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CONCIRCULAR CURVATURE TENSOR ON K-CONTACT MANIFOLDS

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Abstract. The object of the present paper is to study ξ-concircularly flat, ϕ-concircularly flat and concircularly semisymmetric K-contact manifolds. Beside these we also study K-contact manifolds satisfying $Z(\xi, X).S = 0$.

1. Introduction

A transformation of a $(2n+1)$-dimensional Riemannian manifold M, which transforms every geodesic circle of M into a geodesic circle, is called a concircular transformation ([16], [25]). A concircular transformation is always a conformal transformation [16]. Here geodesic circle means a curve in M whose first curvature is constant and whose second curvature is identically zero. Thus the geometry of concircular transformations, i.e., the concircular geometry, is generalization of inversive geometry in the sense that the change of metric is more general than that induced by a circle preserving diffeomorphism. An interesting invariant of a concircular transformation is the concircular curvature tensor Z. It is defined by ([24], [26])

$$Z(X,Y)W = R(X,Y)W - \frac{r}{2n(2n+1)}[g(Y,W)X - g(X,W)Y], \tag{1.1}$$

where $X, Y, W \in T(M)$. Riemannian manifolds with vanishing concircular curvature tensor are of constant curvature. Thus the concircular curvature tensor is a measure of the failure of a Riemannian manifold to be of constant curvature.

Let M be an almost contact metric manifold equipped with an almost contact metric structure (ϕ, ξ, η, g). At each point $p \in M$, decompose the tangent space T_pM into direct sum $T_pM = \phi(T_pM) \oplus \{\xi_p\}$, where $\{\xi_p\}$ is the 1-dimensional linear subspace of T_pM generated by $\{\xi_p\}$. Thus the conformal

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curvature tensor C is a map

$$C\colon T_pM \times T_pM \times T_pM \to \phi(T_pM) \oplus \{\xi_p\},\ p \in M.$$

It may be natural to consider the following particular cases:

(1) $C\colon T_p(M) \times T_p(M) \times T_p(M) \to \{\xi_p\}$, i.e, the projection of the image of C in $\phi(T_p(M))$ is zero.

(2) $C\colon T_p(M)\times T_p(M)\times T_p(M) \to \phi(T_p(M))$, i.e, the projection of the image of C in $L(\xi_p)$ is zero. This condition is equivalent to

$$C(X,Y)\xi = 0. \tag{1.2}$$

(3) $C\colon \phi(T_p(M)) \times \phi(T_p(M)) \times \phi(T_p(M)) \to \{\xi_p\}$, i.e, when C is restricted to $\phi(T_p(M)) \times \phi(T_p(M)) \times \phi(T_p(M))$, the projection of the image of C in $\phi(T_p(M))$ is zero. This condition is equivalent to

$$\phi^2 C(\phi X, \phi Y)\phi W = 0. \tag{1.3}$$

A K-contact manifold satisfying (1.2) and (1.3) are called ξ-conformally flat and ϕ-conformally flat respectively. A K-contact manifold satisfying the cases (1), (2) and (3) are considered in [13], [28] and [7] respectively.

In [28], it is proved that a K-contact manifold is ξ-conformally flat if and only if it is an η-Einstein Sasakian manifold. In [11], the authors studied ξ-conformally flat $N(k)$-contact metric manifold. A compact ϕ-conformally flat K-contact manifold with regular contact vector field has been studied in [7]. Moreover, in [12] the authors studied quasi-conformal curvature tensor in K-contact manifolds and in [27] A. Yildiz et al studied certain curvature properties in Kenmotsu manifolds. Also in [17] C. Özgür studied ϕ-conformally flat, ϕ-conharmonically flat and ϕ-projectively flat Lorentzian para-Sasakian manifolds. Analogous to the considerations of conformal curvature tensor here we define following:

