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NEW CRITERIA FOR UNIVALENCE OF CERTAIN INTEGRAL OPERATORS

B. A. FRASIN

Abstract. In this paper, we obtain the sufficient condition $\left|\frac{zf''(z)}{f'(z)}\right| < \frac{3}{2}\left|\frac{zf'(z)}{f(z)} - 1\right|$ for analytic function f to be univalent and starlike in the open unit disk. Furthermore, we derive new univalence conditions for the integral operators $\left\{\gamma\int\limits_0^z t^{\gamma-1}\left(g'(t)\right)^{\alpha}\left(\frac{g(t)}{t}\right)^{\beta}dt\right\}^{\frac{1}{\gamma}}$, $\left\{\alpha\int\limits_0^z t^{\alpha-1}\left(g'(t)\right)^{\frac{1}{\alpha}}\left(\frac{g(t)}{t}\right)^{\frac{1}{\beta}}dt\right\}^{\frac{1}{\alpha}}$ and $\left\{\alpha\int\limits_0^z t^{\alpha-1}\left(g'(t)\right)^{\alpha}\left(\frac{g(t)}{t}\right)^{\beta}dt\right\}^{\frac{1}{\alpha}}$, where g in the last two integrals satisfies the above inequality.

1. Introduction and definitions

Let $\mathcal{A}$ denote the class of functions of the form:

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n \tag{1.1}$$

which are analytic in the open unit disk $\mathcal{U} = \{z : |z| < 1\}$. Further, by $\mathcal{S}$ we shall denote the class of all functions in $\mathcal{A}$ which are univalent in $\mathcal{U}$. A function $f(z) \in \mathcal{S}$ is said to be starlike if it satisfies

$$\operatorname{Re}\left(\frac{zf'(z)}{f(z)}\right) > 0 \qquad (z \in \mathcal{U}). \tag{1.2}$$

In the last two decades many authors (see for example [1, 5, 10, 8, 9, 11, 13]) have obtained various sufficient conditions for the univalence of the integral operators

$$\left\{\alpha\int\limits_0^z t^{\alpha-1}\left(\frac{g(t)}{t}\right)^{\frac{1}{\beta}}dt\right\}^{\frac{1}{\alpha}}, \left\{\alpha\int\limits_0^z t^{\alpha-1}\left(\frac{g(t)}{t}\right)^{\beta}dt\right\}^{1/\alpha}, \left\{\alpha\int\limits_0^z t^{\alpha-1}\left(\frac{g(t)}{t}\right)^{\alpha-1}dt\right\}^{1/\alpha},$$

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$\left\{\alpha \int_0^z t^{\alpha-1}(g'(t))dt\right\}^{1/\alpha}$ and $\left\{\alpha \int_0^z t^{\alpha-1}(g'(t))^{\beta}dt\right\}^{1/\alpha}$, where the function g belongs to the class $\mathcal{A}$ and the parameters α, β are complex numbers such that the above integrals are exist. Here and throughout in the sequel every many-valued function is taken with the principal branch.

In this paper, we are mainly interested on some integral operators of the type

$$F_{\alpha,\beta}(z) = \left\{\alpha \int_0^z t^{\alpha-1}(g'(t))^{\frac{1}{\alpha}}\left(\frac{g(t)}{t}\right)^{\frac{1}{\beta}} dt\right\}^{\frac{1}{\alpha}}, \tag{1.3}$$

$$G_{\alpha,\beta}(z) = \left\{\alpha \int_0^z t^{\alpha-1}(g'(t))^{\alpha}\left(\frac{g(t)}{t}\right)^{\beta} dt\right\}^{\frac{1}{\alpha}}, \tag{1.4}$$

and

$$H_{\alpha,\beta,\gamma}(z) = \left\{\gamma \int_0^z t^{\gamma-1}(g'(t))^{\alpha}\left(\frac{g(t)}{t}\right)^{\beta} dt\right\}^{\frac{1}{\gamma}}, \tag{1.5}$$

where $g \in \mathcal{A}$ and $\alpha, \beta, \gamma \in \mathbb{C}\backslash\{0\}$. More precisely, we would like to derive new sufficient condition for univalency of the integral operators of the type (1.3), (1.4) and (1.5) by using the results from the proofs of theorems in [10, 8, 9], and the results from [11, 13] and by making use of the following lemmas.

