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MEDIAN PRIME IDEALS OF PSEUDO-COMPLEMENTED DISTRIBUTIVE LATTICES

M. Sambasiva Rao

Abstract. Coherent ideals, strongly coherent ideals, and τ-closed ideals are introduced in pseudo-complemented distributive lattices and their characterization theorems are derived. A set of equivalent conditions is derived for every ideal of a pseudo-complemented distributive lattice to become a coherent ideal. The notion of median prime ideals is introduced and some equivalent conditions are derived for every maximal ideal of a pseudo-complemented distributive lattice to become a median prime ideal which leads to a characterization of Boolean algebras.

Introduction

The theory of pseudo-complements in lattices, and particularly in distributive lattices was developed by M.H. Stone [10], O. Frink [4], and George Grätzer [5]. Later many authors like R. Balbes [1], T.P. Speed [8], and O. Frink [4] etc., extended the study of pseudo-complements to characterize Stone lattices. In [11], P.V. Venkatanarasimham generalized the theory of pseudo-complements in partially ordered sets. In [6], the author introduced the concept of δ-ideals in pseudo-complemented distributive lattices and then Stone lattices are characterized in terms of δ-ideals. In [7], the authors investigated the properties of normal ideals of pseudo-complemented distributive lattices and then characterized the disjunctive lattices with the help of normal ideals.

In this note, the concepts of coherent ideals and strongly coherent ideals are introduced in pseudo-complemented distributive lattices. A set of equivalent conditions is derived for every ideal of a pseudo-complemented distributive lattice to become a coherent ideal. It is showed that every strongly coherent ideal of a pseudo-complemented distributive lattice is coherent. The concepts of τ-closed ideals and weakly Stone lattices are introduced in pseudo-complemented distributive lattices and then the class of all weakly Stone lattices is characterized with the help of τ-closed ideals. So far many authors studied various properties of minimal prime ideals in many a algebraic structures. In this note, we aim to investigate the properties of certain class of prime ideals on the name of median

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prime ideals as a special subclass of minimal prime ideals of pseudo-complemented distributive lattices. Median prime ideals are then characterized and also derived that every median prime ideal of a pseudo-complemented distributive lattice is a coherent ideal. A set of equivalent conditions is derived for every maximal ideal of a pseudo-complemented distributive lattice to become a strongly coherent ideal. Some equivalent conditions are derived for every maximal ideal of a pseudo-complemented distributive lattice to become a median prime ideal which leads to a characterization of Boolean algebras. A set of equivalent conditions is also derived for every prime ideal of a pseudo-complemented distributive lattice to become median which leads to a characterization of relatively complemented lattices.

1. Preliminaries

The reader is referred to [2], [3] and [8] for the elementary notions and notations of pseudo-complemented distributive lattices. However some of the preliminary definitions and results are presented for the ready reference of the reader.

Definition 1.1 ([2])**.** An algebra $(L, \wedge, \vee)$ of type $(2, 2)$ is called a distributive lattice if for all $x, y, z \in L$, it satisfies the following properties $(1), (2), (3)$ and (4) along with (5) or $(5')$

(1) $x \wedge x = x$, $x \vee x = x$,
(2) $x \wedge y = y \wedge x$, $x \vee y = y \vee x$,
(3) $(x \wedge y) \wedge z = x \wedge (y \wedge z)$, $(x \vee y) \vee z = x \vee (y \vee z)$,
(4) $(x \wedge y) \vee x = x$, $(x \vee y) \wedge x = x$,
(5) $x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z)$,
(5′) $x \vee (y \wedge z) = (x \vee y) \wedge (x \vee z)$.

A non-empty subset A of a lattice L is called an ideal (filter) of L if $a \vee b \in A$ $(a \wedge b \in A)$ and $a \wedge x \in A$ $(a \vee x \in A)$ whenever $a, b \in A$ and $x \in L$. The set $(a] = \{x \in L \mid x \leq a\}$ (resp. $[a) = \{x \in L \mid a \leq x\}$) is called a principal ideal (resp. principal filter) generated by a. The set $\mathcal{I}(L)$ of all ideals of a distributive lattice L with 0 forms a complete distributive lattice. The set $\mathcal{F}(L)$ of all filters of a distributive lattice L with 1 forms a complete distributive lattice. A proper filter P of a distributive lattice L is said to be *prime* if for any $x, y \in L$, $x \vee y \in P$ implies $x \in P$ or $y \in P$. A prime filter P of a lattice L is called *maximal* if there exists no proper filter Q of L such that $P \subset Q$. A proper filter P of a distributive lattice is *minimal* [3] if there exists no prime filter Q of L such that $Q \subset P$.

Theorem 1.2 ([3])**.** *A prime ideal P of a distributive lattice L is minimal if and only if to each $x \in P$, there exists $y \notin P$ such that $x \wedge y = 0$.*

The pseudo-complement b^* of an element b is the greatest element disjoint from b, if such an element exists. The defining property of b^* is:

$$a \wedge b = 0 \;\Leftrightarrow\; a \wedge b^* = a \;\Leftrightarrow\; a \leq b^*$$

where $\leq$ is a partial ordering relation on the lattice L.

A distributive lattice L in which every element has a pseudo-complement is called a pseudo-complemented distributive lattice. For any two elements a, b of a pseudo-complemented lattice, we have the following:

(1) $a \leq b$ implies $b^* \leq a^*$,
(2) $a \leq a^{**}$,
(3) $a^{***} = a^*$,
(4) $(a \vee b)^* = a^* \wedge b^*$,
(5) $(a \wedge b)^{**} = a^{**} \wedge b^{**}$.

An element a of a pseudo-complemented distributive lattice L is called a dense element if $a^* = 0$ and the set D of all dense elements of L forms a filter in L. An element $a \in L$ is called a closed element if $x^{**} = x$.

Definition 1.3 ([2])**.** A pseudo-complemented distributive lattice L is called a Stone lattice if $x^* \vee x^{**} = 1$ for all $x \in L$.