Definition 1. A $(2n+1)$-dimensional K-contact manifold is said to be ξ-concircularly flat if

$$Z(X,Y)\xi = 0, \text{ where } X, Y \in T(M). \tag{1.4}$$

Definition 2. A $(2n+1)$-dimensional K-contact manifold is said to be ϕ-concircularly flat if

$$g(Z(\phi X, \phi Y)\phi U, \phi W) = 0. \tag{1.5}$$

Let (M,g) be a Riemannian manifold and let ∇ be the Levi-Civita connection of (M,g). A Riemannian manifold is called locally symmetric [8] if $\nabla R = 0$, where R is the Riemannian curvature tensor of (M,g). A Riemannian manifold $(M,g), n \geq 3$, is called semisymmetric if

$$R.R = 0 \tag{1.6}$$

holds, where R denotes the curvature tensor of the manifold. It is well known that the class of semisymmetric manifolds includes the set of locally symmetric manifolds ($\nabla R = 0$) as a proper subset. Semisymmetric Riemannian manifolds were first studied by E. Cartan, A. Lichnerowich, R. S. Couty and N. S. Sinjukov. A Riemannian manifold $(M, g), n \geq 3$, is said to be Ricci semisymmetric if on M we have

$$R.S = 0, \tag{1.7}$$

where S is the Ricci tensor.

The class of Ricci semisymmetric manifolds includes the set of Ricci-symmetric manifolds ($\nabla S = 0$) as a proper subset. A Riemannian manifold $(M, g), n \geq 3$, is said to be concircularly semisymmetric if on M we have

$$R.Z = 0, \tag{1.8}$$

where Z is the concircular curvature tensor.

In [2] Binh, De and Tamássy have been studied partially pseudo symmetric K-contact Riemannian manifolds. In the case of contact manifolds S. Tanno ([23], [24]) showed among others that there exists no semisymmetric or Ricci semisymmetric K-contact manifold. Motivated by the above studies, in this paper we study ξ-concircularly flat and ϕ-concircularly flat K-contact manifolds. Again in [5], D. E. Blair et al. studied concircular curvature tensor in contact metric manifolds. A $(2n+1)$-dimensional $N(k)$-contact metric manifold satisfying $Z(\xi, X).Z = 0$, $Z(\xi, X).R = 0$ and $R(\xi, X).Z = 0$ has been considered in [5]. B. J. Papantoniou [18] and D. Perrone [19] included the studies of contact metric manifolds satisfying $R(\xi, X).S = 0$, where S is the Ricci tensor. Motivated by these studies, we continue the study of K-contact manifolds with concircular curvature tensor Z satisfying $Z(\xi, X).S = 0$.

The present paper is organized as follows: After preliminaries in section 3, we study ξ-concircularly flat K-contact metric manifolds and prove that a $(2n+1)$-dimensional, $n \geq 1$, ξ-concircularly flat K-contact metric manifold is of positive scalar curvature. Section 4 deals with the study of ϕ-concircularly flat K-contact manifolds. In this section we prove that a $(2n+1)$-dimensional, $n \geq 1$, K-contact manifold is ϕ-concircularly flat if and only if the manifold is Einstein with scalar curvature $2n(2n+1)$. Section 5 is devoted to study a $(2n+1)$-dimensional concircularly semisymmetric K-contact manifold and prove that in this case the manifold is Sasakian. Finally in section 6 we study a $(2n+1)$-dimensional, $n \geq 1$, K-contact manifolds satisfying $Z(\xi, X).S = 0$ and prove that the K-contact manifold satisfies $Z(\xi, X).S = 0$ if and only if the manifold is an Einstein manifold provided $r \neq 2n(2n+1)$. As a consequence of the results we obtain some important Corollaries.

2. Preliminaries

By a contact manifold we mean a $(2n+1)$- dimensional differentiable manifold M^{2n+1} which carries a global 1-form η, there exists a unique vector field ξ, called the characteristic vector field such that, $\eta(\xi) = 1$ and $d\eta(\xi, X) = 0$. A Riemannian metric g on M^{2n+1} is said to be an associated metric if there exists a $(1,1)$ tensor field ϕ such that

$$d\eta(X,Y) = g(X,\phi Y),\ \eta(X) = g(X,\xi),\ \phi^2 = -I + \eta \otimes \xi. \tag{2.1}$$

From these equations we have

$$\phi\xi = 0,\ \eta \circ \phi = 0,\ g(\phi X, \phi Y) = g(X,Y) - \eta(X)\eta(Y). \tag{2.2}$$

The manifold M equipped with the contact structure (ϕ, ξ, η, g) is called a contact metric manifold. If ξ is a Killing vector field, then M^{2n+1} is said to be a K-contact manifold ([3], [4], [20]). K-contact manifolds have been studied by several authors such as S. Tanno ([23], [22], [24]), S. Sasaki ([20], [21]), Y. Hatakeyama, Y. Ogawa and S. Tanno [14], M. C. Chaki and D. Ghosh [9], U. C. De and S. Biswas [10] and many others.

