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MIXED-HYBRID MODEL OF THE FRACTURE FLOW*

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Abstract. Finite element/mixed-hybrid formulation of a discrete fracture network model.

Key words. fracture flow, stochastic discrete fracture network, mixed-hybrid formulation of the finite element method

1. Introduction. Nuclear energy produced on the Earth has reached 16% of the world's energy production and it is generated by almost 500 nuclear reactors. The weakest link in the production of the energy by this way is a safe storage of highly radioactive spent fuel. This text deals with suggestion of a mathematical model describing percolation of groundwater in the fractured matrix of a solid rock, medium supposed as possible repository of dangerous nuclear waste.

In general, there are three main possible accesses to the problem of modelling the fracture flow. When only a large-scale model is required and if there is no need to know detail flow and transport behavior in any site subarea, it is possible to use *equivalent porous medium models*. More complex than single continuum models are *double porosity models*. These models are, however, again not capable adequate interpret small-scale measurements, it is possible to use them only to predict the average behavior at some distance from a repository. As a third possibility, we can generate a statistical model of the fracture network and model the flow and transport in by this way obtained network. Nowadays, due to high computer requirements, it is possible to solve just local problems by using *(stochastic) discrete fracture network models*. For more complex description of the problem, see for instance [1] or [11].

We decided to build all scale models. **Local models** will be developed first to simulate the detail behaviour of the contamination after an accidental breakdown of the waste container. In this case, **stochastic discrete fracture network model** will be constructed under the assumption that the fractures can be represented by circle discs whose frequency, size, aperture and orientation can be statistically derived from field measurements of characteristics of natural fractures. The hydraulic conductivity will be assigned from known hydraulic tests performed in drill holes in crystalline rocks or alternatively approximated by known relations to the other measurements. Also the fracture wall roughness will be statistically characterized. In order to comprehend the channeling effect inside a single fracture, an aperture-distribution function will be introduced.

For **regional models**, representing hydrogeological systems with size of tens to hundreds of km^2, an equivalent porous medium approach will be used. Here,

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we will develop a method of integration of physical and chemical characteristics of the fracture network, obtained from local discrete fracture network models, into the anisotropy tensor of hydraulic permeability of rock blocks, similarly as in [4]. By using local discrete fracture network models, we will also determine mean values of active surfaces of rock in order that we can evaluate the chemical interaction between rock and solution.

In this text, only a stochastic discrete fracture network model, obviously basis for our models, will be presented.

2. Mathematical-Physical Formulation. Let us suppose that in the domain of interest Ω, the system $\mathcal{S}$ of subdomains representing the fractures is given. The subdomains are supposed as bounded parts of plane varieties in R^3. We suppose that we have

$$\mathcal{S} \;=\; \{\,\alpha_\ell,\ \chi_\ell\,;\, \ell \in L\}\,, \tag{2.1}$$

where α_ℓ is the equation of ℓ^{th} variety and χ_ℓ specifies the space limitation of this variety. L is the index set of fractures. Let us suppose that the boundary $\partial\mathcal{S}$ and the boundaries of each fracture are Lipschitzian.

The fluid velocity/mass weighted velocity within a system of fractures $\mathcal{S}$ $\mathbf{u}$, $\mathbf{u}$: $\mathcal{S} \to R^3$, $\mathbf{u} \in \mathcal{S}$ [1], can be characterized by

$$\mathbf{u}' = -\frac{\mathbf{k}}{\mu}\nabla h\,,$$

where tensor $\mathbf{k}$ is a general permeability tensor (second rank symmetric and uniformly positive definite tensor), μ is the liquid dynamic viscosity, the prime indicates vector in the fracture plane (the whole equation is expressed in local 2-D coordinates of an appropriate fracture) and ∇h is a gradient of the piezometric head h defined as $h = \frac{p}{\varrho g} + z$, p is the fluid pressure, g is the gravitational acceleration constant, ϱ is the fluid density, and z is the elevation, positive upward taken vertical coordinate.

