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NUMERICAL SOLUTION OF 2D STEADY AND UNSTEADY FLOWS USING COMPOSITE SCHEME*

R. HONZÁTKO[†], M. JANDA[†] AND K. KOZEL[‡]

1. Abstract. The work deals with using finite volume method and triangular, quadrilateral or combine mesh in the form of cell-centered or cell-vertex scheme. Governing system of equations is the system of Euler equations. Composite scheme has more dissipative part (Lax-Friedrich scheme) and less dissipative part (Lax-Wendroff scheme). Steady solution is solved by time dependent method. Unsteady flow is solved by the same method and unsteady behaviour is caused by a time change of downstream conditions in a channel and a cascade. Authors present 2D inviscid compressible flows in a channel and incompressible flows through a cascade.

2. Mathematical model. Consider inviscid flows for incompressible and compressible problem. The mathematical model is the system of Euler equations in conservation form

$$RW_t + F_x + G_y = 0, \tag{2.1}$$

where

$$\begin{aligned} W &= \|\rho,\, \rho u,\, \rho v,\, e\|^T \\ F &= \|\rho u,\, \rho u^2 + p,\, \rho uv,\, (e+p)u\|^T \\ G &= \|\rho v,\, \rho uv,\, \rho v^2 + p,\, (e+p)v\|^T \\ p &= (\kappa - 1)\left[e - \tfrac{1}{2}\rho(u^2+v^2)\right] \\ R &= diag\|1,\, 1,\, 1,\, 1\| \end{aligned} \tag{2.2}$$

in the case of compressible flows and for incompressible flows we have

$$\begin{aligned} W &= \|\tfrac{p}{\rho},\, u,\, v\|^T \\ F &= \|u,\, u^2 + \tfrac{p}{\rho},\, uv\|^T \\ G &= \|v,\, uv,\, v^2 + \tfrac{p}{\rho}\|^T \\ R &= diag\|0,\, 1,\, 1\|\,. \end{aligned} \tag{2.3}$$

Here ρ is density, p - pressure, (u, v) is velocity vector, e - total energy per unit volume, κ is Poisson constant. Upstream conditions are $W = W_\infty$ or given 3 values of W_∞ in (2.2) resp. 2 values of W_∞ in (2.3) where last value is computed by extrapolation. Downstream condition is only given $p = p_2$. Next values of W_2 are extrapolated. Wall conditions are nonpermeability conditions $(u, v)_n = 0$ (normal component of velocity vector is zero).

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[†]Dept. of Mathematics of Faculty of Nuclear Sciences and Physical Engineering, Czech Technical University.

[‡]Dept. of Technical Mathematics of Faculty of Mechanical Engineering, Czech Technical University.

3. Numerical solution. In our numerical solution we used combination of Lax-Wendroff (LW) and Lax-Friedrichs (LF) schemes in finite volume form of composite scheme

$$C(U) = LW(U) \otimes LF(U) \tag{3.1}$$

and triangular ($m = 3$) or quadrilateral ($m = 4$) grid in cell-centered or cell-vertex forms:

1. cell-centered form
   - LF Predictor:

$$\begin{aligned} W_i^{n+1/2} &= W_i^n - \frac{\Delta t}{2\mu_i} \sum_{k=1}^{m} \left(\bar{F}_k^n \Delta y_k - \bar{G}_k^n \Delta x_k \right) + \\ &+ \frac{\varepsilon}{m} \sum_{k=1}^{m} (W_k - W_i)^n \end{aligned} \tag{3.2}$$

   - LF Corrector:

$$\begin{aligned} W_i^{n+1} &= W_i^{n+1/2} - \frac{\Delta t}{\mu_i} \sum_{k=1}^{m} \left(\bar{F}_k^{n+1/2} \Delta y_k - \bar{G}_k^{n+1/2} \Delta x_k \right) + \\ &+ \frac{\varepsilon}{m} \sum_{k=1}^{m} (W_k - W_i)^{n+1/2} \end{aligned} \tag{3.3}$$

   - LW Predictor is the same as LF Predictor (3.2)
   - LW Corrector:

$$W_i^{n+1} = W_i^n - \frac{\Delta t}{\mu_i} \sum_{k=1}^{m} \left(\bar{F}_k^{n+1/2} \Delta y_k - \bar{G}_k^{n+1/2} \Delta x_k \right) , \tag{3.4}$$

$\bar{F}_k = \frac{1}{2}(F_k + F_{k+1})$, $\bar{G}_k = \frac{1}{2}(G_k + G_{k+1})$, $\varepsilon \in (0, 1\rangle$.

