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OPERATIONAL RULES AND ARBITRARY ORDER TWO-INDEX TWO-VARIABLE HERMITE MATRIX GENERATING FUNCTIONS

M. S. METWALLY

Abstract. The main aim of this paper is to introduce generalized forms of operational rules associated with operators corresponding to a generalized Hermite matrix polynomials expansions. The associated generating functions is reformulated within the framework of an operational formalism and the theory of exponential operators. We obtain to unilateral and bilateral generating functions by using the same procedure. Possible extensions of the technique are also discussed.

1. Introduction and preliminaries

An extension to the matrix framework of the classical families of Hermite, Jacobi, Laguerre, Legendre and Tchebicheff polynomials have been introduced and studied in a number of previous papers [1, 14, 13, 15, 16, 18, 17, 20] in $\mathbb{C}^{N\times N}$. The Hermite matrix polynomials of the associated generating functions is reformulated within the framework of an operational formalism. In [4, 3], by using the monomiality principle, by exploiting operational methods, many properties of ordinary are easily derived and framed in a more general context and to study the properties of new families of special functions. This approach has indeed allowed the derivation of the Burchnall identity and of its extension to the Hermite matrix polynomials [2, 7].

The use of operational identities [6, 8, 12, 21], currently exploited in the theory of algebraic decomposition of exponential operators, may significantly simplify the study of Hermite matrix generating functions and the discovery of new relations, hardly achievable by conventional means. Infinite sums, involving Hermite polynomials, notwithstanding the fact that a systematic investigation on this subject is still lacking and the relevant knowledge is limited [11].

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Before entering into more technical details, we will introduce some identities that will be largely exploited in this work.

If D_0 is the complex plane cut along the negative real axis and $\log(z)$ denotes the principal logarithm of z, then $z^{\frac{1}{2}}$ represents $\exp(\frac{1}{2}\log(z))$. If A is a matrix in $\mathbb{C}^{N\times N}$, its two-norm denoted $||A||_2$ is defined by

$$||A||_2 = \sup_{x\neq 0} \frac{||Ax||_2}{||x||_2},$$

where for a vector y in $\mathbb{C}^N$, $||y||_2$ denotes the usual Euclidean norm of y, $||y||_2 = (y^T y)^{\frac{1}{2}}$. The set of all the eigenvalues of A is denoted by $\sigma(A)$. If $f(z)$ and $g(z)$ are holomorphic functions of the complex variable z, which are defined in an open set Ω of the complex plane, and if A is a matrix in $\mathbb{C}^{N\times N}$ such that $\sigma(A) \subset \Omega$, then from the properties of the matrix functional calculus, it follows that

$$f(A)g(A) = g(A)f(A). \tag{1.1}$$

we say that a matrix A in $\mathbb{C}^{N\times N}$ is a positive stable matrix if [13, 16, 18, 17]

$$Re(z) > 0, \quad \text{for all } \; z \in \sigma(A). \tag{1.2}$$

A sequence of polynomial $p_n(x)$ $(n \in N, x \in C)$ is said a quasi monomial whenever two operators $\widehat{M}$ and $\widehat{P}$, called multiplication and derivative operators on [4, 3, 8, 12], can be defined three operators in such a way that

$$\begin{aligned} \widehat{M} p_n(x) &= p_{n+1}(x), \\ \widehat{P} p_n(x) &= n p_{n-1}(x). \end{aligned} \tag{1.3}$$

By combining the above recurrences, we also find

$$\widehat{M}\widehat{P} p_n(x) = n p_n(x), \tag{1.4}$$

where $\widehat{M}$, $\widehat{P}$ and $\widehat{M}\widehat{P}$ are called respectively the lowering, the raising and the transfer operators associated to the polynomial set $p_n(x)$ and which can be interpreted as the differential equation defining $p_n(x)$ if $\widehat{M}$ and $\widehat{P}$ have a differential realization.

Furthermore, if $p_0(x) = 1$ from the first of (1.5) it follows that

$$\widehat{M}^n 1 = p_n(x). \tag{1.5}$$

We define the Burchnall identity [2]

$$\exp\left(y\frac{\partial^m}{\partial x^m}\right) x^n = \left(x + my\frac{\partial^{m-1}}{\partial x^{m-1}}\right)^n. \tag{1.6}$$

This paper is devoted to a more substantive effort in proofs of some known properties as well as new expansions formulae related to these Hermite matrix polynomials. Many properties of these polynomials have been straight forwardly derived within this new framework, which has allowed the possibility of introducing two-index two-variable generating Hermite function.