If we suppose that the whole system $\mathcal{S}$ is already completely drowned by a contaminated underground water supposed as a sparse homogeneous liquid, we can use the hydraulic conductivity tensor $\mathbf{K} = \frac{\mathbf{k}}{\mu}$ and finally also positive definite tensor $\mathbf{A} = \mathbf{K}^{-1}$, which characterizes medium resistance. We denote $\tilde{p} \equiv \frac{p}{\varrho g}$ as modified pressure and we are ready to sign the general fracture flow equation in a form

$$\mathbf{A}\mathbf{u}' = -(\nabla\tilde{p} + \nabla z)\,. \tag{2.2}$$

The mass balance/continuity equation will be used in the form

$$\nabla \cdot \mathbf{u} = q\,, \tag{2.3}$$

since we suppose an uncompressible fluid and a steady case. Here $\nabla \cdot \mathbf{u}$ means divergence of the searched vector function $\mathbf{u}$ and q represents stationary sources/sinks density.

[1] $\mathbf{u} \in \mathcal{S}$ is a symbolic expressions for the fact that the fracture flow velocity vector has to lie in the fracture plane/planes.

We set the Dirichlet's boundary condition

$$\tilde{p} = \tilde{p}_D \quad \text{in} \quad \Lambda_D \tag{2.4}$$

on a subset Λ_D of $\partial\mathcal{S}$. Here $\tilde{p}_D$ is a known function prescribing the external piezometric head $\tilde{p}$.

We suppose fulfilling of the general Newton's boundary condition on $\Lambda_N \subset \partial\mathcal{S}$

$$\mathbf{u} \cdot \mathbf{n} - \sigma(\tilde{p} - \tilde{p}_D) = u_N \quad \text{in} \quad \Lambda_N\,. \tag{2.5}$$

Here σ is the transmission coefficient function delimited by the constant $\overline{\sigma}_M$, $(0 \leq \sigma \leq \overline{\sigma}_M)$, $\tilde{p}_D$ is function similar to $\tilde{p}_D$ in (2.4), varying only by the support, and $\mathbf{n}$ is the unit outward boundary normal vector. This form of Newton's boundary condition allows us to describe even the situations, when some part of the boundary $\partial\mathcal{S}$ is impervious and a homogeneous Neumann's condition $\mathbf{u} \cdot \mathbf{n} = 0$ should be prescribed. In our case, we only set $\sigma = u_N = 0$.

Finally, we require

$$\Lambda_D \cap \Lambda_N = \emptyset\,,\ \overline{\Lambda}_D \cup \overline{\Lambda}_N = \partial\mathcal{S}\,,\ \Lambda_D \neq \emptyset\,.$$

3. Mixed-hybrid Formulation. In this section, we will present the own finite element, mixed hybrid formulation of the problem of saturated steady flow of a homogeneous incompressible fluid in a discrete fracture network. For the mixed-hybrid formulation, we will need a concrete partition of the system $\mathcal{S}$.

Let us suppose that we have a triangulation $\mathcal{T}_h$ of the system $\mathcal{S}$. We suppose that it holds:

$$\begin{aligned}
&(i) \qquad\qquad \overline{\mathcal{S}} = \cup_{e\in\mathcal{T}_h}\overline{e}\ ;\\
&(ii) \qquad e_i \cap e_j = \emptyset\ , \ \text{ if } \ i \neq j\ ;\\
&(iii)\ e \in \mathcal{T}_h \text{ is an open subset of } \mathcal{S}\,.
\end{aligned}$$

Let us denote

$$\Lambda_{h,D} = \cup_{e\in\mathcal{T}_h}\partial e - \Lambda_D\,,$$

i.e. $\Lambda_{h,D}$ denotes the structure of interelement edges and edges belonging to so part of the boundary $\partial\mathcal{S}$, where the Newton's condition is prescribed. Further, we will need following function spaces:

We extend the space of scalar Lebesgue integrable functions $L^2(\mathcal{S})$ also for the vector functions and denote the appropriate space $\mathbf{L}^2(\mathcal{S})$, $\mathbf{L}^2(\mathcal{S}) = L^2(\mathcal{S}) \times L^2(\mathcal{S})$, again supposed in local coordinates in a given element $e \in \mathcal{S}$. We set a scalar product on this space as $(\mathbf{u}, \mathbf{v})_{0,\mathcal{S}} = \int_{\mathcal{S}} \mathbf{u} \cdot \mathbf{v}\ dS$.