Theorem 1.4 ([2])**.** *The following conditions are equivalent in a pseudo-complemented distributive lattice L:*

(1) *L is a Stone lattice;*
(2) *for $x, y \in L$, $(x \wedge y)^* = x^* \vee y^*$;*
(3) *for $x, y \in L$, $(x \vee y)^{**} = x^{**} \vee y^{**}$.*

For any non-empty subset A of a distributive lattice L, the annihilator [9] of A is defined as $A^* = \{x \in L \mid a \wedge x = 0 \text{ for all } a \in A\}$. For any $\emptyset \neq A \subseteq L$, A^* is an ideal of L such that $A \cap A^* = \{0\}$. In case of $A = \{a\}$, we simply denote $\{a\}^*$ by $(a)^*$. Throughout this note, all lattices are bounded pseudo-complemented distributive lattices unless otherwise mentioned.

2. Coherent ideals

In this section, the concept of coherent ideals and strongly coherent ideals are introduced in lattices. Stone lattices are characterized with help of coherent ideals. A set of equivalent conditions is derived for every ideal of a lattice to become a coherent ideal which leads to a characterization of Boolean algebras.

Definition 2.1. For any non-empty subset A of a lattice L, define

$$A^\tau = \{x \in L \mid a^* \vee x^* = 1 \text{ for all } a \in A\}$$

Clearly $\{0\}^\tau = L$ and $L^\tau = \{0\}$. For any $a \in L$, we denote $(\{a\})^\tau$ by $(a)^\tau$.

Proposition 2.2. *For any non-empty subset A of a lattice L, A^τ is an ideal of L such that $A \cap A^\tau = \{0\}$.*

Proof. Clearly $0 \in A^\tau$. Let $x, y \in A^\tau$. For any $a \in A$, we get $(x \vee y)^* \vee a^* = (x^* \wedge y^*) \vee a^* = (x^* \vee a^*) \wedge (y^* \vee a^*) = 1 \wedge 1 = 1$. Hence $x \vee y \in A^\tau$. Again, let $x \in A^\tau$ and $y \leq x$. Since $y \leq x$, we get $x^* \leq y^*$. For any $c \in A$, we get $1 = x^* \vee c^* \leq y^* \vee c^*$. Hence $x^* \vee c^* = 1$, which gives $y \in A^\tau$. Therefore A^τ is an ideal of L. Now, let $x \in A \cap A^\tau$. Then $x^* = x^* \vee x^* = 1$. Hence $x = 0$. Therefore $A \cap A^\tau = \{0\}$. $\square$

The following lemma is a direct consequence of the above definition.

Lemma 2.3. *For any two non-empty subsets A and B of a lattice L, we have*

(1) $A \subseteq B$ *implies* $B^\tau \subseteq A^\tau$,
(2) $A \subseteq A^{\tau\tau}$,
(3) $A^{\tau\tau\tau} = A^\tau$,
(4) $A^\tau = L$ *if and only if* $A = \{0\}$.

In case of ideals, we have the following result.

Proposition 2.4. *For any two ideals I, J of a lattice L, $(I \vee J)^\tau = I^\tau \cap J^\tau$.*

Proof. Let I, J be two ideals of L. Clearly $(I \vee J)^\tau \subseteq I^\tau \cap J^\tau$. Conversely, let $x \in I^\tau \cap J^\tau$. Let $c \in I \vee J$ be an arbitrary element. Then $c = i \vee j$ for some $i \in I$ and $j \in J$. Now $x^* \vee c^* = x^* \vee (i \vee j)^* = x^* \vee (i^* \wedge j^*) = (x^* \vee i^*) \wedge (x^* \vee j^*) = 1 \wedge 1 = 1$. Thus $x \in (I \vee J)^\tau$ and therefore $I^\tau \cap J^\tau \subseteq (I \vee J)^\tau$. □

The following corollary is a direct consequence of the above results.

Corollary 2.5. *Let L be a lattice. For any $a, b \in L$, the following properties hold:*

(1) $a \leq b$ *implies* $(b)^\tau \subseteq (a)^\tau$,
(2) $(a \vee b)^\tau = (a)^\tau \cap (b)^\tau$,
(3) $(a)^\tau = L$ *if and only if* $a = 0$,
(4) $a \in (b)^\tau$ *implies* $a \wedge b = 0$,
(5) $a^* = b^*$ *implies* $(a)^\tau = (b)^\tau$,
(6) $a \in D$ *implies* $(a)^\tau = \{0\}$.

For any non-empty subset A of a lattice, it can be easily seen that $A^\tau \subseteq A^*$. However, we derive a set of equivalent conditions for every ideal of a lattice to satisfy the reverse inclusion which leads to a characterization of Stone lattices.

Theorem 2.6. *The following assertions are equivalent in a lattice L:*

(1) *L is a Stone lattice;*
(2) *for any ideal I of L, $I^\tau = I^*$;*
(3) *for any $a \in L$, $(a)^\tau = (a)^*$;*
(4) *for any two ideals I, J of L, $I \cap J = \{0\}$ if and only if $I \subseteq J^\tau$;*
(5) *for $a, b \in L, a \wedge b = 0$ implies $a^* \vee b^* = 1$;*
(6) *for $a \in L$, $(a)^{\tau\tau} = (a^*)^\tau$.*

Proof. (1) ⇒ (2): Assume that L is a Stone lattice. Let I be an ideal of L. Clearly $I^\tau \subseteq I^*$. Conversely, let $x \in I^*$. Then $x \wedge y = 0$ for all $y \in I$. Since L is Stone, $x^* \vee y^* = (x \wedge y)^* = 0^* = 1$ for all $y \in I$. Hence $x \in I^\tau$. Therefore $I^* \subseteq I^\tau$.
(2) ⇒ (3): It is clear.
(3) ⇒ (4): Assume condition (3). Let I, J be two ideals of L. Suppose $I \cap J = \{0\}$.