Lemma 1.1 (see [7]). *Let $\alpha \in \mathbb{C}$ with $\mathrm{Re}(\alpha) > 0$. If $f \in \mathcal{A}$ satisfies*

$$\frac{1-|z|^{2\,\mathrm{Re}(\alpha)}}{\mathrm{Re}(\alpha)}\left|\frac{zf''(z)}{f'(z)}\right| \leq 1, \tag{1.6}$$

for all $z \in \mathcal{U}$, then the integral operator

$$F_{\alpha}(z) = \left\{\alpha \int_0^z t^{\alpha-1} f'(t)dt\right\}^{\frac{1}{\alpha}} \tag{1.7}$$

is analytic and univalent in $\mathcal{U}$.

Lemma 1.2 (see [6]). *Let $\alpha \in \mathbb{C}$ with $\mathrm{Re}(\alpha) > 0$. If $f \in \mathcal{A}$ satisfies* (1.6) *for all $z \in \mathcal{U}$, then, for any complex number γ with $\mathrm{Re}(\gamma) \geq \mathrm{Re}(\alpha)$, the integral operator*

$$G_{\gamma}(z) = \left\{\gamma \int_0^z t^{\gamma-1} f'(t)dt\right\}^{\frac{1}{\gamma}} \tag{1.8}$$

is analytic and univalent in $\mathcal{U}$.

Lemma 1.3 (see [4])**.** *If the function $g(z)$ is regular in $\mathcal{U}$, then, for all $\xi \in \mathcal{U}$ and $z \in \mathcal{U}$, $g(z)$ satisfies*

$$\left| \frac{g(\xi) - g(z)}{1 - g(\xi)\overline{g(z)}} \right| \leq \left| \frac{\xi - z}{1 - \xi \overline{z}} \right|, \tag{1.9}$$

$$|g'(z)| \leq \frac{1 - |g(z)|^2}{1 - |z|^2}. \tag{1.10}$$

The equality holds only for $g(z) = \varepsilon((z + u)/(1 + \overline{u}z))$, where $|\varepsilon| = 1$ and $|u| < 1$.

Remark 1.4 (see [4]). For $z = 0$, from the inequality (1.9),

$$\left| \frac{g(\xi) - g(0)}{1 - g(\xi)\overline{g(0)}} \right| \leq |\xi|, \tag{1.11}$$

and hence

$$|g(\xi)| \leq \frac{|\xi| + |g(0)|}{1 + |g(0)|\,|\xi|}. \tag{1.12}$$

considering $g(0) = \delta$ and $\xi = z$, we see that

$$|g(z)| \leq \frac{|z| + |\delta|}{1 + |\delta|\,|z|} \tag{1.13}$$

for all $z \in \mathcal{U}$.

Lemma 1.5 (Schwarz Lemma [4])**.** *Let the function $g(z)$ be regular in $\mathcal{U}$, $g(0) = 0$, and $|g(z)| \leq 1$, for all $z \in \mathcal{U}$, then*

$$|g(z)| \leq |z| \tag{1.14}$$

for all $z \in \mathcal{U}$, and $|g'(0)| \leq 1$. The equality in (1.14) for $z \neq 0$ holds only if $g(z) = \varepsilon z$, where $|\varepsilon| = 1$.