A contact metric structure is said to be normal(Sasakian) if the almost complex structure J on $M \times R$ defined by $J(X, f\frac{d}{dt}) = (\phi X - f\xi, \eta(X)\frac{d}{dt})$, f being a function on M^{2n+1}, is integrable. A contact metric manifold is Sasakian if and only if

$$R(X,Y)\xi = \eta(Y)X - \eta(X)Y. \tag{2.3}$$

Every Sasakian manifold is K-contact, but the converse need not be true, except in dimension three [15]. K-contact manifolds are not too well known, because there is no such a simple expression for the curvature tensor as in the case of Sasakian manifolds. For details we refer to [1, 15, 20].

Besides the above relations in K-contact manifold the following relations hold ([3], [15], [20]):

$$\nabla_X \xi = -\phi X \tag{2.4}$$

$$\tilde{R}(\xi, X, Y, \xi) = \eta(R(\xi, X)Y) = g(X,Y) - \eta(X)\eta(Y) \tag{2.5}$$

$$R(\xi, X)\xi = -X + \eta(X)\xi \tag{2.6}$$

$$S(X,\xi) = 2n\eta(X) \tag{2.7}$$

$$(\nabla_X \phi)Y = R(\xi, X)Y, \tag{2.8}$$

for any vector fields X, Y.

Again a K-contact manifold is called Einstein if the Ricci tensor S is of the form $S = \lambda g$, where λ is a constant and η- Einstein if the Ricci tensor S is of the form $S = ag + b\eta \otimes \eta$, where a, b are smooth functions on M. It is well known [15] that in a K-contact manifold a and b are constants. Also it is known [6] that a compact η-Einstein K-contact manifold is Sasakian provided $a \geq$-2.

In a $(2n+1)$-dimensional almost contact metric manifold, if $\{e_1, \ldots, e_{2n}, \xi\}$ is a local orthonormal basis of vector fields, then $\{\phi e_1, \ldots, \phi e_{2n}, \xi\}$ is also a local orthonormal basis. It is easy to verify that

$$\sum_{i=1}^{2n} g(e_i, e_i) = \sum_{i=1}^{2n} g(\phi e_i, \phi e_i) = 2n \tag{2.9}$$

$$\sum_{i=1}^{2n} g(e_i, W)S(Y, e_i) = \sum_{i=1}^{2n} g(\phi e_i, W)S(Y, \phi e_i) \tag{2.10}$$
$$= S(Y, W) - S(Y, \xi)\eta(W),$$

for $Y, W \in T(M)$. In particular in view of $\eta \circ \phi = 0$, we get

$$\sum_{i=1}^{2n} g(e_i, \phi W)S(Y, e_i) = \sum_{i=1}^{2n} g(\phi e_i, \phi W)S(Y, \phi e_i) = S(Y, \phi W), \tag{2.11}$$

for $Y, W \in T(M)$. If M is a K-contact manifold, then it is known that

$$R(X, \xi)\xi = X - \eta(X)\xi, \ \ X \in T(M) \tag{2.12}$$

and

$$S(\xi, \xi) = 2n. \tag{2.13}$$

Moreover M is Einstein if and only if

$$S = 2ng. \tag{2.14}$$

From (2.13) we get

$$\sum_{i=1}^{2n} S(e_i, e_i) = \sum_{i=1}^{2n} S(\phi e_i, \phi e_i) = r - 2n, \tag{2.15}$$

where r is the scalar curvature. In a K-contact manifold we also have

$$\tilde{R}(\xi, Y, W, \xi) = g(\phi Y, \phi W), \ \ Y, W \in T(M). \tag{2.16}$$

Consequently

$$\sum_{i=1}^{2n} \tilde{R}(e_i, Y, W, e_i) = \sum_{i=1}^{2n} \tilde{R}(\phi e_i, Y, W, \phi e_i) = S(Y, W) - g(\phi Y, \phi W). \tag{2.17}$$

Where $\tilde{R}(X, Y, W, V) = g(R(X, Y)W, V)$, $X, Y, W, V \in T(M)$. For more details we refer to [28], [7].