We denote as $H^1(\mathcal{S})$ the Sobolev space

$$H^1(\mathcal{S}) = \{\varphi \in L^2(\mathcal{S});\ \nabla\varphi \in \mathbf{L}^2(\mathcal{S})\}\,.$$

For the functions $\varphi \in H^1(\mathcal{S})$, it is possible to define the trace $\gamma\varphi$ on the boundary $\partial\mathcal{S}$. We further define

$$H^1_D(\mathcal{S}) = \{\ \varphi \in H^1(\mathcal{S});\ \gamma\varphi = 0 \ \text{ on } \ \Lambda_D\ \}\,.$$

On the the partition $\mathcal{T}_h$ and edges structure $\Lambda_{h,D}$, we define spaces

$$\mathbf{H}(div,\mathcal{T}_h) = \{\mathbf{v} \in \mathbf{L}^2(\mathcal{S})\,;\ \nabla\cdot\mathbf{v}^e \in L^2(e) \quad \forall e \in \mathcal{T}_h\}\,,$$

and

$$H^{\frac{1}{2}}(\Lambda_{h,D}) = \{\mu : \Lambda_{h,D} \to R\,;\ \exists\varphi \in H^1_D(\mathcal{S}),\ \mu = \gamma_h\varphi\}\,,$$

where γ_h is a trace operator defined on the element edges structure $\Lambda_{h,D}$ of selected partition $\mathcal{T}_h$. The upper index e denotes restriction of a function on appropriate element e.

For the weak mixed-hybrid formulation, we suppose $a_{pq}(\mathbf{x}) \in L_\infty(\mathcal{S})$, $q \in L_2(\mathcal{S})$, $p_D \in H^{\frac{1}{2}}(\Lambda_D \cup \Lambda_N)$, $u_N \in H^{-\frac{1}{2}}(\Lambda_N)$ and $\sigma \in L_\infty(\Lambda_N)$. The main idea of this formulation is a weak performance of Darcy's law (2.2), mass balance (2.3) and the boundary condition (2.5) on each element e from the given partition $\mathcal{T}_h$. Using only the Green's formula and summing of the three describing weak equalities over all elements $e \in \mathcal{T}_h$, assuring the weak mass balance between single elements (each inner edge of the partition $\mathcal{T}_h$ is common to two elements and it can happen, that it is common to three or more elements), we obtain a resulting set of equations, which is equivalently rewritten by the following construction and definition:

We define a space

$$\mathbf{W}_D(\mathcal{T}_h) = \mathbf{H}(div,\mathcal{T}_h) \times L^2(\mathcal{S}) \times H^{\frac{1}{2}}(\Lambda_{h,D})\ , \tag{3.1}$$

bilinear form

$$\begin{aligned}\mathcal{B}(\mathcal{T}_h;\widehat{\mathbf{w}},\mathbf{w}) = \sum_{e\in\mathcal{T}_h}\{&(\mathbf{A}\mathbf{u}^e,\mathbf{v}^e)_{0,e} - (p^e,\nabla\cdot\mathbf{v}^e)_{0,e} - (\nabla\cdot\mathbf{u}^e,\phi^e)_{0,e} + \\ &+\langle\lambda^e,\mathbf{v}^e\cdot\mathbf{n}^e\rangle_{\partial e\cap\Lambda_{h,D}} - \langle\mathbf{u}^e\cdot\mathbf{n}^e,\mu^e\rangle_{\partial e\cap\Lambda_{h,D}} - \langle\sigma^e\lambda^e,\mu^e\rangle_{\partial e\cap\Lambda_N}\}\end{aligned} \tag{3.2}$$

and a linear functional

$$\begin{aligned}\mathcal{Q}(\mathcal{T}_h;\mathbf{w}) = \sum_{e\in\mathcal{T}_h}\{&-\langle p_D^e,\mathbf{v}^e\cdot\mathbf{n}^e\rangle_{\partial e\cap\Lambda_D} + \langle u_N^e - \sigma^e p_D^e,\mu^e\rangle_{\partial e\cap\Lambda_N} \\ &+ (z^e,\nabla\cdot\mathbf{v}^e)_{0,e} - \langle z^e,\mathbf{v}^e\cdot\mathbf{n}^e\rangle_{\partial e} - (q^e,\phi^e)_{0,e}\}\,,\end{aligned} \tag{3.3}$$

where $\widehat{\mathbf{w}} = (\mathbf{u},p,\lambda) \in \mathbf{W}_D(\mathcal{T}_h)$ and $\mathbf{w} = (\mathbf{v},\phi,\mu) \in \mathbf{W}_D(\mathcal{T}_h)$.