Lemma 1.6 (see [2])**.** *If $f \in \mathcal{S}$ then*

$$\left| \frac{zf'(z)}{f(z)} \right| < \frac{1 + |z|}{1 - |z|} \qquad (z \in \mathcal{U}). \tag{1.15}$$

Further, we need the following lemma:

Lemma 1.7. *Let $f \in \mathcal{A}$ and $zf'(z)/f(z) \neq 1$ in $\mathcal{U}$ and suppose that*

$$\left| \frac{zf''(z)}{f'(z)} \right| < \frac{3}{2} \left| \frac{zf'(z)}{f(z)} - 1 \right| \qquad (z \in \mathcal{U}), \tag{1.16}$$

then f is univalent and starlike in $\mathcal{U}$.

Proof. Let $w(z)$ be defined by

$$\frac{zf'(z)}{f(z)} = 1 + w(z). \tag{1.17}$$

Then $w(z)$ is analytic in $\mathcal{U}$ with $w(0) = 0$. By the logarithmic differentiations, we get from (1.17) that

$$\frac{zf''(z)}{f'(z)} = w(z) + \frac{zw'(z)}{1 + w(z)}. \tag{1.18}$$

It follows that

$$\left|\frac{zf''(z)/f'(z)}{zf'(z)/f(z) - 1}\right| \geq \operatorname{Re}\left(\frac{zf''(z)/f'(z)}{zf'(z)/f(z) - 1}\right) = \operatorname{Re}\left(1 + \frac{zw'(z)}{w(z)}\frac{1}{1 + w(z)}\right). \tag{1.19}$$

Suppose that there exists $z_0 \in \mathcal{U}$ such that

$$\max_{|z|<|z_0|} |w(z)| = |w(z_0)| = 1, \tag{1.20}$$

and let $w(z_0) = e^{i\theta}(\theta \neq \pi)$, then from Jack's Lemma [3], we have

$$z_0 w'(z_0) = kw(z_0), \tag{1.21}$$

where $k \geq 1$ is a real number. From (1.19), we obtain

$$\left|\frac{z_0 f''(z_0)/f'(z_0)}{z_0 f'(z_0)/f(z_0) - 1}\right| \geq \operatorname{Re}\left(1 + \frac{z_0 w'(z_0)}{w(z_0)}\frac{1}{1 + w(z_0)}\right) = \operatorname{Re}\left(1 + k\frac{1}{1 + e^{i\theta}}\right) \geq \frac{3}{2}$$

which contradicts our assumption (1.16). Therefore, $|w(z)| < 1$ holds for all $z \in \mathcal{U}$, or equivalently,

$$\left|\frac{zf'(z)}{f(z)} - 1\right| < 1.$$

This completes the proof of Lemma 1.7. □

Throughout our present discussion, we assume that $zf'(z)/f(z) \neq 1$ in $\mathcal{U}$ for any analytic function f in $\mathcal{U}$.

2. Univalence conditions

We begin by proving the following theorem:

Theorem 2.1. *Let $g \in \mathcal{A}$ satisfies the condition* (1.16) *and $\alpha, \beta \in \mathbb{C}\backslash\{0\}$ with $|\alpha| \leq |\beta|$. If $\alpha = a + bi$; $a \in (0, 10]$ and*

$$a^4 + a^2b^2 - 25 \geq 0,\ a \in (0, 1/2) \quad a^2 + b^2 - 100 \geq 0,\ a \in [1/2, 10] \tag{2.1}$$

then the integral operator $F_{\alpha,\beta}(z)$ defined by (1.3) *is analytic and univalent in $\mathcal{U}$.*

Proof. Define

$$h(z) = \int_0^z (g'(t))^{\frac{1}{\alpha}} \left(\frac{g(t)}{t}\right)^{\frac{1}{\beta}} dt,$$

so that, obviously

$$h'(z) = (g'(z))^{\frac{1}{\alpha}} \left(\frac{g(z)}{z}\right)^{\frac{1}{\beta}}, \tag{2.2}$$

and so $h(0) = h'(0) - 1 = 0$. Differentiating both sides of (2.2) logarithmically, we obtain

$$\frac{zh''(z)}{h'(z)} = \frac{1}{\alpha}\left(\frac{zg''(z)}{g'(z)}\right) + \frac{1}{\beta}\left(\frac{zg'(z)}{g(z)} - 1\right). \tag{2.3}$$