2. Operational identities and properties of two-index two-variable Hermite matrix polynomials

One of the most direct ways of exploring generalized classes of Hermite matrix polynomials is to start from modified forms of the ordinary Hermite matrix polynomials generating function. We consider therefore the generalized Hermite matrix polynomials $H_{n,m}(x,y,A)$ defined a two-index two-variable by the generating function

$$\sum_{n=0}^{\infty} \frac{t^n}{n!} H_{n,m}(x,y,A) = \exp\left(xt\sqrt{mA} - yt^m I\right) \tag{2.1}$$

with m must be a positive integer. We obtain an explicit representation for the two-variable Hermite matrix polynomials in the form

$$H_{n,m}(x,y,A) = n! \sum_{k=0}^{[\frac{n}{m}]} \frac{(-1)^k y^k (x\sqrt{mA})^{n-mk}}{k!(n-mk)!}. \tag{2.2}$$

Their recurrence properties can be derived either from (2.1) and (2.2). It is indeed easy to prove that

$$\begin{aligned}
&\frac{\partial}{\partial x} H_{n,m}(x,y,A) = n\sqrt{mA} H_{n-1,m}(x,y,A),\\
&\frac{\partial}{\partial y} H_{n,m}(x,y,A) = -\frac{n!}{(n-m)!} H_{n-m,m}(x,y,A), \quad n \geq m,\\
&H_{n+1,m}(x,y,A) = \left[x\sqrt{mA} - my(\sqrt{mA})^{-(m-1)} \frac{\partial^{m-1}}{\partial x^{m-1}}\right] H_{n,m}(x,y,A)
\end{aligned} \tag{2.3}$$

which once combined yield

$$\frac{\partial^m}{\partial x^m} H_{n,m}(x,y,A) + (\sqrt{mA})^m \frac{\partial}{\partial y} H_{n,m}(x,y,A) = 0. \tag{2.4}$$

We use Hermite polynomials to show that the monomiality principle can be exploited to study the properties of the polynomials. Furthermore, according to (2.4), some times called heat polynomials, the $H_{n,m}(x,y,A)$ are said to be under the action of the operators

$$\begin{aligned}
\widehat{P} &= \sqrt{mA}^{-1} \frac{\partial}{\partial x},\\
\widehat{M} &= x\sqrt{mA} - my(\sqrt{mA})^{-(m-1)} \frac{\partial^{m-1}}{\partial x^{m-1}}
\end{aligned} \tag{2.5}$$

which act on $H_{n,m}(x,y,A)$ according to the rules

$$\begin{aligned}
\widehat{P} H_{n,m}(x,y,A) &= nH_{n-1,m}(x,y,A),\\
\widehat{M} H_{n,m}(x,y,A) &= H_{n+1,m}(x,y,A).
\end{aligned} \tag{2.6}$$

Whereas the identity

$$\widehat{M}\widehat{P}H_{n,m}(x,y,A) = nH_{n,m}(x,y,A) \tag{2.7}$$

holds using the explicit definition of $\widehat{M}$ and $\widehat{P}$ given by (2.5), we find that $H_{n,m}(x,y,A)$ satisfies the following differential equation of the m-th order in the form

$$\left[y\frac{\partial^m}{\partial x^m} - \frac{x}{m}(\sqrt{mA})^m\frac{\partial}{\partial x} + \frac{n}{m}(\sqrt{mA})^m\right]H_{n,m}(x,y,A) = 0. \tag{2.8}$$

We also find

$$H_{n,m}(x,y,A) = n!\sum_{k=0}^{[\frac{n}{m}]}\frac{(-1)^k y^k}{k!}(\sqrt{mA})^{-(mk)}\frac{\partial^{mk}}{\partial x^{mk}}(x\sqrt{mA})^n \tag{2.9}$$

which can be used as an alternative to the series (2.2) and which can be viewed as an alternative to Rodrigues's formula (2.9).