2. cell-vertex form using grid of triangles and dual grid (see [3])
   - LF Predictor:

$$W_i^{n+1/2} = \frac{1}{3} \sum_{k=1}^{3} W_k^n - \frac{\Delta t}{2\mu_i} \sum_{k=1}^{3} \left(\bar{F}_k^n \Delta y_k - \bar{G}_k^n \Delta x_k \right) \tag{3.5}$$

   - LF Corrector:

$$\begin{aligned} W_{ok}^{n+1} &= \frac{1}{6} \sum_{i=1}^{6} W_i^{n+1/2} - \\ &- \frac{\Delta t}{2\mu_i} \sum_{i=1}^{6} \left(\bar{F}_i^{n+1/2} \Delta y_i - \bar{G}_i^{n+1/2} \Delta x_i \right) \end{aligned} \tag{3.6}$$

- LW Predictor is the same as LF Predictor (3.5)
- LW Corrector:

$$W_{ok}^{n+1} = W_{ok}^{n} - \frac{\Delta t}{\mu_{ok}} \sum_{i=1}^{6} \left(\bar{F}_i^{n+1/2} \Delta y_i - \bar{G}_i^{n+1/2} \Delta x_i \right) , \tag{3.7}$$

$\bar{F}_i = \frac{1}{2}(F_i + F_{i+1})$, $\bar{G}_i = \frac{1}{2}(G_i + G_{i+1})$.

4. Some numerical results. In the first part we present steady solution in a DCA8% 2D cascade of inviscid incompressible flows in the form of distribution of $q = \sqrt{u^2 + v^2}$ along upper and lower wall surface and periodical boundaries (fig. 4.1). Next fig. 4.2 shows steady results of transonic flows ($M_\infty = 0,675$) in GAMM channel using isolinies of Mach number (fig. 4.3) and distribution of Mach number along upper and lower walls. We can see results with shock wave. Comparison to the other numerical results is in very good agreement (see [2],[4]). Results of fig. 4.1 was achieved by cell-centered method and grid of triangles, results of fig. 4.2,4.3 by cell-vertex method grid of triangles. Next fig. 4.4,4.5 show unsteady flows in a cascade (incompressible flows) using distribution of $q = \sqrt{u^2 + v^2}$ along upper and lower surface. Fig. 4.6 shows unsteady solution in a GAMM channel. Both unsteady solutions were caused by a time change of $p_2 = p_2(t) = p_1(1 + k_1 \sin k_2 \omega t)$.

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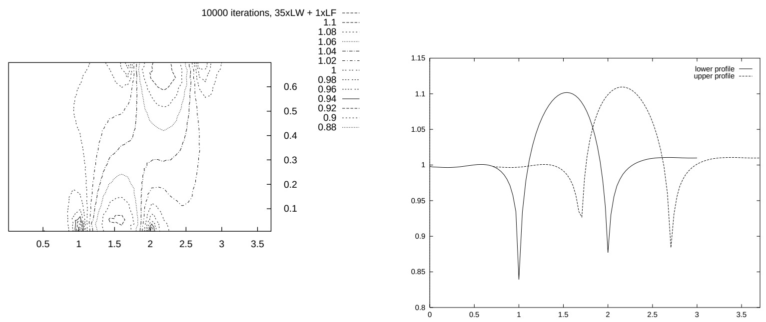


FIG. 4.1. *steady incompressible flows: isolines of* $q = \sqrt{u^2 + v^2}$ *and velocity distribution on the boundaries*