Let $x \in I$. For any $y \in J$, we get $x \wedge y \in I \cap J = \{0\}$. Hence $x \wedge y = 0$. Now

$$\begin{aligned} x \wedge y = 0 \text{ for all } y \in J \quad &\Rightarrow \quad x \in (y)^* \qquad \text{for all } y \in J \\ &\Rightarrow \quad x \in (y)^\tau \qquad \text{for all } y \in J \\ &\Rightarrow \quad x^* \vee y^* = 1 \quad \text{for all } y \in J \end{aligned}$$

which yields that $x \in J^\tau$. Conversely, suppose that $I \subseteq J^\tau$. Let $x \in I \cap J$. Then $x \in I \subseteq J^\tau$ and $x \in J$. Hence $x \in J \cap J^\tau = \{0\}$. Therefore $I \cap J = \{0\}$.
(4) $\Rightarrow$ (5): Assume condition (4). Let $a, b \in L$ be such that $a \wedge b = 0$. Then

$$\begin{aligned} a \wedge b = 0 \quad &\Rightarrow \quad (a] \cap (b] = \{0\} \\ &\Rightarrow \quad (a] \subseteq (b]^\tau \qquad \text{by (4)} \\ &\Rightarrow \quad a \in (b]^\tau \\ &\Rightarrow \quad a^* \vee b^* = 1 \end{aligned}$$

(5) $\Rightarrow$ (6): Assume condition (5). Let $a \in L$. Clearly $a \wedge a^* = 0$. By (5), we get $a^* \vee a^{**} = 1$. Hence $a^* \in (a)^\tau$. Thus $(a)^{\tau\tau} \subseteq (a^*)^\tau$. Conversely, let $x \in (a^*)^\tau$ and $t \in (a)^\tau$. Since $t \in (a)^\tau$, we get $a^* \vee t^* = 1$. Hence $a^{**} \wedge t^{**} = 0$. Thus $t^{**} \leq a^*$. Now

$$\begin{aligned} x \in (a^*)^\tau \quad &\Rightarrow \quad a^{**} \vee x^* = 1 \\ &\Rightarrow \quad a^* \wedge x^{**} = 0 \\ &\Rightarrow \quad t^{**} \wedge x^{**} = 0 \qquad \text{since } t^{**} \leq a^* \\ &\Rightarrow \quad t \wedge x = 0 \qquad \text{since } t \wedge x \leq t^{**} \wedge x^{**} \\ &\Rightarrow \quad t^* \vee x^* = 1 \qquad \text{by (5)} \end{aligned}$$

which holds for all $t \in (a)^\tau$. Hence $x \in (a)^{\tau\tau}$. Therefore $(a^*)^\tau \subseteq (a)^{\tau\tau}$.
(6) $\Rightarrow$ (1): Assume condition (6). Let $a \in L$. Since $a \in (a)^{\tau\tau} = (a^*)^\tau$, we get $a^* \vee a^{**} = 1$. Therefore L is a Stone lattice. □

The concept of coherent ideals is now introduced in lattices.

Definition 2.7. An ideal I of a lattice L is called a *coherent ideal* if for all $x, y \in L, (x)^\tau = (y)^\tau$ and $x \in I$ imply that $y \in I$.

Clearly each $(x)^\tau, x \in L$ is a coherent ideal. It is evident that any ideal I is a coherent ideal if it satisfies $(x)^{\tau\tau} \subseteq I$ for all $x \in I$.

Theorem 2.8. *The following assertions are equivalent in a lattice L:*

(1) *L is a Boolean algebra;*
(2) *every principal ideal is a coherent ideal;*
(3) *every ideal is a coherent ideal;*
(4) *every prime ideal is a coherent ideal;*
(5) *for $a, b \in L, (a)^\tau = (b)^\tau$ implies $a = b$;*
(6) *for $a, b \in L, a^* = b^*$ implies $a = b$.*

Proof. (1) $\Rightarrow$ (2): Assume that L is a Boolean algebra. By [[6], Theorem 2.15], every element is closed. Let $(x]$ be a principal ideal of L. Then $x^{**} = x$. Let $a, b \in L$

be such that $(a)^\tau = (b)^\tau$ and $a \in (x]$. Since L is Boolean algebra, we get $a \vee a^* = 1$. Now

$$\begin{aligned} a \vee a^* = 1 \quad &\Rightarrow \quad x \vee a^* = 1 \qquad \text{since } a \in (x] \\ &\Rightarrow \quad a^* \vee x^{**} = 1 \\ &\Rightarrow \quad x^* \in (a)^\tau = (b)^\tau \\ &\Rightarrow \quad b^* \vee x^{**} = 1 \\ &\Rightarrow \quad (b^* \vee x^{**})^* = 0 \\ &\Rightarrow \quad b^{**} \wedge x^* = 0 \\ &\Rightarrow \quad b \wedge x^* = 0 \qquad \text{since } b \leq b^{**} \\ &\Rightarrow \quad b \leq x^{**} \end{aligned}$$

which yields $b \in (x^{**}] = (x]$. Therefore $(x]$ is a coherent ideal of L.

(2) $\Rightarrow$ (3): Assume condition (2). Let I be an ideal of L. Choose $a, b \in L$. Suppose $(a)^\tau = (b)^\tau$ and $a \in I$. Then clearly $(a] \subseteq I$. Since $a \in (a]$ and $(a]$ is a coherent ideal, we get that $b \in (a] \subseteq I$. Therefore I is a coherent ideal.

(3) $\Rightarrow$ (4): It is clear.

(4) $\Rightarrow$ (5): Assume that every prime ideal of L is a coherent ideal. Let $a, b \in L$ such that $(a)^\tau = (b)^\tau$. Suppose $a \neq b$. Then there exists a prime ideal P of L such that $a \in P$ and $b \notin P$. By the hypothesis, P is a coherent ideal of L. Since $(a)^\tau = (b)^\tau$ and $a \in P$, we get $b \in P$, which is a contradiction. Therefore $a = b$.

(5) $\Rightarrow$ (6): By Corollary 2.5 (5), it is direct.

(6) $\Rightarrow$ (1): Assume condition (6). Then L has a unique dense element. Therefore L is a Boolean algebra. □

Definition 2.9. For any ideal I of a lattice L, define $\xi(I)$ as follows:

$$\xi(I) = \{x \in L \mid (x)^\tau \vee I = L\}$$

The following lemma is an immediate consequence of the above definition.

Lemma 2.10. *For any two ideals I, J of a lattice L, the following properties hold:*

(1) $I \subseteq J$ *implies* $\xi(I) \subseteq \xi(J)$,
(2) $\xi(I \cap J) = \xi(I) \cap \xi(J)$,
(3) $\xi(I) \subseteq I$.

