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ECG-BASED HEART BEAT DETECTION USING RATIONAL FUNCTIONS

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Dedicated to Professor Ferenc Schipp on the occasion of his 75th birthday, to Professor William Wade on the occasion of his 70th birthday and to Professor Péter Simon on the occasion of his 65th birthday.

Abstract. The aim of this paper is to present a novel heart beat detection algorithm using rational modelling of ECG signals. The algorithm considers several candidate beat locations. For a given candidate a rational model is fitted to the ECG signal by means of numerical optimization and Fourier partial sums with respect to the Malmquist-Takenaka system. The resultant model parameters are used as a basis of classification. The classification is performed by an SVM classifier, which is trained on annotated ECG records of the PhysioNet database.

1. Introduction

ECG is a widely used tool in cardiology for inspecting heart condition and diagnosing malfunction. Computer aided analysis of ECG signals, which provides additional diagnostic apparatus to cardiologists, is an active research area of biomedical signal processing.

An important aspect of ECG processing is the problem of detecting heart beat locations. Several analysis algorithms operate on beats, hence these procedures rely on beat locations found by a previous processing stage. In this paper a novel beat detection method based on rational modeling of QRS complexes [5] is presented and compared to other well-known ones, such as the gqrs and gqpost functions [11] of the PhysioNet [7] software package and the Pan-Tompkins algorithm [13].

The novel method operates as follows. The peaks of the input signal are considered as candidate beat locations, then rational models are fitted to the neighbourhood of each one of these locations. The candidate locations are

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found by selecting local maxima in the input signal. The candidates have to be at least 200 ms apart; if two candidates are closer than this threshold, the one with lower peak value is discarded. The parameters of the rational model serve as features describing that particular candidate, which are then used by a classifier algorithm to distinguish heart beats from non-beat locations. The mathematical background of the rational model is described briefly in Section 2. The details of extracting signal features using the rational model are elaborated in Section 3. The extracted features are passed to a classification phase, which is the main focus of Section 4. The results obtained by running the algorithm on the PhysioNet CinC challenge 2014 set-p dataset are presented in Section 5. Section 6 concludes the paper with a summary and a discussion of possible development directions.

The algorithm described above was originally motivated by the PhysioNet CinC challenge 2014 contest [6], however it wasn't submitted as a solution due to run-time performance problems. Although speeding up the algorithm is possible, the main focus of this paper is not performance-related. Instead, the main goal here is to demonstrate the usability of the rational model as a tool in ECG beat detection.

The idea of rational modelling has been applied recently in the field of biomedical signal processing. The rational model has already been shown to describe heart beats well regardless of the ECG lead used: the numerical optimization of the Malmquist-Takenaka basis parameters (see Section 2 and 3) yield approximately the same results for different leads [4]. Another application of rational signal modelling is epileptic seizure classification in EEG signals [10].

2. Rational modeling of ECG signals

This section briefly outlines the process of rational ECG modelling along with the mathematical apparatus required to understand the beat detection algorithm. The interested reader is encouraged to refer to [5] for more details.

Let's consider the sequence $a_0, a_1, \ldots, a_n, \ldots$ of complex numbers in the open unit disc, i.e. $a_n \in \mathbb{C}$, $|a_n| < 1$ $(n \in \mathbb{N})$. The rational functions

$$\Phi_n(z) = \frac{\sqrt{1-|a_n|^2}}{1-\bar{a}_n z} \prod_{j=0}^{n-1} \frac{z-a_n}{1-\bar{a}_n z}$$

are called *Malmquist-Takenaka functions* (see e.g. [8], [14]), the parameters a_n are referred to as *inverse poles*. These functions form an orthogonal system with respect to the scalar product

$$\langle f, g \rangle = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(e^{it}) \overline{g(e^{it})} \mathrm{d}t,$$

where the functions f and g are square integrable on the complex unit circle. QRS complexes of ECG signals can be approximated using Fourier partial sums with respect to the system $(\Phi_n,\, n \in \mathbb{N})$:

$$\tilde{f}(z) = \sum_{n=0}^{N} \langle f, \Phi_n \rangle \, \Phi_n(z)$$

where the complex unit circle is parameterized by $z = e^{it}$; $t \in [-\pi, \pi)$ is the normalized time variable.

Since the Malmquist-Takenaka basis is a complex-valued system, one has to find an appropriate imaginary part for the real-valued signal, or construct a real-valued orthogonal system from the Malmquist-Takenaka basis. Both approaches are viable. Here the former method is followed, since the RAIT toolbox [9] for MATLAB, upon which the beat detection algorithm has been built, provides tools to construct the imaginary part.

Figure 1 shows a concrete example of QRS complexes approximated by rational functions.