To this aim we remind that the $H_{n,m}(x,y,A)$ are the natural solutions of the heat partial differential equation

$$\begin{aligned}(\sqrt{mA})^{-m}\frac{\partial^m}{\partial x^m}F(x,y,A) + \frac{\partial}{\partial y}F(x,y,A) = 0,\\ F(x,0,A) = (x\sqrt{mA})^n.\end{aligned} \tag{2.10}$$

According to (2.10), we can define the $H_{n,m}(x,y,A)$ through the operational rule

$$H_{n,m}(x,y,A) = \exp\left(-y(\sqrt{mA})^{-m}\frac{\partial^m}{\partial x^m}\right)\left(x\sqrt{mA}\right)^n \tag{2.11}$$

which can be used as an alternative to the series (2.1).

According to the previous discussion can be viewed as a differential problem to

$$x\frac{\partial}{\partial x}(x\sqrt{mA})^n = (x\sqrt{mA})\sqrt{mA}^{-1}\frac{\partial}{\partial x}(x\sqrt{mA})^n = n(x\sqrt{mA})^n, \tag{2.12}$$

where $x\sqrt{mA}$ and $\sqrt{mA}^{(-1)}\frac{\partial}{\partial x}$ are the multiplicative and derivative operators for $(x\sqrt{mA})^n$. We also underline that (2.8) can be obtained from (2.12) by just applying the previous exponential operator to both sides. Namely,

$$\begin{aligned}\exp\left(-y(\sqrt{mA})^{-m}\frac{\partial^m}{\partial x^m}\right)\left(x\frac{\partial}{\partial x}(x\sqrt{mA})^n\right)\\ = n\exp\left(-y(\sqrt{mA})^{-m}\frac{\partial^m}{\partial x^m}\right)\left(x\sqrt{mA}\right)^n\end{aligned} \tag{2.13}$$

from which (2.8) follows, after using (2.11) and after noting that

$$
\begin{aligned}
&\exp\left(-y(\sqrt{mA})^{-m}\frac{\partial^m}{\partial x^m}\right)\left(x\frac{\partial}{\partial x}(x\sqrt{mA})^n\right) = \\
&= \left[\exp\left(-y(\sqrt{mA})^{-m}\frac{\partial^m}{\partial x^m}\right)x\sqrt{mA}\exp\left(y(\sqrt{mA})^{-m}\frac{\partial^m}{\partial x^m}\right)\right] \\
&\times\left[\exp\left(-y(\sqrt{mA})^{-}m\frac{\partial^m}{\partial x^m}\right)\sqrt{mA}^{-1}\frac{\partial}{\partial x}\exp\left(y(\sqrt{mA})^{-m}\frac{\partial^m}{\partial x^m}\right)\right] \\
&\times\left[\exp\left(-y(\sqrt{mA})^{-m}\frac{\partial^m}{\partial x^m}\right)\left(x\sqrt{mA}\right)^n\right] \\
&= \left(x\sqrt{mA} - my(\sqrt{mA})^{-(m-1)}\frac{\partial^{m-1}}{\partial x^{m-1}}\right)\sqrt{mA}^{-1}\frac{\partial}{\partial x}H_{n,m}(x,y,A)
\end{aligned}
\tag{2.14}
$$

as a consequence of the fact that

$$
\begin{aligned}
&\exp\left(-y(\sqrt{mA})^{-m}\frac{\partial^m}{\partial x^m}\right)x\sqrt{mA}\exp\left(y(\sqrt{mA})^{-m}\frac{\partial^m}{\partial x^m}\right) \\
&= x\sqrt{mA} - my(\sqrt{mA})^{-(m-1)}\frac{\partial^{m-1}}{\partial x^{m-1}} \\
&\exp\left(-y(\sqrt{mA})^{-m}\frac{\partial^m}{\partial x^m}\right)\sqrt{mA}^{-1}\frac{\partial}{\partial x}\exp\left(y(\sqrt{mA})^{-m}\frac{\partial^m}{\partial x^m}\right) \\
&= \sqrt{mA}^{(-1)}\frac{\partial}{\partial x}.
\end{aligned}
\tag{2.15}
$$

We will complete the aforementioned analysis to infinite series of ordinary Hermite matrix polynomials. Using (1.7) and (1.8), we get the Burchnall identity in the form

$$
\begin{aligned}
&\exp\left(-y(\sqrt{mA})^{-m}\frac{\partial^m}{\partial x^m}\right)(x\sqrt{mA})^n \\
&\qquad = \left[x\sqrt{mA} - my(\sqrt{mA})^{-(m-1)}\frac{\partial^{m-1}}{\partial x^{m-1}}\right]^n.
\end{aligned}
\tag{2.16}
$$