Proof. (1) and (2) can be obtained by routine verification.
(3) Let $x \in \xi(I)$. Then $(x)^\tau \vee I = L$. Since $x \in L$, we get $x = a \vee b$ for some $a \in (x)^\tau$ and $b \in I$. Then $x \wedge a = 0 \in I$ and $x \wedge b \in I$. Thus $x = x \wedge x = x \wedge (a \vee b) = (x \wedge a) \vee (x \wedge b) = x \wedge b \in I$. Therefore $\xi(I) \subseteq I$. □

Proposition 2.11. *For any ideal I of a lattice L, $\xi(I)$ is an ideal L.*

Proof. Clearly $0 \in \xi(I)$. Let $x, y \in \xi(I)$. Then $(x)^\tau \vee I = (y)^\tau \vee I = L$. Hence $(x \vee y)^\tau \vee I = \{(x)^\tau \cap (y)^\tau\} \vee I = \{(x)^\tau \vee I\} \cap \{(y)^\tau \vee I\} = L$. Hence $x \vee y \in \xi(I)$.

Again let $x, \in \xi(I)$ and $y \leq x$. Then $L = (x)^\tau \vee I \subseteq (y)^\tau \vee I$. Thus $y \in \xi(I)$. Therefore $\xi(I)$ is an ideal of L. □

Definition 2.12. An ideal I of a lattice L is called *strongly coherent* if $I = \xi(I)$.

Proposition 2.13. *Every strongly coherent ideal of a lattice is a coherent ideal.*

Proof. Let I be a strongly coherent ideal of a lattice L. Let $x, y \in L$ be such that $(x)^\tau = (y)^\tau$ and $x \in I = \xi(I)$. Then clearly $(x)^\tau \vee I = L$. Hence $(y)^\tau \vee I = L$ and so $y \in \xi(I) = I$. Therefore I is a coherent ideal of L. □

Definition 2.14. An ideal I of a lattice L is called τ-closed if $I = I^{\tau\tau}$.

Clearly $\{0\}$ is the smallest τ-closed ideal and L is the largest τ-closed ideal.

Proposition 2.15. *Every τ-closed ideal of a lattice is a coherent ideal.*

Proof. Let I be a τ-closed ideal of a lattice L. Let $x, y \in L$ be such that $(x)^\tau = (y)^\tau$. Suppose $x \in I$. Then $y \in (y)^{\tau\tau} = (x)^{\tau\tau} \subseteq I^{\tau\tau} = I$. Therefore I is coherent. □

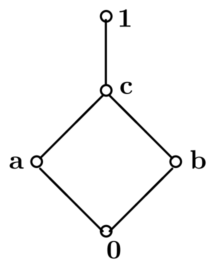
Definition 2.16. A lattice L is called *weakly Stone lattice* if $(x)^\tau \vee (x)^{\tau\tau} = L$ for all $x \in L$.

Theorem 2.17. *Every Stone lattice is a weakly Stone lattice.*

Proof. Assume that L is a Stone lattice. Let $x \in L$. Since L is a Stone lattice, we get $(x)^* \vee (x)^{**} = L$. For any $a \in L$, we get $a = b \vee c$ for some $b \in (x)^* = (x)^\tau$ and $c \in (x)^{**}$. Since $c \in (x)^{**}$, we get $c \wedge t = 0$ for all $t \in (x)^* = (x)^\tau$. Since L is Stone, we get $c^* \vee t^* = 1$ for all $t \in (x)^\tau$. Hence $c \in (x)^{\tau\tau}$. Thus $a \in (x)^\tau \vee (x)^{\tau\tau}$. Therefore $L \subseteq (x)^\tau \vee (x)^{\tau\tau}$, which means that L is a weakly Stone lattice. □

The converse of the above theorem is not true. For, consider

Example 2.18. Consider the following bounded and finite distributive lattice $L = \{0, a, b, c, 1\}$ whose Hasse diagram is given by:



Clearly L is a pseudo-complemented lattice. It can be easily observed that $(a)^\tau = (b)^\tau = (c)^\tau = \{0\}$. Hence $(a)^{\tau\tau} = (b)^{\tau\tau} = (c)^{\tau\tau} = L$. Observe that L is a weakly Stone lattice. But L is not a Stone lattice because of $a^* \vee a^{**} = b \vee a = c \neq 1$.

Theorem 2.19. *The following assertions are equivalent in a lattice L:*

(1) *L is a weakly Stone algebra;*

(2) *every τ-closed ideal is strongly coherent;*

(3) *for each $x \in L, (x)^{\tau\tau}$ is strongly coherent.*

Proof. (1) $\Rightarrow$ (2): Assume that L is a weakly Stone lattice. Let I be a τ-closed ideal of L. Then $I^{\tau\tau} = I$. Clearly $\xi(I) \subseteq I$. Conversely, let $x \in I$. It can be easily verified that $(x)^{\tau\tau} \subseteq I^{\tau\tau}$. Hence $L = (x)^{\tau} \vee (x)^{\tau\tau} \subseteq (x)^{\tau} \vee I^{\tau\tau} = (x)^{\tau} \vee I$. Thus $x \in \xi(I)$. Therefore I is strongly coherent.

(2) $\Rightarrow$ (3): Since each $(x)^{\tau\tau}$ is τ-closed, it is obvious.

(3) $\Rightarrow$ (1): Assume condition (3). Let $x \in L$. Then we get $\xi((x)^{\tau\tau}) = (x)^{\tau\tau}$. Since $x \in (x)^{\tau\tau}$, we get $(x)^{\tau} \vee (x)^{\tau\tau} = L$. Therefore L is a weakly Stone lattice. □

3. Median prime ideals

In this section, the notion of median prime ideals is introduced in lattices. Characterization theorems of median prime ideals are derived for every prime ideal to become a median prime and every maximal ideal to become a median prime.

Lemma 3.1. *Let P be a prime ideal of a lattice L. For any $x \in L$, we have*

$$x \notin P \quad \textit{implies} \quad (x)^{\tau} \subseteq P$$

Proof. Suppose $x \notin P$. Let $a \in (x)^{\tau}$. Then $a^* \vee x^* = 1$. Hence $a^{**} \wedge x^{**} = 0$. Thus $a \wedge x = 0 \in P$. Since $x \notin P$, we must have $a \in P$. Therefore $(x)^{\tau} \subseteq P$. □

Definition 3.2. A prime ideal P of a lattice L is called *median* if to each $x \in P$, there exists $y \notin P$ such that $x^* \vee y^* = 1$.