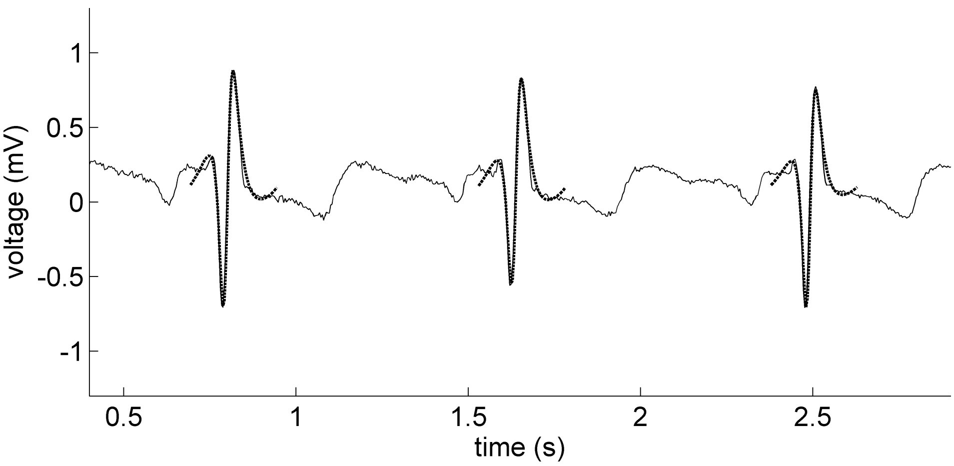

Figure 1. Rational approximation of QRS complexes. The dashed curves are graphs of the real part of Malmquist-Takenaka partial sums.

3. Feature extraction

To be able to distinguish QRS complexes from other parts of the ECG signal, a concise description of a particular location in the ECG signal is needed. This section describes the process of obtaining this description using the rational model introduced in Section 2.

The parameters $(a_0, a_1, \ldots, a_N)$ of the Malmquist-Takenaka system make the model very general and adaptable, however, for practical reasons, some constraints are needed to be introduced. Firstly, to be able to represent constant signals, setting a_0 to 0 is required. Secondly, to keep the implementation simple and the run-time performance manageable, the rest of the parameters are set to be equal. Formally:

$$a_0 = 0, \quad a_1 = a_2 = \ldots = a_N = a.$$

With these constraints in place, the only remaining free parameter is a. To find the optimal value of this parameter for a given ECG signal, a numerical optimization algorithm is used in the following way. For a given candidate location a small window centered at the candidate is considered. Although the length of a normal QRS complex is at most 200 ms, a bit wider 250 ms window is used in order to help the periodic model to assume the same value at the interval ends. Then a Nelder-Mead simplex algorithm [12] is used to optimize the parameter a by minimizing the ℓ_2 norm of the approximation error measured at the signal samples in the window. The initial value of the parameter a is chosen to align the peak of the Malmquist-Takenaka basis functions with the candidate location, which promotes faster convergence of the optimization algorithm.

Finding a good rational approximation is important in order to represent the ECG signal faithfully. The approximation error can be reduced by increasing the number of terms in the Fourier partial sums or letting the optimizer perform more iterations. Both of these modifications however increase computational costs. To find suitable values for the number N of Fourier-terms corresponding to the inverse pole a (the *order* of a) and the maximal number of Nelder-Mead iterations M, a subset of the set-p database containing 10 randomly chosen records are selected. The reference beats of the selected records are approximated by a rational model using the parameters $N = 2, 3, 4$ and $M = 2, 4, 8, 16, 32$.

Figure 2 compares the distributions of the approximation errors in terms of PRD (percentage root-mean-square difference) [1] using a box plot: on each box, the central mark is the median, the edges of the box are the lower and upper quartiles, and the whiskers extend to the most extreme data points not considered outliers. The PRD of a sample $\hat{x}[k]$ $(k = 1, 2, \ldots, K)$ with respect to the reference $x[k]$ $(k = 1, 2, \ldots, K)$ is calculated using the following formula

$$\mathrm{PRD} = \sqrt{\frac{\sum_{k=1}^{K}(x[k] - \hat{x}[k])^2}{\sum_{k=1}^{K}(x[k] - \bar{x})^2}},$$

where $\bar{x}$ is the sample mean.

One can conclude that setting M greater than 16 doesn't perceivably improve the accuracy, therefore the proposed algorithm uses $M = 16$. Increasing the parameter N does decrease approximation error, however, as described in Section 5, it does not improve classification accuracy. Therefore $N = 2$ is used.