3. ξ-concircularly flat K-contact manifolds

In this section we study ξ-concircularly flat K-contact manifolds. Let M be a $(2n+1)$-dimensional, $n \geq 1$, ξ-concircularly flat K-contact manifold. Putting $W = \xi$ in (1.1) and applying (1.4) and $g(X, \xi) = \eta(X)$, we have

$$R(X, Y)\xi = \frac{r}{2n(2n+1)}[\eta(Y)X - \eta(X)Y]. \tag{3.1}$$

Putting $Y = \xi$ in (3.1), we obtain

$$R(X, \xi)\xi = \frac{r}{2n(2n+1)}[X - \eta(X)\xi]. \tag{3.2}$$

Using (2.6) in (3.2), yields

$$(1 - \frac{r}{2n(2n+1)})[X - \eta(X)\xi] = 0. \tag{3.3}$$

Now $[X - \eta(X)\xi] \neq 0$ in a contact metric manifold, in general. Therefore (3.3) gives

$$r = 2n(2n+1). \tag{3.4}$$

Since $r = 2n(2n+1)$ is positive. Therefore we can state the following:

Theorem 3.1. *A* $(2n+1)$*-dimensional* ξ*-concircularly flat* K*-contact manifold is of positive scalar curvature.*

Now,let us consider $r = 2n(2n+1)$ and the manifold is Sasakian. Then from (1.1) and (2.3) we easily obtain

$$Z(X,Y)\xi = 0.$$

In the view of the above discussion we have the following:

Theorem 3.2. *A* $(2n+1)$*-dimensional Sasakian manifold is* ξ*-concircularly flat if and only if* $r = 2n(2n+1)$

4. ϕ-concircularly flat K-contact manifolds

This section is devoted to study ϕ-concircularly flat K-contact manifolds. Let M be a $(2n+1)$-dimensional ϕ-concircularly flat K-contact manifold.
Using (1.1) in (1.3) we obtain

$$g(R(\phi X, \phi Y)\phi W, \phi V) = \frac{r}{2n(2n+1)}[g(\phi Y, \phi W)g(\phi X, \phi V) - g(\phi X, \phi W)g(\phi Y, \phi V)]. \tag{4.1}$$

Let $\{e_1, e_2, \ldots, e_n, \phi e_1, \phi e_2, \ldots, \phi e_n, \xi\}$ be an orthonormal ϕ-basis of the tangent space. Putting $X = V = e_i$ in (4.1) and taking summation over $i = 1$ to $2n$ and using (2.17), we obtain

$$S(\phi Y, \phi W) = [\frac{r(2n-1)}{2n(2n+1)} + 1]g(\phi Y, \phi W). \tag{4.2}$$

Replacing Y and W by ϕY and ϕW in (4.2) and using (2.1), (2.7) yields

$$S(Y,W) = Ag(Y,W) + B\eta(Y)\eta(W), \tag{4.3}$$

where A and B are given by the following relations:

$$A = \frac{r(2n-1)}{2n(2n+1)} + 1 \text{ and } B = (2n-1) - \frac{r(2n-1)}{2n(2n+1)}. \tag{4.4}$$

Let $\{e_i\}, i = 1, 2, \ldots, (2n+1)$ be the orthonormal basis of the tangent space at any point of the manifold. Putting $X = Y = e_i$ in (4.3) and taking summation over $1 \leq i \leq (2n+1)$, we obtain

$$r = A(2n+1) + B. \tag{4.5}$$

Again, putting $Y = W = \xi$ in (4.3) yields

$$2n = S(\xi, \xi) = A + B. \tag{4.6}$$

Solving (4.5), (4.6) we get

$$A = \frac{r}{2n} - 1 \text{ and } B = (2n+1) - \frac{r}{2n}. \tag{4.7}$$

Now, equating the value of A or B from (4.4) and (4.7) we obtain $r = 2n(2n+1)$. Therefore the equation (4.3) becomes

$$S(Y, W) = 2ng(Y, W). \tag{4.8}$$

Hence the manifold is Einstein with scalar curvature $r = 2n(2n+1)$.