The following examples to illustrate the usefulness of the above procedure. The property allows us to derive the identity

$$
\begin{aligned}
H_{n+r,m}(x,y,A) &= \left[x\sqrt{mA} - my(\sqrt{mA})^{-(m-1)}\frac{\partial^{m-1}}{\partial x^{m-1}}\right]^n \\
&\quad\times\left[x\sqrt{mA} - my(\sqrt{mA})^{-(m-1)}\frac{\partial^{m-1}}{\partial x^{m-1}}\right]^r \\
&= \left[x\sqrt{mA} - my(\sqrt{mA})^{-(m-1)}\frac{\partial^{m-1}}{\partial x^{m-1}}\right]^n H_{r,m}(x,y,A)
\end{aligned}
\tag{2.17}
$$

which can be exploited to investigate further properties of Hermite matrix polynomials

$$
\begin{aligned}
(2.18)\quad H_{n,m}(x,y,A) &= \left[x\sqrt{mA} - my(\sqrt{mA})^{-(m-1)}\frac{\partial^{m-1}}{\partial x^{m-1}}\right]^n H_{0,m}(x,y,A)\\
&= \left[x\sqrt{mA} - my(\sqrt{mA})^{-(m-1)}\frac{\partial^{m-1}}{\partial x^{m-1}}\right]^n \exp\left(-y(\sqrt{mA})^{-m}\frac{\partial^{m}}{\partial x^{m}}\right)
\end{aligned}
$$

allows us to write (2.18) as the generating function

$$
\begin{aligned}
(2.19)\quad \sum_{n=0}^{\infty}\frac{t^n}{n!}&\left[x\sqrt{mA} - my(\sqrt{mA})^{-(m-1)}\frac{\partial^{m-1}}{\partial x^{m-1}}\right]^n \exp\left(-y(\sqrt{mA})^{-m}\frac{\partial^{m}}{\partial x^{m}}\right)\\
&= \exp\left[t\left(x\sqrt{mA} - my(\sqrt{mA})^{-(m-1)}\frac{\partial^{m-1}}{\partial x^{m-1}}\right)\right]\\
&\qquad\times \exp\left(-y(\sqrt{mA})^{-m}\frac{\partial^{m}}{\partial x^{m}}\right).
\end{aligned}
$$

From (2.17) and putting $r = n$

$$
(2.20)\quad H_{2n,m}(x,y,A) = \left[x\sqrt{mA} - my(\sqrt{mA})^{-(m-1)}\frac{\partial^{m-1}}{\partial x^{m-1}}\right]^n H_{n,m}(x,y,A)
$$

By recalling that Hermite matrix polynomials $H_{n,m}(x,y,A)$ are also defined through the operational identity. The use of the inverse of (2.11) allows to conclude that

$$
(2.21)\qquad (x\sqrt{mA})^n = \exp\left(y(\sqrt{mA})^{-m}\frac{\partial^{m}}{\partial x^{m}}\right)H_{n,m}(x,y,A).
$$

The second of the identities in (2.1) and (2.11) allows us to obtain the following results

$$
(2.22)\quad
\begin{aligned}
\sum_{n=0}^{\infty}\frac{t^n}{n!}H_{2n,m}(x,y,A) &= \sum_{n=0}^{\infty}\frac{t^n}{n!}\exp\left(-y(\sqrt{mA})^{-m}\frac{\partial^{m}}{\partial x^{m}}\right)(x\sqrt{mA})^{2n}\\
&= \exp\left(-y(\sqrt{mA})^{-m}\frac{\partial^{m}}{\partial x^{m}}\right)\exp\left((x\sqrt{mA})^2 t\right)\\
&= \sum_{n=0}^{\infty}\frac{t^n}{n!}\left[x\sqrt{mA} - my(\sqrt{mA})^{-(m-1)}\frac{\partial}{\partial x}\right]^{2n}\\
&= \exp\left[t\left(x\sqrt{mA} - my(\sqrt{mA})^{-(m-1)}\frac{\partial^{m-1}}{\partial x^{m-1}}\right)^2\right].
\end{aligned}
$$

In this paper, we have touched many points and examples that deserve a deeper analysis to the generating functions.