Making use of Lemma 1.7 and Lemma 1.6, from (2.3), we obtain

$$\begin{aligned} \left|\frac{zh''(z)}{h'(z)}\right| &\leq \frac{3}{2\left|\alpha\right|}\left|\frac{zg'(z)}{g(z)} - 1\right| + \frac{1}{\left|\beta\right|}\left|\frac{zg'(z)}{g(z)} - 1\right| \\ &\leq \left(\frac{3}{2\left|\alpha\right|} + \frac{1}{\left|\beta\right|}\right)\left|\frac{zg'(z)}{g(z)} - 1\right| \\ &\leq \frac{5}{2\left|\alpha\right|}\left(\left|\frac{zg'(z)}{g(z)}\right| + 1\right) \\ &\leq \frac{5}{2\left|\alpha\right|}\left(\frac{1+|z|}{1-|z|} + 1\right) \end{aligned}$$

which readily shows that

$$\frac{1-|z|^{2a}}{a}\left|\frac{zh''(z)}{h'(z)}\right| \leq \frac{5(1-|z|^{2a})}{2a\sqrt{a^2+b^2}}\left(\frac{1+|z|}{1-|z|} + 1\right).$$

Thus, we have

$$\frac{1-|z|^{2a}}{a}\left|\frac{zh''(z)}{h'(z)}\right| \leq \frac{5}{a\sqrt{a^2+b^2}}\frac{1-|z|^{2a}}{1-|z|} \tag{2.4}$$

for all $z \in \mathcal{U}$.

Define the function $\Psi : (0,1) \to \mathbb{R}$, by

$$\Psi(x) = \frac{1-x^{2a}}{1-x}, \quad (x = |z|\,,\ a > 0).$$

Then it easy to prove that

$$\Psi(x) = \begin{cases} 1, & \text{if } a \in (0, \frac{1}{2}), \\ 2a, & \text{if } a \in [\frac{1}{2}, \infty). \end{cases} \tag{2.5}$$

For $a \in (0, 10]$, from (2.4), (2.5) and the hypothesis (2.1), it follows that

$$\frac{1-|z|^{2a}}{a}\left|\frac{zh''(z)}{h'(z)}\right| \leq 1$$

for all $z \in \mathcal{U}$. Applying Lemma 1.1 for the function $h(z)$, we prove that $F_{\alpha,\beta}(z)$ is analytic and univalent in $\mathcal{U}$. $\square$

Next, we prove

Theorem 2.2. *Let* $\alpha, \beta \in \mathbb{C}\backslash\{0\}$ *with* $|\alpha| \geq |\beta|$ *and* $\mathrm{Re}(\alpha) = a > 0$. *If* $g \in \mathcal{A}$ *satisfies the condition* (1.16) *and*

$$\left|\frac{zg'(z)}{g(z)} - 1\right| \leq 1 \qquad (z \in \mathcal{U}). \tag{2.6}$$

Then, for

$$|\alpha| \leq \frac{(2a+1)^{(2a+1)/2a}}{5} \tag{2.7}$$

the integral operator $G_{\alpha,\beta}(z)$ *defined by* (1.4) *is analytic and univalent in* $\mathcal{U}$.