Definition 3.1. *As a weak solution of the mixed-hybrid formulation of the problem of steady fracture flow described by (2.2) and (2.3) with boundary conditions defined by (2.4) and (2.5) on the partition $\mathcal{T}_h$ of the system of varieties $\mathcal{S}$, we understand a function $\widehat{\mathbf{w}} = (\mathbf{u},p,\lambda) \in \mathbf{W}_D(\mathcal{T}_h)$, which satisfies the integral identity*

$$\mathcal{B}(\mathcal{T}_h;\widehat{\mathbf{w}},\mathbf{w}) = \mathcal{Q}(\mathcal{T}_h;\mathbf{w}) \tag{3.4}$$

for all $\mathbf{w} = (\mathbf{v},\phi,\mu) \in \mathbf{W}_D(\mathcal{T}_h)$.

Our three unknowns are the finally searched fracture flow velocity $\mathbf{u}$[2], the external piezometric head p and the solution will yield also directly approximation λ of trace of p on the element edges structure $\Lambda_{h,D}$. Functions $\mathbf{v},\phi$ and μ are the testing functions.

[2]We suppose here $\mathbf{u}$ directly as a vector in an appropriate fracture plane 2-D local coordinates and we do not repeat the notation $\mathbf{u}'$ used by the formulation of the problem. Similarly, we use single p instead of $\tilde{p}$ for the simplification.

4. Finite-dimensional Approximation. The main idea of the final approximation is to transform the problem from the definition 3.1 into a finite dimensional one. We have to prepare appropriate finite dimensional function spaces first.

We define a 3-dimensional space of vector functions $\mathbf{RT}^0(e)$ linear on a given element e with the base $\mathbf{v}_l^e$, $l \in \{1,2,3\}$, where

$$\mathbf{v}_1^e = k_1^e \begin{bmatrix} x_1 - \alpha_{11}^e \\ x_2 - \alpha_{12}^e \end{bmatrix}, \quad \mathbf{v}_2^e = k_2^e \begin{bmatrix} x_1 - \alpha_{21}^e \\ x_2 - \alpha_{22}^e \end{bmatrix}, \quad \mathbf{v}_3^e = k_3^e \begin{bmatrix} x_1 - \alpha_{31}^e \\ x_2 - \alpha_{32}^e \end{bmatrix}.$$

We set the parameters $\alpha_{11}^e - \alpha_{32}^e$ so that each base function $\mathbf{v}_l^e$ expresses flux through one edge of the element e, i.e. we require that

$$\int_{f_l^e} \mathbf{n}_l^e \cdot \mathbf{v}_m^e \, dl \;=\; \delta_{lm} \quad l,m = 1,2,3\,,$$

where f_l^e is l-th edge of the element e and $\mathbf{n}_l^e$ is the unite outward normal vector of this edge.

We define now the Raviart-Thomas space $\mathbf{RT}_{-1}^0(\mathcal{T}_h)$ of on each element linear vector functions,

$$\mathbf{RT}_{-1}^0(\mathcal{T}_h) \equiv \{\mathbf{v} \in \mathbf{L}^2(\mathcal{S})\,;\; \mathbf{v}|_e \in \mathbf{RT}^0(e) \quad \forall e \in \mathcal{T}_h\}\,,$$

an index set $I_h = 1,2,\ldots,|I_h|$ for global numbering of basic functions of $\mathbf{RT}_{-1}^0(\mathcal{T}_h)$ and we set following relation between local and global numbering: $\mathbf{v}_l^j \equiv \mathbf{v}_{3(j-1)+l}$.

