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PÁL GYULA – JULIUS PAL (1881-1946), THE HUNGARIAN – DANISH MATHEMATICIAN

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Abstract. Julius Pal (Pál Gyula in Hungarian usage) was a noted mathematician of the first half of twentieth century. The first part of his life is connected to Hungary, while the second to Denmark. As a consequence of this fact, and to turbulent history, very few exact information concerning his life and work survived. We started our common research work some years ago on Pal, and now we are able to give an overview of his path of life, and some inspection into his scientific work. For studying his life we could mainly used only primary sources from state and primary archives both in Hungary and Denmark.

1. Years in Hungary (1881–1919)

Gyula Pál was born in the Hungarian city of Győr as Gyula Perl from a Jewish family. Later, in 1909, he "magyarized" his family name to Pál and religion to Roman catholic. The evidences are in Győr's Town Archives. In 1940 he managed somehow to get a false birth certificate with these latter data clearly to defend himself and his family from Nazis' pursuit. Burning a lot of letters and documents in 1944 had probably the same reason.

According to the birth register of Jewish church his father's name was Dávid Perl (merchant, later carrier) and mother's name Berta Perl. He attended the local renowned Benedictine grammar school. Among his schoolmates were the two Riesz brothers: Frederic (Frigyes) and Marcell, with whom Pál remained in close and friendly connection later on, too.

From grammar school's reports we can see that he was a talented pupil getting the best marks in nearly all subjects. Two other things also included in the reports that were characteristic of Pál and affected his whole life: poor health (exempt from gymnastics) and "non-regular" behaviour. His school leaving report (GCE) shows also univocal "excellent" in all the six subjects: Hungarian, Latin, Greek philosophy, Hungary's history, Mathematics, Physics.

After finishing grammar school in 1890 he continued his study at Budapest University. He got his degree there only in 1908 but meantime went to study at universities of Göttingen, Munich, and perhaps Paris. In Göttingen he was student of C. Charatheodory and worked together with A. Haar.

In the period of 1908–1909 he was a teacher at the vocational high school (Oberrealschule in German) of the Transylvanian town, Székelyudvarhely. Even as young man he made deep

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(sometimes controversial) impression on people because of his "non-regular" character. The noted Hungarian novelist Dezső Szabó included him in his autobiographical novel. Szabó also was a teacher at Székelyudvarhely and describes Pál as an intelligent, educated, and erudite personal, who at the same time perhaps too ambitious.

Besides teaching he made intensive research work under the guidance of F. Riesz, who was then professor of the nearby University of Kolozsvár (now Cluj in Romania). Between 1912 and 1915 Gyula Pál published nine papers in leading periodicals. In 1916 he got his doctorate from Kolozsvár University under F. Riesz.

Pál tried to get a job in a city with university having better conditions for research work. First in 1912 he applied for a high school job in Budapest supported by letters of recommendations from A. Haar and F. Riesz (both at Kolozsvár University). Two years later he tried a high school in Pozsony (now Bratislava in Slovakia) with recommendation letters not only from Haar and Riesz but from H. Tietze (Brünn Technische Hochschule). These applications were unsuccessful, but at the end of war in 1918 or 1919 he managed to get a job in Pozsony.

FIGURE 1. Pál Gyula, the talented young scientist.

During the First World War he did military service in Hungary's army as a volunteer officer in the Italian front. There he was wounded and a bullet remained in his back which made sitting difficult to him and affected badly his temper. For his wound and bravery he received a Hungarian army award in 1922.