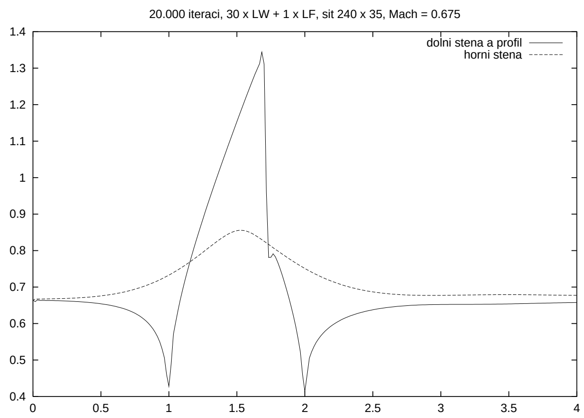


FIG. 4.2. *steady transonic flows, Mach number distribution on the walls*

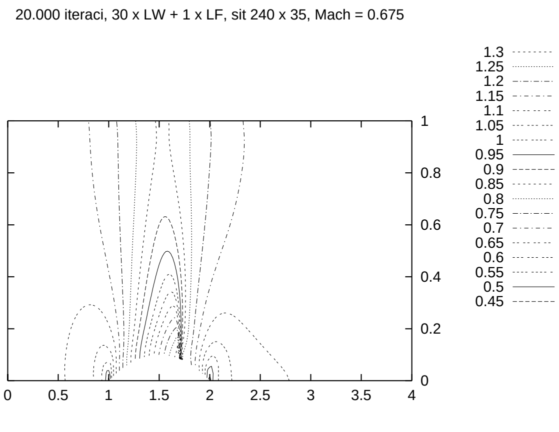


FIG. 4.3. *steady transonic flows: isolines of Mach number*