We need first the space $M^0(e)$ of scalar functions constant on a given element e for the definition of the space $M_{-1}^0(\mathcal{T}_h)$,

$$M_{-1}^0(\mathcal{T}_h) \;\equiv\; \{\phi_h \in L^2(\mathcal{S})\,;\; \phi_h|_e \in M^0(e) \quad \forall e \in \mathcal{T}_h\}\;.$$

The functions ϕ_j, $\phi_j(\mathbf{x}) = 1$ for $\mathbf{x} \in e_j$ and $\phi_j(\mathbf{x}) = 0$ for $\mathbf{x} \notin e_j$, $j \in J_h$, where $J_h = 1,2,\ldots,|J_h|$ is the index set of elements, create the base of $M_{-1}^0(\mathcal{T}_h)$.

If $f \in \Lambda_{h,D}$ is some edge, we define first the space $M^0(f)$ of functions constant on the edge f and finally

$$M_{-1}^0(\Lambda_{h,D}) \;\equiv\; \{\mu_h : \Lambda_{h,D} \to R\,;\; \mu_h|_f \in M^0(f) \quad \forall f \in \Lambda_{h,D}\}\;.$$

The basic functions of $M_{-1}^0(\Lambda_{h,D})$ are $\mu_k(\mathbf{x}) = 1$ for $\mathbf{x} \in f_k$ and $\mu_k(\mathbf{x}) = 0$ for $\mathbf{x} \notin f_k$, where f_k is k-th edge from $\Lambda_{h,D}$, $k \in K_h$, $K_h = 1,2,\ldots,|K_h|$ is the index set of element edges from $\Lambda_{h,D}$.

For more proper definition of the finite dimensional spaces, see for example [2]. We are ready now to define the own finite-dimensional approximation of the problem of the fracture flow.

DEFINITION 4.1. *As a Raviart-Thomas approximation of the mixed-hybrid formulation of the problem of steady fracture flow described by (2.2), (2.3) with boundary conditions (2.4) and (2.5) on the partition $\mathcal{T}_h$ of the system of varieties $\mathcal{S}$, we understand a function $\widehat{\mathbf{w}}^h = (\mathbf{u}_h, p_h, \lambda_h) \in \mathbf{W}_D^h(\mathcal{T}_h)$, which satisfies the integral identity*

$$\mathcal{B}(\mathcal{T}_h; \widehat{\mathbf{w}}^h, \mathbf{w}^h) = \mathcal{Q}(\mathcal{T}_h; \mathbf{w}^h) \tag{4.1}$$

for all $\mathbf{w}^h = (\mathbf{v}_h, \phi_h, \mu_h) \in \mathbf{W}_D^h(\mathcal{T}_h)$.

The function space $\mathbf{W}_D^h(\mathcal{T}_h)$ is defined by

$$\mathbf{W}_D^h(\mathcal{T}_h) = \mathbf{RT}_{-1}^0(\mathcal{T}_h) \times M_{-1}^0(\mathcal{T}_h) \times M_{-1}^0(\Lambda_{h,D})\,. \tag{4.2}$$

If we substitute for $\widehat{\mathbf{w}}^h$ and $\mathbf{w}^h$ into the equation (4.1), using the finite dimension of used spaces and expressing $\mathbf{u}_h(\mathbf{x}) = \sum_{i\in I_h} U^i \mathbf{v}_i(\mathbf{x})$, $p_h(\mathbf{x}) = \sum_{j\in J_h} P^j \phi_j(\mathbf{x})$ and $\lambda_h(\mathbf{x}) = \sum_{k\in K_h} \Lambda^k \mu_k(\mathbf{x})$, realizing that the basic function $\mathbf{v}_a$ is not zero only on the element e_d, where $d = (a-1) \div 3 + 1$ (the expression $\div$ means integral division), ϕ_b has only a triangle e_b as its support and μ_c is non-zero only on edge of such element, that this elements contains the edge f_c (we denote this set of triangles for given μ_c as $\mathcal{T}_c$), and using the linearity of the scalar product and all operators (involving the divergence and integral ones), we obtain set of linear equations