According to Lone Pal and Nørlund's book [23] he participated in the revolutionary movement in 1918–1919. But it could not have been the main reason of his emigration (as in most cases), since he applied for a Danish visa before April 5, 1919 when the Hungarian Soviet Republic just started. Due to Nørlund the motive was his job loss as a consequence of Czechoslovakian occupation of Pozsony. He went to Copenhagen to have a job, as it turns out from the consulate's letter:

> *Das Kgl. Dänische Generalkonsulat bestätigt hierdurch, daß Herr Prof. Phil. Julius Pål aus Györ (Ungarn) von der dänischen Universität in Kopenhagen eingeladen wurde, dort für dänische Studenten Vorlesungen aus Mathematik abzuhalten.*
>
> *Das Generalkonsulat ersucht hiermit die kompetenten Behörden, einen Reisepass für Dänemark für Herrn Doktor Pal ausstellen zu wollen.*

The invitation letter must have come from Harald Bohr whom Pál met earlier. Pál mentioned in a "Julfest" speech in 1939 that his first visit to Denmark was 38 years ago. Further, he asked for an official certificate from the major of Székelyudvarhely in 1912 needed for his planned marriage in Copenhagen. The marriage was not realized then, but he went again a state budgeted study trip to Germany and France proved by a trilingual recommendation letter from Hungary's ministry of education. In 1914 he was in Göttingen sending the paper [3] from there. One footnote in [10] refers to talk with H. Lebesgue in Paris. Other sources confirm his stay there adding that the Danish mathematician Harald Bohr (brother of the Nobel-prize winner physicist Niels Bohr) was also in Paris and they studied Lebesgue integral together.

2. Copenhagen years (1919–1946)

The "Reisepass" meant a cornerstone in his life. His Hungarian name Pál Gyula changed to Julius Pal loosing also the accent mark on *a* in his family name. It should be noted that he kept Pál Gyula as signature in all his letters and felt home sickness for a long time.

The exact arrival date of Pal to Copenhagen is not known, but it must have been before August, since at the end of [10] we find: København August 1919. Besides H. Bohr, J. Møllerup, the president of Danish Mathematical Society helped him a lot at the beginning.

His teaching carrier started as temporary teacher at Skt. Jørgens Gymnasium (grammar school), where Børge Jessen was one of his pupil. Jessen later became a leading figure of Danish mathematical life and kept up friendly relations with many Hungarian mathematicians (F. Riesz, L. Kalmár, A. Rényi, P. Turán, etc.) Julius Pal seemed to be a mediator of relations between Hungarian and Danish scientists through Bohr brothers and Jessen.

Jessen and some other pupils of Pal wrote on their former teacher in their books [22,23,24, 25]. From these books and from some other sources (e.g. from his letters to B. Jessen and F. Riesz) one can form a picture of Pal as a teacher and personality. He was an enthusiastic and effective, as well as a high demanding, "tough" teacher. Because of his painful wound sometimes he was nervous and impatient, but at the same time helpful and friendly with a good sense of humour. His different "paprika-like" Hungarian temper caused also problems in a country of "cooler" mentality. Some people were offended by his behaviour misunderstanding it: taking his frankness and "fiery soul" as rudeness or abrasiveness, his self-respect as self-conceit, etc. Moreover, Pal had to work hard to care for his wife and daughter by teaching in a foreign country in a foreign language. He complained about it in a letter to F. Riesz.

In spite of the above mentioned problems most of his former students formed a positive opinion on him: " ... we are many who feel a great depth of gratitude toward him." [22]; "He became a very close friend of me, ... " [25]; " ... he left his stamp on me forever." [23].

Beside mathematics Pal had deep interest in politics, poetry and painting. His favourite poet was Goethe and one of his favourite painting was: Pipacsok a búzában by the Hungarian painter P. Szinnyei-Merse. When B. Jessen travelled to Hungary in 1929 Pal asked to see this picture in museum.

Pal himself could visit Hungary only two times. First in 1931 with his family spending several months not only there (Győr at his brother's) but in Austria, too. Secondly alone in 1935 proved by a postcard to Jessen from Szeged.

FIGURE 2. Julius Pal, the experienced teacher.

Pal married probably in 1921 (there was no state marriage certification system in Denmark then) with the daughter of the Danish painter Rudolf Bissen. Their only child, Ilona Birgit Pal, was born in 1922.

In 1925 Pal joined Polyteknisk Lareanstalt (Polytechnic) and remained there until his death. Beside his main job there he sometimes undertook temporary part time jobs, too. His polytechnics carrier started with an assistantship to Harald Bohr in 1925, followed by a lector (lecturer) post in 1926, and culminated with a king nominated docentship (associate professor) in 1929. As a precondition he was given Danish citizenship one year earlier. He taught mainly analysis and wrote a bulky and excellent textbook on it published in 1931 and reprinted in 1941.