From Example 2.18, we initially observe that every prime ideal of a lattice need not be median, For consider the prime ideal $P = \{0, a\}$ of L. Notice that, for $a \in P$, there is no $x \notin P$ such that $a^* \vee x^* = 1$. Therefore P is not median.

Lemma 3.3. *Let P be a median prime ideal of a lattice L. For any $x \in L$, we have*

$$x \in P \quad \textit{if and only if} \quad (x)^{\tau\tau} \subseteq P\,.$$

Proof. Suppose $x \in P$. Let $a \in (x)^{\tau\tau}$. Then $(x)^{\tau} \subseteq (a)^{\tau}$. Since $x \in P$ and P is median, there exists $y \notin P$ such that $x^* \vee y^* = 1$. Then $y \in (x)^{\tau} \subseteq (a)^{\tau}$. Since $y \notin P$, we get $(y)^{\tau} \subseteq P$. Hence $a \in (a)^{\tau\tau} \subseteq (y)^{\tau} \subseteq P$. Therefore $(x)^{\tau\tau} \subseteq P$. □

In the following, we derive a characterization theorem of median prime ideals.

Theorem 3.4. *A prime ideal P of a lattice L is median if and only if it satisfies*

$$x \notin P \textit{ if and only if } (x)^{\tau} \subseteq P.$$

Proof. Assume that P is a median prime ideal of L and $x \in L$. Suppose $x \notin P$. By Lemma 3.1, we have $(x)^{\tau} \subseteq P$. On the other hand, assume that $(x)^{\tau} \subseteq P$. Suppose $x \in P$. Since P is median, there exists $y \notin P$ such that $x^* \vee y^* = 1$. Hence $y \in (x)^{\tau} \subseteq P$, which is a contradiction. Therefore $x \notin P$.

Conversely, assume that P satisfies the condition. Suppose $x \in P$. By the assumed condition, we get that $(x)^{\tau} \nsubseteq P$. Then there exists $y \in (x)^{\tau}$ such that $y \notin M$. Hence $x^* \vee y^* = 1$ where $y \notin P$. Therefore P is median. □

Theorem 3.5. *Every median prime ideal of a lattice is a coherent ideal.*

Proof. Let P be a median prime ideal of a lattice L. Suppose $x, y \in L$ be such that $(x)^\tau = (y)^\tau$ and $x \in P$. Since P is median, there exists $a \notin P$ such that $x^* \vee a^* = 1$. Hence $a \in (x)^\tau = (y)^\tau$. Since $a \in (y)^\tau$, we get $0 \in y \wedge a \in P$. Since P is prime and $a \notin P$, it yields that $y \in P$. Therefore P is a coherent ideal. $\square$

Definition 3.6. For any prime ideal P of a lattice L, define

$$\omega(P) = \{x \in L \mid (x)^\tau \nsubseteq P\}.$$

Lemma 3.7. *For any prime ideal P of a lattice L, $\omega(P)$ is an ideal contained in P.*

Proof. Clearly $0 \in \omega(P)$. Let $x, y \in \omega(P)$. Then $(x)^\tau \nsubseteq P$ and $(y)^\tau \nsubseteq P$. Since P is prime, we get $(x \vee y)^\tau = (x)^\tau \cap (y)^\tau \nsubseteq P$. Hence $x \vee y \in \omega(P)$. Let $x \in \omega(P)$ and $y \leq x$. Then $(x)^\tau \notin P$ and $(x)^\tau \subseteq (y)^\tau$. Since $(x)^\tau \nsubseteq P$, we get $(y)^\tau \nsubseteq P$. Thus $y \in \omega(P)$. Therefore $\omega(P)$ is an ideal of L. Now, let $x \in \omega(P)$. Then, we get $(x)^\tau \nsubseteq P$. Hence there exists $a \in (x)^\tau$ such that $a \notin P$. Since $a \in (x)^\tau$, we get $a \wedge x = 0 \in P$. Since $a \notin P$, we must have $x \in P$. Therefore $\omega(P) \subseteq P$. $\square$

Let us denote that $\mathcal{P}$ is the set of all prime ideals of a lattice L. For any $a \in L$, we also denote $\mathcal{P}_a = \{P \in \mathcal{P} \mid a \notin P\}$.

Theorem 3.8. *Let L be a lattice and $a \in L$. Then $(a)^\tau \subseteq \bigcap\limits_{P \in \mathcal{P}_a} \omega(P)$.*

Proof. Let $x \in (a)^\tau$ and $P \in \mathcal{P}_a$. Then $x^* \vee a^* = 1$ and $a \notin P$. Hence $a \in (x)^\tau$ such that $a \notin P$. Thus $(x)^\tau \nsubseteq P$. Hence $x \in \omega(P)$. Thus $(a)^\tau \subseteq \omega(P)$ which is true for all $P \in \mathcal{P}_a$. Therefore $(a)^\tau \subseteq \bigcap\limits_{P \in \mathcal{P}_a} \omega(P)$. $\square$

Corollary 3.9. *Let L be a lattice and $a \in L$. Then $a \notin P$ implies $(a)^\tau \subseteq \omega(P)$.*

Proposition 3.10. *Every median prime ideal of a lattice is a minimal prime ideal.*

Proof. Let P be a median prime ideal of a lattice L and $x \in L$. Suppose $x \in P$. Since P is median, there exists $y \notin P$ such that $x^* \vee y^* = 1$. Hence $x \wedge y \leq x^{**} \wedge y^{**} = 0$. By Theorem 1.2, P is a minimal prime ideal of L. $\square$

The converse of Proposition 3.10 is not true. For consider the minimal prime ideal $P = \{0, a\}$ of the lattice given in Example 2.18. Consider the element a of P. It can be observed that there is no $x \notin P$ such that $a^* \vee x^* = 1$. Therefore P is not median.