$$\begin{aligned}\sum_{i\in I_h} U^i(\mathbf{A}\mathbf{v}_i^{e_d}, \mathbf{v}_a)_{0,e_d} - \sum_{j\in J_h} P^j(\phi_j^{e_d}, \nabla\cdot\mathbf{v}_a)_{0,e_d} + \sum_{k\in K_h} \Lambda^k \langle \mu_k^{e_d}, \mathbf{v}_a\cdot\mathbf{n}^{e_d}\rangle_{\partial e_d \cap \Lambda_{h,D}} = \\ = -\langle p_D^{e_d}, \mathbf{v}_a\cdot\mathbf{n}^{e_d}\rangle_{\partial e_d\cap\Lambda_D} + (z^{e_d}, \nabla\cdot\mathbf{v}_a)_{0,e_d} - \langle z^{e_d}, \mathbf{v}_a\cdot\mathbf{n}^{e_d}\rangle_{\partial e_d}\,, \\ \forall\, \mathbf{v}_a \in \{\mathbf{v}_i\}_{i=1}^{|I_h|}\,,\end{aligned} \tag{4.3}$$

$$\begin{aligned}-\sum_{i\in I_h} U^i(\nabla\cdot\mathbf{v}_i^{e_b}, \phi_b)_{0,e_b} = -(q^{e_b}, \phi_b)_{0,e_b}\,, \\ \forall\, \phi_b \in \{\phi_j\}_{j=1}^{|J_h|}\,,\end{aligned} \tag{4.4}$$

$$\begin{aligned}\sum_{e\in\mathcal{T}_c}\left[\sum_{i\in I_h} U^i\langle \mathbf{v}_i^e\cdot\mathbf{n}^e, \mu_c\rangle_{\partial e\cap\Lambda_{h,D}} - \sum_{k\in K_h}\Lambda^k\langle\sigma^e\mu_k^e, \mu_c\rangle_{\partial e\cap\Lambda_N}\right] = \\ = \sum_{e\in\mathcal{T}_c}\langle u_N^e - \sigma^e p_D^e, \mu_c\rangle_{\partial e\cap\Lambda_N}\,, \\ \forall\, \mu_c \in \{\mu_k\}_{k=1}^{|K_h|}\,,\end{aligned} \tag{4.5}$$

where $\mathbf{v}_a$ is a-th basic function of $\mathbf{RT}_{-1}^0(\mathcal{T}_h)$, the superscript e_a denotes restriction on the a-th element and correspondingly in the two remaining set of equations.

If we analyze in detail the structure of the system of equations (4.3)-(4.5) describing the mixed-hybrid model of the fracture flow, we can express the linear algebra problem in a form

$$\begin{array}{lllllll} \mathbb{A}\mathbf{u} & + & \mathbb{B}\mathbf{p} & + & \mathbb{C}\boldsymbol{\lambda} & = & \mathbf{q}_1 \\ \mathbb{B}^T\mathbf{u} & & & & & = & \mathbf{q}_2 \\ \mathbb{C}^T\mathbf{u} & + & & & \mathbb{S}\boldsymbol{\lambda} & = & \mathbf{q}_3 \end{array}\;, \tag{4.6}$$

the unknowns $\mathbf{u} = (U^1, \ldots, U^{|I_h|})^T$, $\mathbf{p} = (P^1, \ldots, P^{|J_h|})^T$ and $\boldsymbol{\lambda} = (\Lambda^1, \ldots, \Lambda^{|K_h|})^T$. For more details, see [10] or [5].

5. Stochastic Discrete Fracture Network Generation. In order to generate the stochastic discrete fracture networks, an original software called **Fracture Network Generator** was developed. Each fracture (geological 3-D object) is in the generator approximated by a flat circle characterized by its *middle coordinates*, *radius*, *orientation*, possibly *hydraulic conductivity*, *aperture distribution* and *roughness*. Fractures are divided into four sets, *fractures in fracture zones*, *deterministically measured single fractures*, *hydraulically important fractures* and *other (common) fractures*. Fractures are further supposed to be divided into three types according to their mean orientation [0,0,1], [0,1,0] or [1,0,0].

Description of all fracture characteristics if fully statistical. Number and spacing of fractures in each set and type determines *fracture frequency*, defined as amount of fractures per one depth meter in each part of the simulated domain Ω. Fracture lengths for each combination of set and type are supposed longnormally distributed, i.e. with the probability density function (p.d.f.) $f(x) = \frac{1}{\sigma\sqrt{2\pi x}}\exp(-(\ln x - \mu)^2/2\sigma^2)$.