Proof. Consider the function

$$f(z) = \int_0^z (g'(t))^\alpha \left(\frac{g(t)}{t}\right)^\beta dt. \tag{2.8}$$

Then the function

$$h(z) = \left(\frac{2}{5\alpha}\right)\frac{zf''(z)}{f'(z)} \tag{2.9}$$

is regular in $\mathcal{U}$, where the constant $|\alpha|$ satisfies inequality (2.7). From (2.8), it follows that

$$\frac{zf''(z)}{f'(z)} = \alpha\left(\frac{zg''(z)}{g'(z)}\right) + \beta\left(\frac{zg'(z)}{g(z)} - 1\right) \tag{2.10}$$

Applying Lemma 1.7, we have that

$$\begin{aligned}\left|\frac{zf''(z)}{f'(z)}\right| &\leq |\alpha|\left|\frac{zg''(z)}{g'(z)}\right| + |\beta|\left|\frac{zg'(z)}{g(z)} - 1\right| \\ &\leq \frac{3|\alpha|}{2}\left|\frac{zg'(z)}{g(z)} - 1\right| + |\alpha|\left|\frac{zg'(z)}{g(z)} - 1\right| \\ &\leq \frac{5|\alpha|}{2}\left|\frac{zg'(z)}{g(z)} - 1\right|\end{aligned} \tag{2.11}$$

using (2.6) , (2.9) and (2.11), we obtain

$$|h(z)| \leq 1 \quad (z \in \mathcal{U}).$$

Since $h(0) = 0$, applying the Schwarz lemma for $h(z)$, we get

$$\frac{2}{5|\alpha|}\left|\frac{zf''(z)}{f'(z)}\right| \leq |z| \quad (z \in \mathcal{U}).$$

Thus, we have that

$$\frac{1-|z|^{2a}}{a}\left|\frac{zf''(z)}{f'(z)}\right| \leq \frac{5\,|\alpha|}{2}\left(\frac{1-|z|^{2a}}{a}\right)|z| \quad (z \in \mathcal{U}). \tag{2.12}$$

Because

$$\max_{|z|\leq 1}\left(\frac{1-|z|^{2a}}{a}\,|z|\right) = \frac{2}{(2a+1)^{(2a+1)/2a}},$$

from (2.12) and (2.7), we obtain

$$\frac{1-|z|^{2a}}{a}\left|\frac{zf''(z)}{f'(z)}\right| \leq 1. \tag{2.13}$$

Applying Lemma 1.1, from (2.13), it follows that the integral operator $G_{\alpha,\beta}(z)$ defined by (1.4) is analytic and univalent in $\mathcal{U}$. □

Finally, we prove

Theorem 2.3. *Let $\alpha, \beta \in \mathbb{C}\backslash\{0\}$ with $|\alpha| \leq |\beta|$ and $\operatorname{Re}(\gamma) \geq \operatorname{Re}(\alpha)$. If $g \in \mathcal{A}$ satisfies*

$$\left|\frac{g''(z)}{g'(z)}\right| + \left|\frac{zg'(z)-g(z)}{zg(z)}\right| \leq 1 \quad (z \in \mathcal{U}). \tag{2.14}$$

Then, for

$$|\alpha| \geq \max_{|z|\leq 1}\left\{\left(\frac{1-|z|^{2\operatorname{Re}(\alpha)}}{\operatorname{Re}(\alpha)}\right)|z|\left(\frac{|\alpha\beta z| + |a_2(\alpha+2\beta)|}{|\alpha\beta| + |a_2(\alpha+2\beta)|\,|z|}\right)\right\}, \tag{2.15}$$

the integral operator $H_{\alpha,\beta,\gamma}(z)$ defined by (1.5) is analytic and univalent in $\mathcal{U}$.

Proof. Define the regular function $f(z)$ in $\mathcal{U}$ by

$$f(z) = \int_0^z (g'(t))^{\frac{1}{\alpha}}\left(\frac{g(t)}{t}\right)^{\frac{1}{\beta}} dt. \tag{2.16}$$