Theorem 3.11. *Let L be a Stone lattice and P be a prime ideal of L. Then P contains a unique median prime ideal $\omega(P)$.*

Proof. Let P be the given prime ideal. Clearly P contains a minimal prime ideal, say Q. Let $x \in Q$. Since Q is minimal, there exists $y \notin Q$ such that $x \wedge y = 0$. Since L is Stone, $x^* \vee y^* = 1$. Hence Q is median. Therefore P contains a median prime ideal Q. We now prove the uniqueness. Suppose P contains two distinct median prime ideals, say Q_1 and Q_2. Choose $a \in Q_1$ and $a \notin Q_2$. Since $a \wedge a^* = 0 \in Q_2$ and $a \notin Q_2$, we must have $a^* \in Q_2$. Since Q_1 is minimal, we get that $L - Q_1$ is a maximal filter such that $a \notin L - Q_1$. Since $L - Q_1$ is maximal, we get $(L - Q_1) \vee [a) = L$. Hence $0 = c \wedge a$ for some $c \notin Q_1$. Now

$$\begin{aligned} c \wedge a = 0 &\Rightarrow c \leq a^* \\ &\Rightarrow a^* \in L - Q_1 \qquad \text{since } c \in L - Q_1 \\ &\Rightarrow a^* \notin Q_1 \\ &\Rightarrow a^{**} \in Q_1 \qquad \text{since } a^* \wedge a^{**} = 0 \end{aligned}$$

Hence $1 = a^* \vee a^{**} \in Q_1 \vee Q_2 \subseteq P$, which is a contradiction. Therefore P contains a unique median prime ideal. Since L is Stone and $\omega(P) \subseteq P$, we get that P contains the unique median prime ideal, precisely $\omega(P)$. □

Theorem 3.12. *The following assertions are equivalent in a lattice L:*

(1) *L is a Stone lattice;*
(2) *for any $P \in \mathcal{P}$, $\omega(P)$ is prime;*
(3) *for any $a, b \in L$, $a \wedge b = 0$ implies $(a)^\tau \vee (b)^\tau = L$;*
(4) *for any $a \in L$, $(a)^\tau \vee (a^*)^\tau = L$.*

Proof. (1) $\Rightarrow$ (2): Assume that L is a Stone lattice. Let P be a prime ideal of L. By Theorem 3.11, we get $\omega(P) = P$ is prime.
(2) $\Rightarrow$ (3): Assume condition (2). Let $a, b \in L$ be such that $a \wedge b = 0$. Suppose $(a)^\tau \vee (b)^\tau \neq L$. Then there exists a prime ideal P such that $(a)^\tau \vee (b)^\tau \subseteq P$. Then

$$\begin{aligned} (a)^\tau \vee (b)^\tau \subseteq P &\Rightarrow (a)^\tau \subseteq P \text{ and } (b)^\tau \subseteq P \\ &\Rightarrow a \notin \omega(P) \text{ and } b \notin \omega(P) \\ &\Rightarrow 0 = a \wedge b \notin \omega(P) \qquad \text{since } \omega(P) \text{ is prime} \end{aligned}$$

which is a contradiction. Therefore $(a)^\tau \vee (b)^\tau = L$.
(3) $\Rightarrow$ (4): Let $a \in L$. Since $a \wedge a^* = 0$, by (3), we are through.
(4) $\Rightarrow$ (1): Assume condition (4). Let $x \in L$. Then by (4), we have $(x)^\tau \vee (x^*)^\tau = L$. Hence $1 \in (x)^\tau \vee (x^*)^\tau$. Then $1 = a \vee b$ for some $a \in (x)^\tau$ and $b \in (x^*)^\tau$. Since $a \vee b = 1$, we get $a^* \wedge b^* = 0$. Hence $a^* \leq b^{**}$. Since $b \in (x^*)^\tau$, we get $b^* \vee x^{**} = 1$, and so $b^{**} \wedge x^* = 0$. Thus $b^{**} \leq x^{**}$. Now

$$\begin{aligned} 1 &= a^* \vee x^* && \text{since } a \in (x)^\tau \\ &\leq b^{**} \vee x^* && \text{since } a^* \leq b^{**} \\ &\leq x^{**} \vee x^* && \text{since } b^{**} \leq x^{**} \end{aligned}$$

which gives that $x^* \vee x^{**} = 1$. Therefore L is a Stone lattice. □

Let us denote the set of all maximal ideals of a lattice L by $\mathcal{M}$. For any ideal I of a lattice L, we also denote $\mathcal{M}_I = \{M \in \mathcal{M} \mid I \subseteq M\}$.

Theorem 3.13. *For any ideal I of a lattice $L, \xi(I) = \bigcap_{M \in \mathcal{M}_I} \omega(M)$.*

Proof. Let $x \in \xi(I)$ and $I \subseteq M$ where $M \in \mathcal{M}$. Then $L = (x)^\tau \vee I \subseteq (x)^\tau \vee M$. Suppose $(x)^\tau \subseteq M$, then $M = L$, which is a contradiction. Hence $(x)^\tau \nsubseteq M$. Thus

$x \in \omega(M)$ for all $M \in \mathcal{M}_I$. Therefore $\xi(I) \subseteq \bigcap_{M \in \mathcal{M}_I} \omega(M)$.

Conversely, let $x \in \bigcap_{M \in \mathcal{M}_I} \omega(M)$. Then, we get $x \in \omega(M)$ for all $M \in \mathcal{M}_I$. Suppose $(x)^\tau \vee I \neq L$. Then there exists a maximal ideal M_0 such that $(x)^\tau \vee I \subseteq M_0$. Hence $(x)^\tau \subseteq M_0$ and $I \subseteq M_0$. Since $I \subseteq M_0$, by hypothesis, we get $x \in \omega(M_0)$. Thus $(x)^\tau \nsubseteq M_0$, which is a contradiction. Therefore $(x)^\tau \vee I = L$. Thus $x \in \xi(I)$. Therefore $\bigcap_{M \in \mathcal{M}_I} \omega(M) \subseteq \xi(I)$. □

From Example 2.18, we can see that the proper ideal $M = \{0, a, b, c\}$ is a maximal ideal in L but not the median ideal because for $a \in M$, there is no $x \notin M$ such that $a^* \vee x^* = 1$. However, in the following theorem, we establish a set of equivalent conditions for every maximal ideal to become a median prime ideal.