For each type (according to the orientation) and set, Fisher-von-Mises distribution $f(\alpha) = \frac{k}{\exp(k)}\exp(k\cos\alpha)\sin\alpha$ of angles α between fracture normal vectors and vectors of mean orientations is entered. Here k is the parameter of the Fisher-von-Mises distribution probably various for various combinations of types and sets.

The generator accepts required deterministically measured single fractures, generates a network of hydraulically important fractures into the whole entered domain, generates fractures into prescribed fracture zones and into the remaining part of the whole domain (except fracture zones) generates network of other, common fractures. The computer generation of random numbers with required distribution is based on principles described in [7], usually used to Monte Carlo simmulations. The generator is also prepared for additional accepting of fractures of a given set, if needed for change of characteristics of already generated fractures. Finally fracture intersections are computed.

A plenty of control tests was provided in order to validate used generating algorithms. We have validated the generator especially from the mathematical (statistical) point, but the sensibility of all results was checked too. All tests methods and their results are in detail described in [9]. In general, we can say that all used generating methods function without any significant mistakes, we only have to put an emphasis on strictly distinguishing between real and in exploration boreholes measured distributions, the latter of them are affected by a selective effect.

We can see an example of a generated fracture network on the figure 5.1.

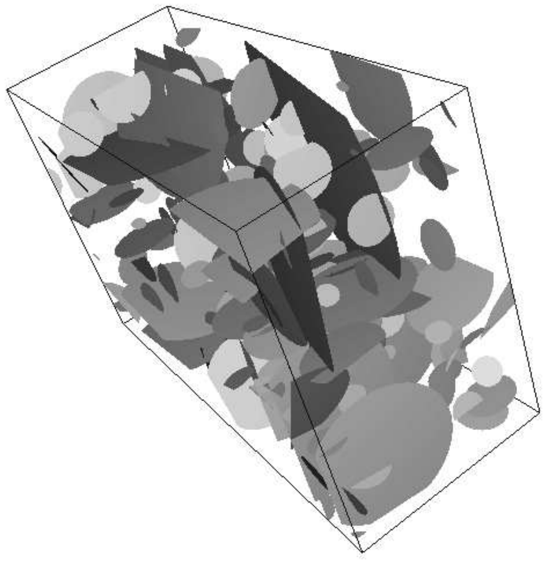

FIG. 5.1. *Generated fracture network*

6. Final Triangular Mesh Construction. Discretization of approximating circles into triangle elements has occurred as a crucial point in final triangular mesh preparation. Although many algorithms solving the discretization of a given 2-D domain are known, only few of them are able to involve pre-defined interface lines (intersections of approximating circles in our case) randomly distributed in the considered domain. The searched algorithm is, however, in addition required not to be designed for adaptive meshing (because of sparing of computer storage) and should be on the contrary capable to simplify the given geometrical situation, i.e. it can be only approximative. In the **Fracture Network Generator**, originally developed discretization algorithm is implemented. It contains of a preliminary phase and of an algorithm for triangulation of an arbitrary polygonally bounded domain with pre-defined interface lines (triangulation algorithm).

In the preliminary phase, identification and various geometrical simplifications of the structure of intersections in single fractures even of the whole fracture network are made. Close, almost parallel fractures are removed from the fracture population or equivalently replaced, because when kept, they could create close intersections in another fracture and this could cause either numerical unstability in the triangulation algorithm or very ill-conditioned resulting matrix due to increase of the chunkiness parameter. 2-D geometrical adaptations (moving and stretching intersections in fracture planes) simplifying the situation in fractures are provided before the start of the triangulation algorithm. However, using these 2-D geometrical simplifications has not trivial consequences for the whole 3-D representation.

The own triangulation algorithm is based on combining the *Domain Decomposition Conception* and adapted *Advancing Front Method.* Many user's setting influencing mainly precision/complexity of the final triangulation are possible.

An example of final triangulated circle disk is on the fig. 6.1.