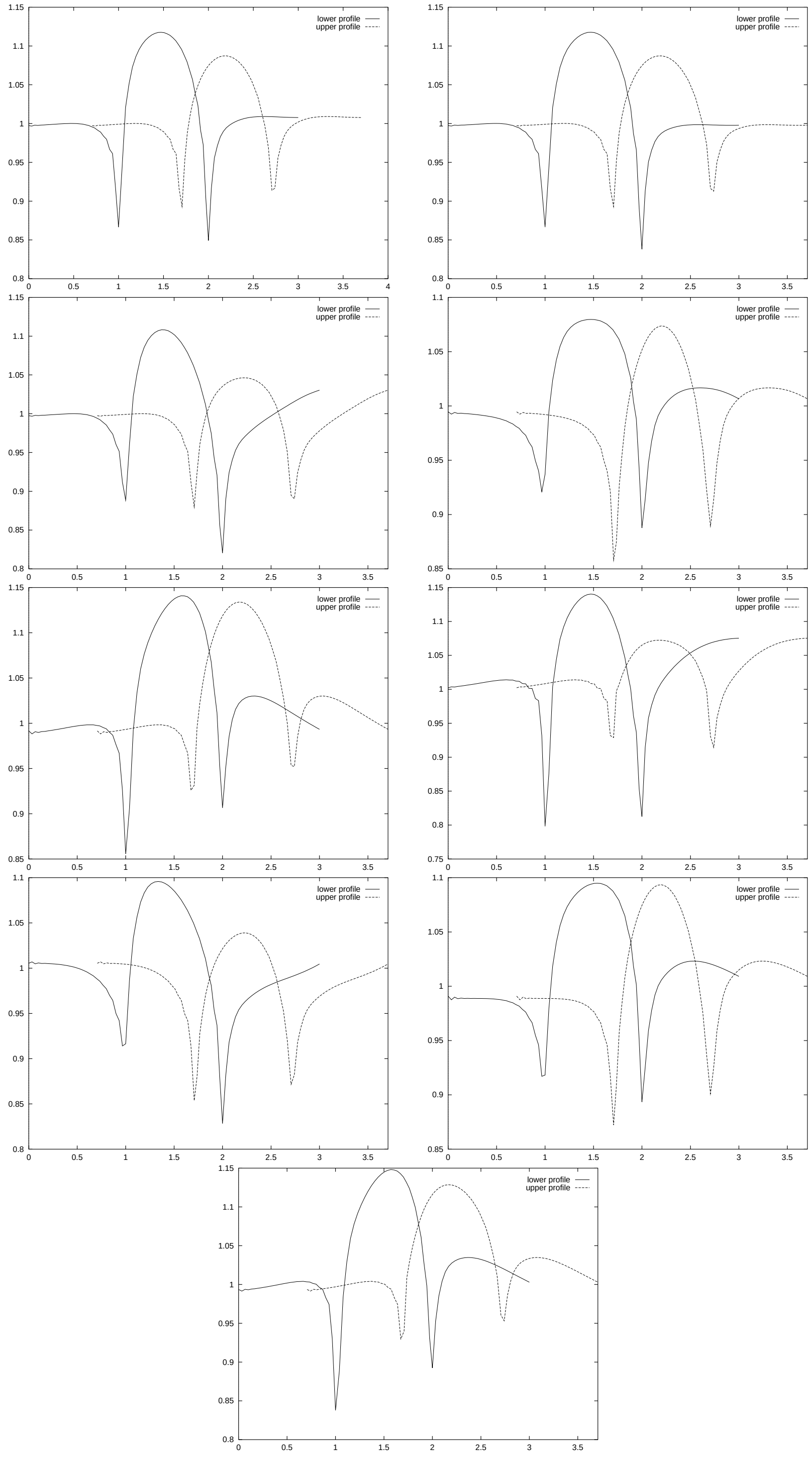


FIG. 4.4. *unsteady incompressible flows, quadrilateral cells, distribution of* $q = \sqrt{u^2 + v^2}$ *on the boundaries*

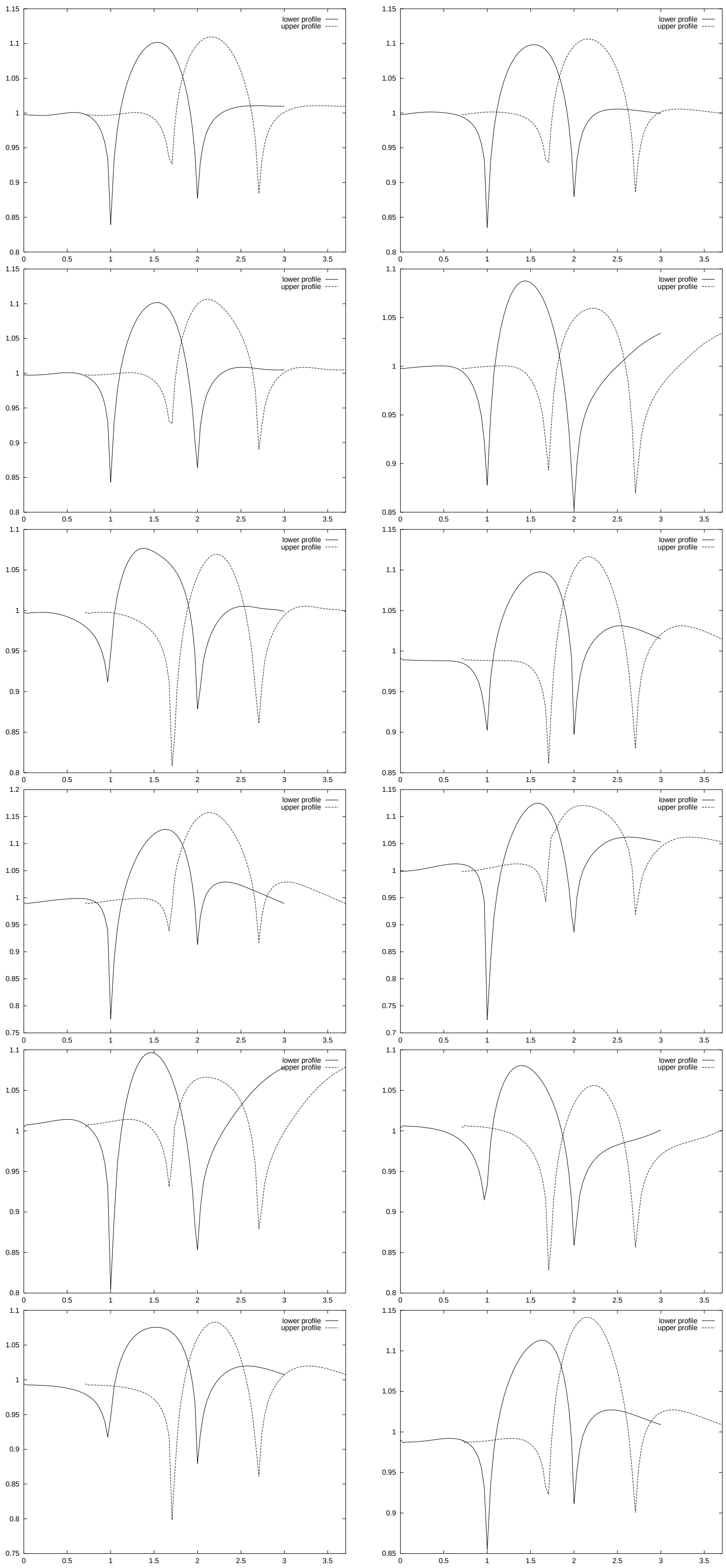
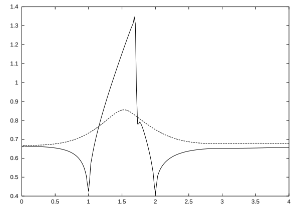