Theorem 3.14. *The following conditions are equivalent in a lattice L.*

(1) *L is a Boolean algebra;*
(2) *every maximal ideal is median;*
(3) *for any $M \in \mathcal{M}$, $\omega(M)$ is maximal;*
(4) *for any $I, J \in \mathcal{I}(L)$, $I \vee J = L$ implies $\xi(I) \vee \xi(J) = L$;*
(5) *for any $I, J \in \mathcal{I}(L)$, $\xi(I) \vee \xi(J) = \xi(I \vee J)$;*
(6) *for any two distinct maximal ideals M, N, $\omega(M) \vee \omega(N) = L$;*
(7) *for any $M \in \mathcal{M}$, M is the unique member of $\mathcal{M}$ such that $\omega(M) \subseteq M$.*

Proof. (1) $\Rightarrow$ (2): Assume that L is Boolean. Let M be a maximal ideal of L and $x \in L$. Clearly M is a prime ideal of L. Let $x \in M$. Suppose $x^* \in M$. Then $1 = x \vee x^* \in M$, which is a contradiction. Hence $x^* \notin M$. Since L is a Boolean algebra, we get $1 = x^* \vee x \leq x^* \vee (x^*)^*$. Therefore M is median.

(2) $\Rightarrow$ (3): Assume condition (2) holds. Let $M \in \mathcal{M}$. Clearly $\omega(M) \subseteq M$. Conversely, let $x \in M$. Since M is median, there exists $y \notin M$ such that $x^* \vee y^* = 1$. Hence $y \in (x)^\tau$ and $y \notin M$. Thus $(x)^\tau \nsubseteq \omega(M)$. Hence $x \in \omega(M)$. Thus $M = \omega(M)$. Therefore $\omega(M)$ is maximal.

(3) $\Rightarrow$ (4): Assume condition (3). Clearly $\omega(M) = M$ for all $M \in \mathcal{M}$. Let $I, J \in \mathcal{I}(L)$ be such that $I \vee J = L$. Suppose $\xi(I) \vee \xi(J) \neq L$. Then there exists a maximal ideal M such that $\xi(I) \vee \xi(J) \subseteq M$. Hence $\xi(I) \subseteq M$ and $\xi(J) \subseteq M$. Now

$$\begin{aligned} \xi(I) \subseteq M \quad &\Rightarrow \quad \bigcap_{M \in \mathcal{M}_I} \omega(M) \subseteq M \qquad \text{by Theorem 3.13} \\ &\Rightarrow \quad \omega(M_i) \subseteq M \quad \text{for some } M_i \in \mathcal{M}_I \quad (\text{since } M \text{ is prime}) \\ &\Rightarrow \quad M_i \subseteq M \qquad \text{since } \omega(M_i) = M_i \\ &\Rightarrow \quad I \subseteq M \end{aligned}$$

Similarly, we can get $J \subseteq M$. Hence $L = I \vee J \subseteq M$, which is a contradiction. Therefore $\xi(I) \vee \xi(J) = L$.

(4) $\Rightarrow$ (5): Assume condition (4). Let $I, J \in \mathcal{I}(L)$. Clearly $\xi(I) \vee \xi(J) \subseteq \xi(I \vee J)$. Let $x \in \xi(I \vee J)$. Then $((x)^\tau \vee I) \vee ((x)^\tau \vee J) = (x)^\tau \vee I \vee J = L$. Hence by condition

(3), we get $\xi((x)^\tau \vee I) \vee \xi((x)^\tau \vee J) = L$. Thus $x \in \xi((x)^\tau \vee I) \vee \xi((x)^\tau \vee J)$. Hence $x = r \vee s$ for some $r \in \xi((x)^\tau \vee I)$ and $s \in \xi((x)^\tau \vee J)$. Now

$$\begin{aligned} r \in \xi((x)^\tau \vee I) &\Rightarrow (r)^\tau \vee (x)^\tau \vee I = L \\ &\Rightarrow L = ((r)^\tau \vee (x)^\tau) \vee I \subseteq (r \wedge x)^\tau \vee I \\ &\Rightarrow (r \wedge x)^\tau \vee I = L \\ &\Rightarrow r \wedge x \in \xi(I) \end{aligned}$$

Similarly, we can get $s \wedge x \in \xi(J)$. Hence

$$\begin{aligned} x &= x \wedge x \\ &= (r \vee s) \wedge x \\ &= (r \wedge x) \vee (s \wedge x) \in \xi(I) \vee \xi(J) \end{aligned}$$

Hence $\xi(I \vee J) \subseteq \xi(I) \vee \xi(J)$. Therefore $\xi(I) \vee \xi(J) = \xi(I \vee J)$.

(5) $\Rightarrow$ (6): Assume condition (5). Let M, N be two distinct maximal ideals of L. Choose $x \in M - N$ and $y \in N - M$. Since $x \notin N$, there exists $x' \in N$ such that $x \vee x' = 1$. Similarly, there exists $y' \in M$ such that $y \vee y' = 1$. Hence $(x \vee y') \vee (x' \vee y) = 1$. Now

$$\begin{aligned} L &= \xi(L) \\ &= \xi((1]) \\ &= \xi\big(((x \vee y') \vee (x' \vee y)]\big) \\ &= \xi\big((x \vee y'] \vee (x' \vee y]\big) \\ &= \xi\big((x \vee y']\big) \vee \xi\big((x' \vee y]\big) \qquad \text{by condition (5)} \\ &\subseteq \omega(M) \vee \omega(N) \qquad \text{since } (x \vee y'] \subseteq M, (x' \vee y] \subseteq N\,. \end{aligned}$$

Therefore $\omega(M) \vee \omega(N) = L$.

(6) $\Rightarrow$ (7): Assume condition (6). Let $M \in \mathcal{M}$. Clearly $\omega(M) \subseteq M$. Suppose $N \in \mathcal{M}$ such that $N \neq M$ and $\omega(M) \subseteq N$. Since $\omega(N) \subseteq N$, by hypothesis, we get $L = \omega(M) \vee \omega(N) \subseteq N$, which is a contradiction. Therefore M is the unique maximal ideal of L such that $\omega(M)$ is contained in M.

