved that Floer homology groups $HF(H,J)$ are independent on the choices of a Hamiltonian and almost complex structure up to natural isomorphism. This proof is also based on the ingredients (i), (ii), (iii) needed to define Floer complex.

The last step can described the following way: In the case of time-independent J and H which is also assumed to be C^2-small, the relevant Floer gradient trajectories are gradient flow lines of the function $-H$, and hence the Floer chain complex reduces to the Morse complex of $-H$. Then the sum of Betti numbers of M with $\mathbb{Z}_2$-coefficients provides a lower bound for the number of critical points of $\mathcal{A}_H$, which implies a version of Theorem 1.3 with $\mathbb{Z}_2$-coefficients for symplectic manifolds with vanishing π_2.

Acknowledgement. The author is grateful to the organizers of the 40th Winter School on Geometry and Physics in Srni for the hospitality.

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