The function

$$p(z) = \alpha\frac{f''(z)}{f'(z)}, \tag{2.17}$$

is regular in $\mathcal{U}$, where the constant $|\alpha|$ satisfies inequality (2.15). From (2.17) and (2.16), we have that

$$\frac{f''(z)}{f'(z)} = \frac{1}{\alpha}\left(\frac{g''(z)}{g'(z)}\right) + \frac{1}{\beta}\left(\frac{zg'(z)-g(z)}{zg(z)}\right) \tag{2.18}$$

and so

$$\left|\frac{f''(z)}{f'(z)}\right| \leq \left|\frac{1}{\alpha}\right|\left|\frac{g''(z)}{g'(z)}\right| + \left|\frac{1}{\alpha}\right|\left|\frac{zg'(z)-g(z)}{zg(z)}\right|$$

$$\leq \left|\frac{1}{\alpha}\right| \left[\left|\frac{g''(z)}{g'(z)}\right| + \left|\frac{zg'(z)-g(z)}{zg(z)}\right|\right].$$

Using the hypothesis (2.14), we obtain

$$|p(z)| \leq 1 \quad (z \in \mathcal{U})$$

and from (2.18), we have $|p(0)| = \left|\frac{a_2(\alpha+2\beta)}{\alpha\beta}\right|$. Consequently, from (1.13), we obtain

$$\left|\alpha\frac{f''(z)}{f'(z)}\right| \leq \frac{|z| + \left|\frac{a_2(\alpha+2\beta)}{\alpha\beta}\right|}{1+\left|\frac{a_2(\alpha+2\beta)}{\alpha\beta}\right||z|} \quad (z \in \mathcal{U}). \tag{2.19}$$

It follows that

$$\frac{1-|z|^{2\operatorname{Re}(\alpha)}}{\operatorname{Re}(\alpha)}\left|\frac{zf''(z)}{f'(z)}\right| \tag{2.20}$$
$$\leq \left|\frac{1}{\alpha}\right|\left(\frac{1-|z|^{2\operatorname{Re}(\alpha)}}{\operatorname{Re}(\alpha)}\right)|z|\left(\frac{|\alpha\beta z| + |a_2(\alpha+2\beta)|}{|\alpha\beta| + |a_2(\alpha+2\beta)|\,|z|}\right).$$

Define the function $\Phi : [0,1] \to \mathbb{R}$, by

$$\Phi(x) = \left(\frac{1-x^{2\operatorname{Re}(\alpha)}}{\operatorname{Re}(\alpha)}\right)\left(\frac{|\alpha\beta|\,x^2 + |a_2(\alpha+2\beta)|\,x}{|\alpha\beta| + |a_2(\alpha+2\beta)|\,x}\right) \quad (x = |z|), \tag{2.21}$$

we have $\Phi(1/2) > 0$, and thus

$$\max_{x\in[0,1]} \Phi(x) > 0. \tag{2.22}$$

Using (2.22) , from (2.20) we obtain

$$\frac{1-|z|^{2\operatorname{Re}(\alpha)}}{\operatorname{Re}(\alpha)}\left|\frac{zf''(z)}{f'(z)}\right| \leq$$
$$\leq \frac{1}{|\alpha|}\max_{|z|\leq 1}\left\{\left(\frac{1-|z|^{2\operatorname{Re}(\alpha)}}{\operatorname{Re}(\alpha)}\right)|z|\left(\frac{|\alpha\beta z| + |a_2(\alpha+2\beta)|}{|\alpha\beta| + |a_2(\alpha+2\beta)|\,|z|}\right)\right\} \tag{2.23}$$

from (2.4), (2.5) and the hypothesis (2.1), it follows that

$$\frac{1-|z|^{2\operatorname{Re}(\alpha)}}{\operatorname{Re}(\alpha)}\left|\frac{zf''(z)}{f'(z)}\right| \leq 1$$

for all $z \in \mathcal{U}$. Applying Lemma 1.2 for the function $f(z)$, we prove that the integral operator $H_{\alpha,\beta,\gamma}(z)$ defined by(1.5) is analytic and univalent in $\mathcal{U}$. □

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Department of Mathematics,
Faculty of Science,
Al al-Bayt University,
P.O. Box: 130095 Mafraq,
Jordan.
E-mail address: bafrasin@yahoo.com