(7) $\Rightarrow$ (1): Assume condition (7) holds. Let $x \in L$. Suppose $(x] \vee (x)^\tau \neq L$. Then there exists a maximal ideal of L such that $(x] \vee (x)^\tau \subseteq M$. Then $x \in M$ and $(x)^\tau \subseteq M$. Hence $x \notin \omega(M)$. Then there exists a maximal ideal M_0 such that $x \notin M_0$ and $\omega(M) \subseteq M_0$. By the uniqueness of M, we get $M = M_0$. Hence $x \in M_0 = M$, which is a contradiction. Hence $(x] \vee (x)^\tau = L$. Then $1 \in (x] \vee (x)^\tau$. Hence $1 = x \vee a$ for some $a \in (x)^\tau$. Since $a \in (x)^\tau$, we get $a^* \vee x^* = 1$. Hence $a \wedge x = 0$. Thus a is the complement of x in L. Therefore L is a Boolean algebra. $\square$

Proposition 3.15. *Every prime strongly coherent ideal of a lattice is a minimal prime ideal.*

Proof. Let P be a prime strongly coherent ideal of a lattice L. Let $x \in P$. Then $x \in \xi(P)$ and hence $(x)^\tau \vee P = L$. Thus there exist $a \in (x)^\tau$ and $b \in P$ such that $a \vee b = 1$. Since $a \in (x)^\tau$, we get $a \wedge x = 0$. Suppose $a \in P$. Since $b \in P$, we get $1 = a \vee b \in P$ which is a contradiction. Hence $a \notin P$. Thus, for any $x \in P$, there exists $a \notin P$ such that $a \wedge x = 0$. Therefore P is minimal. □

Every maximal ideal of a lattice need not be a strongly coherent ideal. For consider the lattice L given in Example 2.18. The ideal $M = \{0, a, b, c\}$ is a maximal ideal but not a strongly coherent ideal in L because of $(c)^\tau \vee M = \{0\} \vee M \neq L$. In the following, a set of equivalent conditions is established for every maximal ideal of a lattice to become a strongly coherent ideal.

Theorem 3.16. *The following assertions are equivalent in a lattice L:*

(1) *L is a Boolean algebra;*
(2) *every maximal ideal is a strongly coherent;*
(3) *every maximal ideal is a minimal prime ideal.*

Proof. (1) ⇒ (2): Assume that L is a Boolean algebra. Let M be a maximal ideal of L. By the above theorem, $\omega(M) = M$. Let $x \in \xi(M)$. Then $(x)^\tau \vee M = L$. Suppose $(x)^\tau \subseteq M$. Then $M = L$, which is a contradiction. Hence $(x)^\tau \nsubseteq M$. Thus $x \in \omega(M)$. Therefore $\xi(M) \subseteq \omega(M)$. Conversely, let $x \in \omega(M)$. Then $(x)^\tau \nsubseteq M$. Since M is maximal, we get $(x)^\tau \vee M = L$. Thus $x \in \xi(M)$. Hence $\omega(M) = \xi(M) = M$. Therefore M is strongly coherent.

(2) ⇒ (3): Assume that every maximal ideal of L is strongly coherent. Then every maximal ideal of L is a prime strongly coherent ideal. By Proposition 3.15, every maximal ideal is a minimal prime ideal.

(3) ⇒ (1): Assume that every maximal ideal of L is a minimal prime ideal. Let $x \in L$. Suppose $1 \notin (x] \vee (x)^*$. Then there exist a maximal ideal M such that $(x] \vee (x)^* \subseteq M$. Hence $x \in M$ and $(x)^* \subseteq M$. By (3), M is a minimal prime ideal. Since M is minimal, and $(x)^* \subseteq M$, we get $x \notin M$ that leads to a contradiction. Thus $1 \in (x] \vee (x)^*$. Then there exist $a \in (x)^*$ such that $1 = x \vee a$. Hence $x \wedge a = 0$ and $x \vee a = 1$. Thus a is the complement of x in L. Therefore L is a Boolean algebra. □

Theorem 3.17. *The following assertions are equivalent in a lattice L:*

(1) *L is relatively complemented;*
(2) *every ideal is strongly coherent;*
(3) *every prime ideal is strongly coherent;*
(4) *every prime ideal is median.*

Proof. (1) ⇒ (2): Assume that L is relatively complemented. Hence L is a Boolean algebra and thus by Theorem 3.14, every maximal ideal is median. Let I be an ideal of L. Clearly $\xi(I) \subseteq I$. Conversely, let $x \in I$. Suppose $(x)^\tau \vee I \neq L$. Then there exists a maximal ideal M such that $(x)^\tau \vee I \subseteq M$. Hence $(x)^\tau \subseteq M$ and $x \in I \subseteq M$. Clearly M is median. Since $(x)^\tau \subseteq M$, we get $x \notin M$ which is a contradiction. Thus $(x)^\tau \vee I = L$. Therefore I is a strongly coherent ideal.

(2) $\Rightarrow$ (3): It is obvious.

(3) $\Rightarrow$ (4): Assume condition (3). Let P be a prime ideal of L. By (3), we get $\xi(P) = P$. Let $x \in P = \xi(P)$. Then $(x)^\tau \vee P = L$. Hence $1 \in (x)^\tau \vee P$. Then $1 = a \vee b$ for some $a \in (x)^\tau$ and $b \in P$. Clearly $a \notin P$, otherwise $1 = a \vee b \in P$. Thus $x^* \vee a^* = 1$ for $a \notin P$. Therefore P is median.

(4) $\Rightarrow$ (1): Assume that every prime ideal is median. Let $a, b \in L$ and $a \in [0, b]$. Suppose $b \notin (a] \vee (a)^\tau$. Then there exists a prime ideal P such that $(a] \vee (a)^\tau \subseteq P$. Thus $a \in P$ and $(a)^\tau \subseteq P$. Since P is median, we get $a \notin P$ which is a contradiction. Hence $b \in (a] \vee (a)^\tau$. Then $b = s \vee t$ for some $s \in (a]$ and $t \in (a)^\tau$. Since $t \in (a)^\tau$, we get $a \wedge t = 0$. Hence $a = a \wedge b = a \wedge (s \vee t) = (a \wedge s) \vee (a \wedge t) = (a \wedge s) \vee 0 = a \wedge s = s$ because of $s \in (a]$. Hence $b = s \vee t = a \vee t$. Also, $t \leq a \vee t = b$. Hence $t \in [0, b]$. Thus t is the complement of a in $[0, b]$. Therefore L is relatively complemented. $\square$

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Department of Mathematics,
MVGR College of Engineering,
Chintalavalasa, Vizianagaram, Andhra Pradesh, India-535005
E-mail: mssraomaths35@rediffmail.com