From 1932 Pal was teaching assistant of H. Bohr at the university, too, and helped him to organize the mathematical institute there that was inaugurated in 1934 and bears H. Bohr's name today. Pal's role was also significant in the foundation of the new institute especially in forming its administrative structure. Besides he was the first librarian of the institute. Unfortunately in 1938 he left the university because of the worsened personal contacts with H. Bohr, B. Jessen and some other people.

Pal was frequently ill that can be concluded from his friends' correspondence and confirmed by his daughter. In spite of this he participated in the resistance during Nazi occupation. His bad health state became even worse when he got the news after the war on the death of his relatives in Hungary. This dreadful news surely contributed to his early death that happened in a Copenhagen hospital on September 6, 1946.

3. SCIENTIFIC WORK OF JULIUS PAL

To analyse his mathematical results and their impact need more research and space. Here we can only give a draft picture about his main research fields.

First of all he was a topologist studying mainly Jordan curves both in plane and space. The starting point of his investigations was the following problem raised by L. Fejér in his famous 1900 *Comptes Rendus Paris* paper:

Let the functions $\phi(t)$ and $\psi(t)$ be continuous in $[0, 2\pi]$ for which $\phi(0) = \phi(2\pi)$ and $\psi(0) = \psi(2\pi)$. Does there exist a complementary function $\aleph(t)$ such that ψ, ψ and $\aleph$ give a Jordan curve of circum-run time 2π?

In [6] Pal showed if ϕ and ψ have common period of form $2\pi/n$ ($n > 1$ integer), then complementary function does not exist. After he gave a sufficient condition for the existence of $\aleph$ when ϕ and ψ have no common period:

Für die Existenz einer Ergänzungsfunktion ist es hinreichend, daß es drei Werte: $a < b < c < a + 2\pi$ geben soll mit folgenden Eigenschaften:

1. *der Punkt $t = a$ der Kurve $x = \phi(t)$, $y = \psi(t)$ hat eine von Null verschiedene Entfernung vom Bogen $b \leq t \leq c$ der Kurve;*
2. *der Punkt $t = b$ ist ein einfacher Punkt des Bogens $a \leq t \leq b$ und der Punkt $t = c$ ein einfacher Punkt des Bogens $c \leq t \leq a + 2\pi$ [6, p.7]*

In [13] he proved the well-known theorem concerning Jordan-arcs stated that these arcs do not cut the plane into pieces.

Perhaps the most important paper of Pal is [11] which deals with the Kakeya problem: find a figure of the least area on which a segment of length 1 can be turned through 360° by a continuous movement. The Japanese mathematician S. Kakeya raised this problem in 1917 and conjectured the deltoid with area $\pi/8$ as solution. An other Japanese, Fujiwara supposed another solution: the equilateral triangle of height 1 as another solution. Fujiwara called the attention of Pal to this problem who proved this conjecture and gave a complete solution to it under the restriction that the required area should be convex using his so called Pal joins. Since then many researchers have investigated this problem generalizing it and raising new ones. The problem proved to be a starting point of a new branch of mathematics: geometric measure theory.

Pal generalized Weierstrass's approximation theorem in [3] giving condition for the existence of an approximation polynomial with integer coefficients:

Es sei $f(x)$ eine für $-1 \leq x \leq +1$ definierte stetige Funktion und sei $c_0 = f(0)$. Werden die Zahlen $c_1, c_2, \dots c_n$, beliebig gegeben, so gibt es ein Polynom $P(x)$, das im Intervall $(-1, +1)$ die Funktion $f(x)$ mit vorgeschriebener Genauigkeit ε approximirt und deren Anfangskoeffizienten $c_0, c_1, c_2, \dots, c_n$ sind.

For this theorem L. Fejér gave another proof using complex functions. And M. Fekete proved an important consequence of Pal's theorem.

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