Conversely, let the manifold is Einstein i.e. $S(Y, W) = 2ng(Y, W)$ with the scalar curvature $r = 2n(2n+1)$.Then we have

$$\begin{aligned} g(Z(\phi X, \phi Y)\phi W, \phi V) = g(R(\phi X, \phi Y)\phi W, \phi V) - \\ [g(\phi Y, \phi W)g(\phi X, \phi V) - g(\phi X, \phi W)g(\phi Y, \phi V)] \end{aligned} \tag{4.9}$$

Let $\{e_1, e_2, \ldots, e_n, \phi e_1, \phi e_2, \ldots, \phi e_n, \xi\}$ be an orthonormal ϕ-basis of the tangent space. Putting $X = V = e_i$ in (4.9) and taking summation over $i = 1$ to $2n$ and using (2.17), we obtain

$$g(Z(\phi X, \phi Y)\phi W, \phi V) = S(\phi Y, \phi W) - 2ng(\phi Y, \phi W) \tag{4.10}$$

Putting $Y = \phi Y$,$W = \phi W$ in (4.8) yields

$$S(\phi Y, \phi W) = 2ng(\phi Y, \phi W). \tag{4.11}$$

Using (4.11) in (4.10) we easily obtain

$$g(Z(\phi X, \phi Y)\phi W, \phi V) = 0. \tag{4.12}$$

Therefore we can state the following:

Theorem 4.1. *A $(2n+1)$-dimensional K-contact manifold is ϕ-concircularly flat if and only if the manifold is Einstein with scalar curvature $2n(2n+1)$.*

In [23] S. Tanno proved that a K- contact manifold is Ricci semisymmetric ($R.S = 0$) if and only if the manifold is an Einstein manifold with scalar curvature $r = 2n(2n+1)$. Thus in the view of the above result we can state the following:

Corollary 4.1. *A $(2n+1)$-dimensional K-contact manifold is ϕ-concircularly flat if and only if the manifold is Ricci semisymmetric.*

Lemma 4.1 ([6])**.** *A compact K-contact Einstein manifold is Sasakian.*

Therefore from the Theorem 4.1 and Lemma 4.1 we have the following:

Corollary 4.2. *A $(2n+1)$-dimensional ϕ-concircularly flat compact K-contact manifold is Sasakian.*

5. Concircularly semisymmetric K-contact manifolds

In this section we study concircularly semisymmetric K-contact manifolds, where Z is the concircular curvature tensor. Therefore we have

$$(R(X,Y).Z)(U,V)W = 0. \tag{5.1}$$

This implies

$$R(X,Y)Z(U,V)W - Z(R(X,Y)U,V)W - Z(U,R(X,Y)V)W - Z(U,V)R(X,Y)W = 0. \tag{5.2}$$

Putting $U = \xi$, $X = \xi$ and $W = \xi$ in (5.2) we have

$$R(\xi,Y)Z(\xi,V)\xi - Z(R(\xi,Y)\xi,V)\xi - Z(\xi,R(\xi,Y)V)\xi - Z(\xi,V)R(\xi,Y)\xi = 0. \tag{5.3}$$

Now,

$$R(\xi,Y)Z(\xi,V)\xi = (1 - \frac{r}{2n(2n+1)})\{\eta(V)\eta(Y) - \eta(V)Y - R(\xi,Y)V\}, \tag{5.4}$$

using (1.1), (2.6). Similarly we have

$$Z(R(\xi,Y)\xi,V)\xi = (1 - \frac{r}{2n(2n+1)})\{\eta(V)\eta(Y)\xi - \eta(Y)V\} - R(Y,V)\xi + \frac{r}{2n(2n+1)}\{\eta(V)Y - \eta(Y)V\}. \tag{5.5}$$

$$Z(\xi,R(\xi,Y)V)\xi = (1 - \frac{r}{2n(2n+1)})\{g(Y,V)\xi - \eta(V)\eta(Y)\xi - R(\xi,Y)V\}, \tag{5.6}$$

using (2.5), and

$$Z(\xi,V)R(\xi,Y)\xi = (1 - \frac{r}{2n(2n+1)})\{\eta(Y)\eta(V)\xi - \eta(Y)V\} - R(\xi,V)Y + \frac{r}{2n(2n+1)}\{g(V,Y)\xi - \eta(Y)V\}. \tag{5.7}$$