FIG. 4.5. *unsteady incompressible flows, triangular cells, distribution of* $q = \sqrt{u^2 + v^2}$ *on the boundaries*



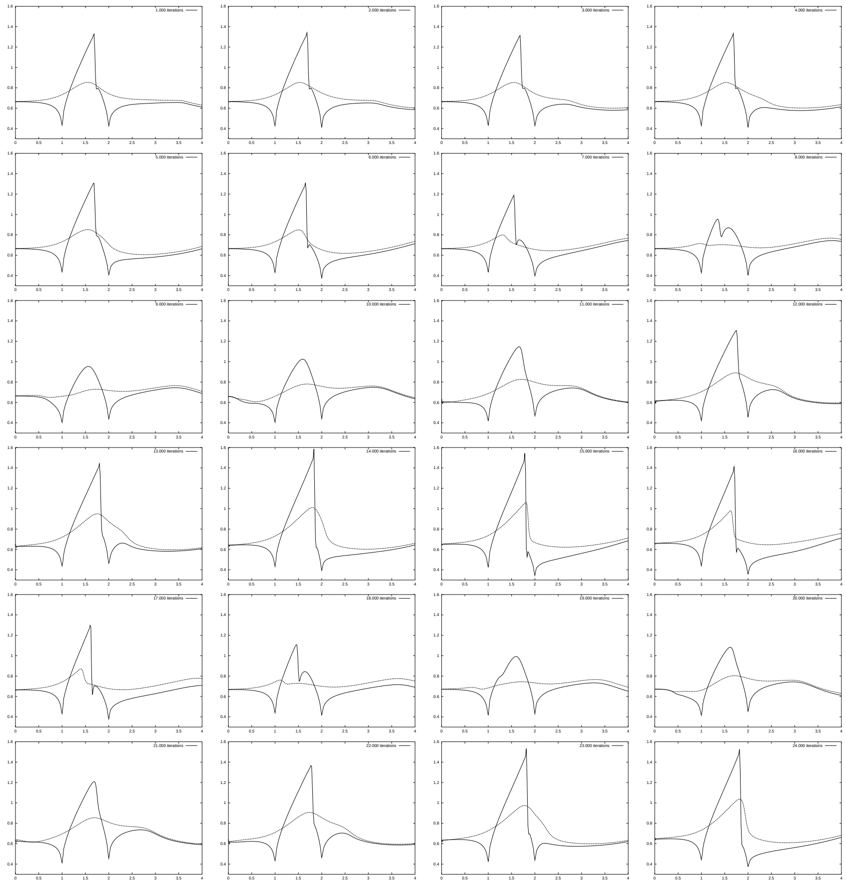


FIG. 4.6. *unsteady solution in a GAMM channel, distribution of Mach number on the walls*