Example 2.1. Let us now consider the infinite sum, involving Hermite matrix polynomials, namely

$$F(x,y,t,A)=\sum_{n=0}^{\infty}t^{n}H_{n,m}(x,y,A). \tag{2.23}$$

By following the same procedure, leading to (2.3), we can write the sum with the series (2.23)

$$\begin{aligned}F(x,y,t,A)&=\sum_{n=0}^{\infty}t^{n}\left[x\sqrt{mA}-my(\sqrt{mA})^{-(m-1)}\frac{\partial^{m-1}}{\partial x^{m-1}}\right]^{n}\\&=[I-t(x\sqrt{mA}-my(\sqrt{mA})^{-(m-1)}\frac{\partial^{m-1}}{\partial x^{m-1}})]^{-1}\\&=\int_{0}^{\infty}e^{-s}e^{st(x\sqrt{mA}-my(\sqrt{mA})^{-(m-1)}\frac{\partial^{m-1}}{\partial x^{m-1}})}ds.\end{aligned} \tag{2.24}$$

Example 2.2. Let A be a matrix in $\mathbb{C}^{N\times N}$ satisfying the condition (1.2), we discuss is the generating function in the elegant form

$$W(x,y,t,A)=\sum_{n=0}^{\infty}\frac{t^{n}}{n!}H_{n+r,m}(x,y,A). \tag{2.25}$$

By exploiting the identity (2.17), we can write (2.25) as

$$\begin{aligned}&W(x,y,t,A)\\&=\sum_{n=0}^{\infty}\frac{t^{n}}{n!}\left[x\sqrt{mA}-my(\sqrt{mA})^{-(m-1)}\frac{\partial^{m-1}}{\partial x^{m-1}}\right]^{n}H_{r,m}(x,y,A)\\&=\exp\left[t\left(x\sqrt{mA}-my(\sqrt{mA})^{-(m-1)}\frac{\partial^{m-1}}{\partial x^{m-1}}\right)\right]H_{r,m}(x,y,A).\end{aligned} \tag{2.26}$$

Now, we give genuine examples of how operational calculus applies to the generating function for products of generalized Hermite matrix polynomials.

Example 2.3. We will discuss involves products, namely

$$\begin{aligned}&\sum_{n=0}^{\infty}\frac{t^{n}}{n!}H_{n,m}(x,y,A)H_{n,m}(z,w,A)=\\&=\sum_{n=0}^{\infty}\frac{t^{n}}{n!}\exp\left(-y(\sqrt{mA})^{-m}\frac{\partial^{m}}{\partial x^{m}}\right)\\&\times\exp\left(-w(\sqrt{mA})^{-m}\frac{\partial^{m}}{\partial z^{m}}\right)\left(x\sqrt{mA}\right)^{n}\left(z\sqrt{mA}\right)^{n}\\&=\exp\left(-y(\sqrt{mA})^{-m}\frac{\partial^{m}}{\partial x^{m}}-w(\sqrt{mA})^{-m}\frac{\partial^{m}}{\partial z^{m}}\right)\exp\left(xzt(\sqrt{mA})^{2}\right).\end{aligned} \tag{2.27}$$

Example 2.4. It is important to underline that the present operational method indicates that infinite series of the type

(2.28)
$$\sum_{n=0}^{\infty} \frac{t^n}{n!} H_{2n,m}(x, y, A) H_{n,m}(z, w, A)$$
$$= \exp\left(-y(\sqrt{mA})^{-m} \frac{\partial^m}{\partial x^m} - w(\sqrt{mA})^{-m} \frac{\partial^m}{\partial z^m} \right)$$
$$\sum_{n=0}^{\infty} \frac{t^n}{n!} (x\sqrt{mA})^{2n} (z\sqrt{mA})^n$$
$$= \exp\left(-y(\sqrt{mA})^{-m} \frac{\partial^m}{\partial x^m} - w(\sqrt{mA})^{-m} \frac{\partial^m}{\partial z^m} \right) \exp\left(tzx^2(\sqrt{mA})^3 \right).$$

Hence, the generating function of products (2.27) and (2.28) are established.

Further examples proving the usefulness of the present method can be easily worked out, but are not reported here for conciseness. The results we have obtained, show the flexibility of the operational methods associated with the theory of generalized polynomials. The general results established in the previous section lead to a number of special cases for selected values of the parameters.

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Department of Mathematics,
Faculty of Science (Suez Branch),
Suez Canal University,
Suez, Egypt
E-mail address: met6458@hotmail.com