*Acta Mathematica Academiae Paedagogicae Nyíregyháziensis*
**20** (2004), 127–129
www.emis.de/journals
ISSN 1786-0091

# GREETINGS FROM RUSSIA

VALENTIN SKVORTSOV

It is a great pleasure for me to greet Professor Wade on the occasion of his sixtieth birthday. I am greeting him on behalf of my colleagues in Moscow University who, like myself, have vivid and happy memories of his visit to Russia.

It was 26 years ago when I met Bill Wade in Moscow where he spent the academic year 1977-78 as a visiting professor at the Department of Mathematics of Moscow State University.

It was rather adventurous of him to come to Russia at that "cold war" period and to bring his family with him. But he came without prejudice, with an open heart, with great sympathy to Russian people, and even with some knowledge of Russian. I am confident that he and his wife Cherri found many friends in Moscow.

Above all, he came with a very good knowledge of the Russian literature in Harmonic Analysis. In fact he had used this knowledge in his very first publications. Being a representative of the V.L. Shapiro school in Harmonic Analysis, Wade was the first who managed to bring together methods developed both in this school and in the Russian school of Real Analysis. More precisely we should talk about the Soviet (or Lusin) school because besides Russians, mathematicians from many Soviet republics (now independent states such as Armenia, Kazakhstan, Georgia) formed this school.

By the time W.R.Wade started his research in the theory of uniqueness of Walsh and Haar series, some progress was made in this field by both schools. In the middle of the sixties, after pioneering papers by Fine, Vilenkin, Shneider, the uniqueness problem in the case of the Walsh series convergent to the Lebesgue integrable sums was solved independently by Arutunyan and Talalyan (1964) and by Crittenden and Shapiro (1965). By combining the American and Soviet techniques, Wade obtained in his first published paper [1] a theorem which generalized the results of Arutunyan and Talalyan, but which had a form similar to the theorem of Crittenden and Shapiro. In particular, following the idea of Arutunyan and Talalyan he proved first the result for Haar series and obtained the Walsh series result as a corollary. He also adapted successfully the Arutunyan-Talalyan techniques to get in [3] a uniqueness theorem for Cesaro summable Haar series. Another paper by Arutunyan was used by Wade in some of his papers on the uniqueness problem for the Walsh series convergent to sums which are integrable in a more general sense than Lebesgue integration (see [4] and [5]). Scneider elementary sets of uniqueness were used by Wade in [7] to produce some new examples of uniqueness sets for the group of integers of a $p$-series field.

In the opposite direction the papers of Wade which were focused on the growth conditions in the uniqueness (for example [5], [6], [8] ) brought this topic to the attention of Russian mathematicians. In some of my papers and in papers of my students, as well as in the subsequent papers of Wade (see [15], [11]) those conditions of growth or finiteness of the partial sums of the Walsh series were modified or weakened.

So our cooperation started in fact before Wade's visit to Russia. At the time Bill arrived to Moscow, he was most interested in the new concept of the dyadic derivative, which had been introduced a few years earlier by Butzer and Wagner. Some publications on this topic, including papers by F.Schipp, had appeared by that time. So Bill chose this topic as a subject for the course of his lectures at Moscow University. The course was very instructive, he prepared his lectures with great care, and, remarkably, the lectures were presented in Russian. We also started joint research on this topic, generalizing a theorem of Schipp on dyadic differentiation of Walsh functions. Our results were published later in a joint paper [14].

The contacts with Bill during his visit and later during my short visits to Tennessee were invariably very fruitful for me; first of all because of his deep engagement with any subject with which he is involved, and because of his thorough knowledge of all related literature.

This characteristic thoroughness and seriousness about whatever he is doing is vividly shown in his numerous very instructive surveys (see [9], [10], [12], [13]) and in his contribution to the encyclopedic monograph on Walsh series written jointly with Schipp, Simon and Pal.

Mathematics was not the only field of interest for Bill during his visit to Russia. Music and Russian religious life were other subjects which he and Cherri took to heart.

In the later years Wade's papers and his further results continue to influence our research in Moscow. One of the most frequently cited of Wade's results is his theorem on the countable union of closed sets of uniqueness for Walsh series (see [2]). This theorem was extended to the case of relatively closed sets by Kholshchevnikova who was a student at the time of Bill's visit to Moscow and attended his lecture there. Wade's theorem on $U$-sets for the group of integers of $p$-series field (see [7]) was extended by Bokaev (my former student from Kazakhstan) to the case of an arbitrary Vilenkin system. Nurkhanov, another student of mine from Kazakhstan, also used Wade's results in his research. So Kazakhstan, where Bokaev is at the moment Dean of the Department of Mathematics in Karaganda University, seems to be a place where Bill Wade would be warmly welcomed if he decided to extend his travelling further to the East from Budapest and Moscow (perhaps on the way to Japan?).

Let me conclude by wishing Bill Wade a long and interesting life in mathematics; and every happiness to his family.

## References

[1] W.R. Wade, A uniqueness theorem for Haar and Walsh series, *Trans. Amer. Math. Soc.* **141** (1969), 187–194.

[2] W.R. Wade, Summing closed $U$-sets for Walsh series, *Proc. Amer. Math. Soc.* **29** (1971), 123–125.

[3] W.R. Wade, Uniqueness theory for Cesaro summable Haar series, *Duke. Math. J.* **38** (1971), 221–227.

[4] W.R. Wade, Uniqueness of Haar series which are $(C, 1)$ summable to Denjoy integrable functions, *Trans. Amer. Math. Soc.* **176** (1973), 489–498.

[5] W.R. Wade, Growth of Haar series on the dyadic rational and uniqueness, *Proc. Amer. Math. Soc.* **50** (1975), 198–201.

[6] W.R. Wade, Growth condition and uniqueness for Walsh series, , *Mich. Math. J.* **24** (1977), 153–155.

[7] W.R. Wade, Sets of uniqueness for the group of interges of a $p$-series field, *Canad. J. Math.* **31** (1979), 858–866.

[8] W.R. Wade, Uniqueness of Walsh series which satisfy an averaged growth condition, SIAM J.Math.Anal.11 (1980),933-937.

[9] W.R. Wade, Recent developments in the theory of Walsh series, International J. Math.5 (1982),625-673

[10] W.R. Wade, Recent developments in the theory of Haar series, *Colloq. Math.* **52** (1987), 213–238.

[11] W.R. Wade, A unified approach to uniqueness of Walsh and Haar series, *Proc. Amer. Math. Soc.* **99** (1987), 61–65.

[12] W.R. Wade, Vilenkin-Fourier series and approximation, *Colloq. Math. Soc. Janos Bolyai* **58** (1990), 699–734.

[13] W.R. Wade, Dyadic harmonic analysis, *Contemp. Math* **208** (1997), 313–350.

[14] V.A.Skvortsov and W.R. Wade, Generalization of some results concerning Walsh series and the dyadic derivative, *Analysis Math.* **5** (1979), 249–255.

[15] W.R. Wade and K. Yoneda, Uniqueness and quasi-measure on the group of integers of a $p$-series field, *Proc. Amer. Math. Soc.* **84** (1982), 202–206.