The resultant parameter a and the corresponding Fourier coefficients form the feature vector of the candidate. The coefficient corresponding to $a_0 = 0$ is omitted, since this term in the Fourier partial sum is constant, which has no

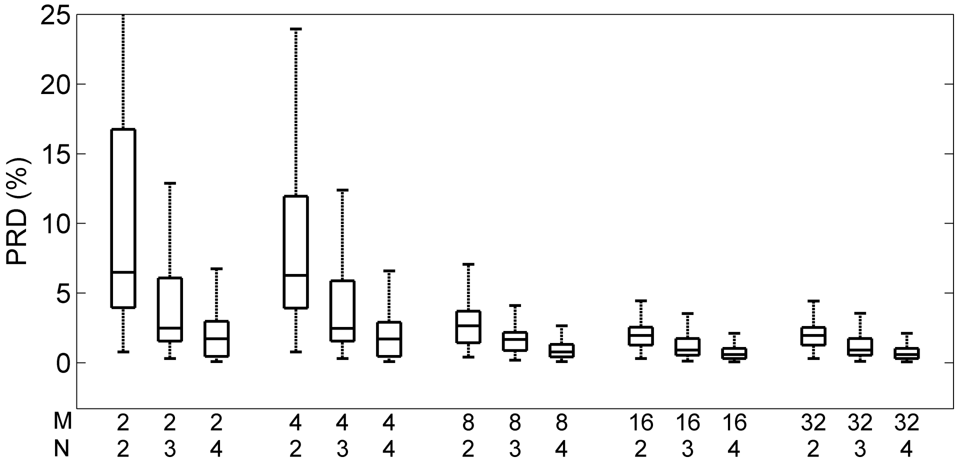

Figure 2. Box plot of PRD approximation errors with respect to the parameters M (top) and N (bottom). The outliers are not shown.

influence on the QRS waveform. The ℓ_2 approximation error is also added as a feature.

4. Classification

To find the heart beats in the ECG signal, each candidate is classified as beat or non-beat location using the features introduced in Section 3. The classification is performed by a Support Vector Machine (SVM) [3] classifier implemented by the libsvm library [2].

Only the signals similar to beat waveforms encountered during training should be accepted as beats. To this end the proposed algorithm uses the

$$K(x, x') = \exp\left(\gamma\|x - x'\|^2\right)$$

so called RBF kernel, which acts as a similarity function between the feature vectors x and x'. This way the distance of candidate and ground truth features will determine the outcome of the classification.

The classifier is trained on a subset of the PhysioNet CinC challenge set-p database consisting of 10% of the total 100 records. The features were normalized to zero mean and unity standard deviation, hence the same transformation is needed before beat detection. The standard RBF γ and soft margin C parameters are selected using grid search based on the performance on a separate set of cross-validation data containing 10% of the records. The parameter grid follows an exponential pattern.

The final performance of the algorithm is evaluated on the remaining 80% of the records (test records) of the database (see Section 5).

5. Results

Table 1 summarizes the results obtained using different N parameters while also comparing them to the gqrs, gqpost and Pan-Tompkins beat detection algorithms. The algorithms were run on 80% of the PhysioNet CinC challenge set-p records, as described in Section 4.

The basis of comparison is sensitivity (Se) and positive predictivity (+P). These values are computed using the following formulas:

$$\mathrm{Se} = 100\frac{\mathrm{TP}}{\mathrm{TP}+\mathrm{FN}},$$
$$+\mathrm{P} = 100\frac{\mathrm{TP}}{\mathrm{TP}+\mathrm{FP}},$$

where TP, FP and FN are the number of true positive, false positive and false negative detections. The gross and average (avg) values represent overall scores and per-record averages respectively.

algorithm	gross Se	gross +P	avg Se	avg +P
gqrs	99.90%	99.20%	99.91%	99.29%
gqrs + gqpost	99.90%	99.52%	99.90%	99.57%
Pan-Tompkins	99.35%	99.73%	99.44%	99.75%
rational, N=2	99.59%	99.73%	99.61%	99.75%
rational, N=3	99.66%	99.51%	99.68%	99.55%
rational, N=4	99.53%	99.55%	99.54%	99.59%

Table 1. Results of the gqrs, gqpost, Pan-Tompkins and the proposed rational algorithm with different values of the order N of the inverse pole a.

The results show that the rational algorithm performs competitively as the classical ones with slight improvements either in sensitivity or positive predictivity. The parameter N can be set to 2, as higher values increase computational costs without noticeable accuracy improvement.

6. Conclusion

A novel ECG heart beat detection method based on rational modeling of QRS complexes has been proposed. The results are competitive to classic algorithms tailored to this task, but the rational model and the extracted features enable a much broader set of applications, such as beat type classification and the extraction of diagnostic parameters.

Run-time performance improvements are also possible, e.g. switching to the conjugate gradient method to optimize the inverse poles or using a discrete real Malmquist-Takenaka system could speed up the algorithm.

The elaboration of the aforementioned applications and optimization strategies remain future work.

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