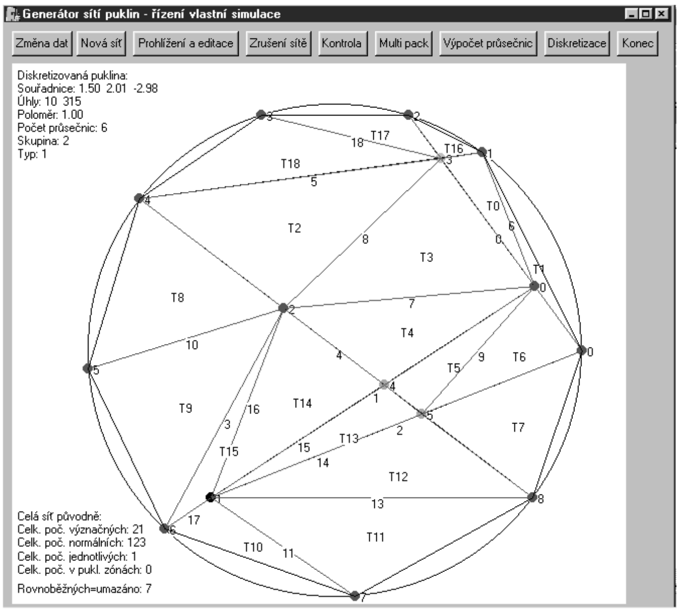

FIG. 6.1. *Final discretization of a circle disk*

An on-element aperture distribution function, derived again from the Fisher-von-Mises distribution p.d.f., but depending also on the size of given fracture and emplacement of the element inside a fracture, is used after the discretization in order to assign to each triangle element an imaginary aperture. Based on this aperture and on a parameter describing roughness of the fracture walls, the element hydraulic conductivity can be later set, if it is not entered directly from results of experimental measurements as unique for the whole fracture; the fracture is, however, still supposed as planar. Data files with complete information about elements in the final triangular network (emplacement, aperture, roughness, hydraulic conductivity and connection to the other elements) are the final results of the **Fracture Network Generator**.

We can see part of a final triangular mesh on the figure 6.2.

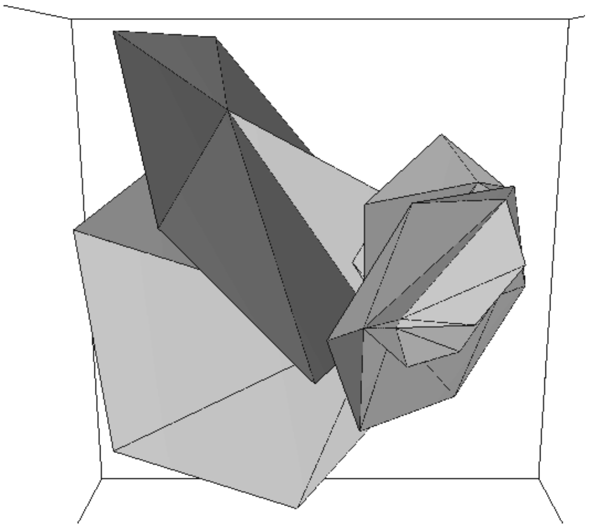

FIG. 6.2. *Final triangular mesh*

7. Conclusion. In the submitted text, possible approaches to the description of the fracture flow and transport are presented first. Consequently, detailed suggestion of a finite element/mixed-hybrid discrete fracture network model is presented. In this model, fractures are supposed as bounded parts of plane varieties, later discretized into triangle elements, the flow is supposed as governed by the Darcy's law. Complete formulation of the Raviart-Thomas approximation of the problem of steady fracture flow leading to final linear-algebra problem is the main result of sections 3 and 4. Later, a brief description of a stochastic fracture network generated by the originally developed software **Fracture Network Generator** is suggested and also the most valuable part of the **Fracture Network Generator**, the algorithm for approximate two-dimensional triangular mesh generation in a circle disk with pre-defined interface lines, is introduced.

The main emphasis is put on the discrete fracture network model, since it is a basis for by us proposed models. A mixed hybrid formulation of the finite element method was chosen for the discretization of the problem, because it, unlike the primal formulation, assures mass balance on each element and its data structures are though suitable for finite volume contaminant transport models, which are perspectively our main goal. A wide work will have to be done in implementation of presented model and later all scale models, but a solid base already exists in a form of the **Fracture Network Generator**, software preparing complex data structures.

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