Using (5.4), (5.5), (5.6) and (5.7) in (5.3) we obtain

$$2\eta(Y)V - \eta(V)Y + g(Y,V)\xi + R(Y,V)\xi + R(\xi,V)Y = 0. \tag{5.8}$$

Interchanging Y and V and then subtracting the result from (5.8) we obtain by the virtue of Bianchi identity that

$$R(V, Y)\xi = \eta(Y)V - \eta(V)Y. \tag{5.9}$$

In the view of the above result we can state the following:

Theorem 5.1. *A* $(2n+1)$*-dimensional concircularly semisymmetric* K*-contact manifold satisfies is Sasakian.*

6. K-contact manifolds satisfying $Z(\xi, X).S = 0$

This section deals with a $(2n + 1)$-dimensional, $n \geq 1$, K-contact manifold satisfying $Z(\xi, X).S = 0$. Now, the relation $Z(\xi, X).S = 0$ implies

$$S(Z(\xi, X)Y, W) + S(Y, Z(\xi, X)W) = 0. \tag{6.1}$$

Using (1.1) in (6.1), we have

$$\begin{aligned} S(R(\xi, X)Y, W) + S(Y, R(\xi, X)W) - \frac{r}{2n(2n+1)}[g(X, Y)S(\xi, W) \\ - \eta(Y)S(X, W) + g(X, W)S(Y, \xi) - \eta(W)S(X, Y)] = 0. \end{aligned} \tag{6.2}$$

Putting $W = \xi$ in (6.2) and using (2.1), (2.7) we obtain

$$\begin{aligned} S(R(\xi, X)Y, \xi) + S(Y, R(\xi, X)\xi) - \frac{r}{2n(2n+1)}[2ng(X, Y) \\ - S(X, Y)] = 0. \end{aligned} \tag{6.3}$$

Now

$$S(R(\xi, X)Y, \xi) = 2n\eta(R(\xi, X)Y) = 2ng(R(X, \xi)\xi, Y). \tag{6.4}$$

Using (2.6) in (6.4) yields

$$S(R(\xi, X)Y, \xi) = g(X, Y) - \eta(X)\eta(Y) \tag{6.5}$$

Again using (2.6), (6.5) in (6.3), we get

$$(S(X, Y) - 2ng(X, Y))(1 - \frac{r}{2n(2n+1)}) = 0 \tag{6.6}$$

Therefore either $S(X, Y) = 2ng(X, Y)$ or, $r = 2n(2n + 1)$.

Conversely, let us consider the manifold is Einstein. Then we have

$$S(X, Y) = 2ng(X, Y) \tag{6.7}$$

Now,

$$(Z(\xi, X).S)(Y, W) = S(Z(\xi, X)Y, W) + S(Y, Z(\xi, X)W) \tag{6.8}$$

Using (6.7) in (6.8), we obtain

$$(Z(\xi, X).S)(Y, W) = 2n[g(Z(\xi, X)Y, W) + g(Y, Z(\xi, X)W)] \tag{6.9}$$

Again using (1.1) in (6.9) yields

$$(Z(\xi, X).S)(Y, W) = 0 \tag{6.10}$$

Therefore we can state the following:

Theorem 6.1. *A $(2n+1)$-dimensional K-contact manifold satisfies $Z(\xi, X).S = 0$ if and only if the manifold is Einstein provided $r \neq 2n(2n+1)$.*

By the Lemma 4.1 and the Theorem 6.1 we can state the following:

Corollary 6.1. *A $(2n+1)$-dimensional compact K-contact manifold satisfies $Z(\xi, X).S = 0$ is Sasakian provided $r \neq 2n(2n+1)$.*

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