Acta Math. Univ. Comenianae
Vol. LXII, 1(1993), pp. 109–115

ON MARCZEWSKI SETS AND SOME IDEALS

M. BALCERZAK

Abstract. Using the methods of Brown and Walsh, we get condition guaranteeing that, for an ideal $\mathcal{I}$ of sets in a perfect Polish space some (s^0) sets are not in $\mathcal{I}$. A few examples and corollaries are given.

0. Introduction

Papers [**Br**], [**W1**], [**W2**] and [**C**] made a significant progress in the studying of (s^0) sets introduced by Marczewski in [**Sz**]. One of the main results states that there exists a nonmeasurable (s^0) set without the Baire property. That was proved in [**Br**] under CH and in [**W1**], [**W2**], [**C**] within ZFC. We analyse the schemes from [**Br**] and [**W1**], [**W2**] and get two criteria for an ideal $\mathcal{I}$ (of sets in a perfect Polish space X) to satisfy $\mathcal{I}_0 \setminus \mathcal{I} \neq \emptyset$ where $\mathcal{I}_0$ denotes the ideal of all (s^0) sets (in X). The original proofs we base on need only a slight modification. However, we give new versions in full. We describe some applications.

Throughout the paper, we fix a perfect Polish space X. A set which has no perfect subset is called totally imperfect. A set $E \subseteq X$ is called an (s^0) set if each perfect set has a perfect subset disjoint from E (see [**Sz**]). Obviously, (s^0) sets are totally imperfect and, moreover, they form an ideal (see [**Sz**]) which will be written as $\mathcal{I}_0$.

For any ideal $\mathcal{I} \subseteq \mathcal{P}(X)$, we always assume that $X \notin \mathcal{I}$ (here $\mathcal{P}(X)$ is the power set of X). The cardinality of continuum is denoted by c.

Further, the following lemma will be useful.

0.1. Lemma. *Let $A \subseteq X$ be an uncountable analytic set and $E \subseteq X$. If $|A \cap E| < c$, then there exists a perfect set $P \subseteq A$ missing E.*

Proof. Find a perfect $P \subseteq A$ (cf. [**Kr**, §39.I]) and c pairwise disjoint perfect subsets of P. At least one of them misses E. $\square$

Received August 16, 1991.
1980 *Mathematics Subject Classification* (1991 *Revision*). Primary 04A15, 54H05.
Key words and phrases. Perfect set, (s^0) set, ideal.

1. The First Criterion

A family $\tilde{\mathcal{I}}$ is called a base of an ideal $\mathcal{I} \subseteq \mathcal{P}(X)$ if $\tilde{\mathcal{I}} \subseteq \mathcal{I}$ and, for each $A \in \mathcal{I}$, there exists $B \in \tilde{\mathcal{I}}$ containing A. We denote

$$\operatorname{cof}(\mathcal{I}) = \min\{\, |\tilde{\mathcal{I}}| : \tilde{\mathcal{I}} \text{is base of } \mathcal{I}\,\}\,,$$
$$\operatorname{cov}(\mathcal{I}) = \min\{\, |\mathcal{H}| : \mathcal{H} \subseteq \mathcal{I} \text{ and } \cup\mathcal{H} = X\,\}\,.$$

It is evident that $\operatorname{cov}(\mathcal{I}) \le \operatorname{cof}(\mathcal{I})$.

We say that $\mathcal{I}$ has property (P) if each perfect set in X has a perfect set belonging to $\mathcal{I}$ (cf. [**Ba1**]).

The following proposition generalizing the method from Example 3 in [**Br**] has been inspired by some comment contained in [**C**].

1.1. Proposition (Criterion 1). *Let $\mathcal{I} \subseteq \mathcal{P}(X)$ be an ideal such that*

(a) $\operatorname{cov}(\mathcal{I}) = \operatorname{cof}(\mathcal{I}) = c$,

(b) *$\mathcal{I}$ has property* (P).

Then $\mathcal{I}_0 \setminus \mathcal{I} \neq \emptyset$.

Proof. Since $\operatorname{cof}(\mathcal{I}) = c$, there is a base $\tilde{\mathcal{I}}$ of $\mathcal{I}$ with $|\tilde{\mathcal{I}}| = c$. Let $\{A_\alpha : \alpha < c\}$ be an enumeration of sets A such that $X \setminus A \in \tilde{\mathcal{I}}$. Let $\{P_\alpha : \alpha < c\}$ be an enumeration of all perfect subsets of X. By virtue of (b), choose a perfect $Q_0 \in \mathcal{I}$ contained in P_0. Pick any x_0 in $A_0 \setminus Q_0$. If $0 < \alpha < c$ and if x_β, for $\beta < \alpha$, are defined, choose a perfect subset $Q_\alpha \in \mathcal{I}$ of P_α and let

$$\mathcal{F}_\alpha = \{Q_\beta : \beta \le \alpha\} \cup \{x_\beta : \beta < \alpha\}\,.$$

Observe that $A_\alpha \setminus \cup\mathcal{F}_\alpha \notin \mathcal{I}$. Indeed, if it is not the case, then for

$$\mathcal{H} = \mathcal{F}_\alpha \cup \{A_\alpha \setminus \cup\mathcal{F}_\alpha\} \cup \{X \setminus A_\alpha\}\,,$$

we would get $|\mathcal{H}| < c$, $\cup\mathcal{H} = X$, which contradicts $\operatorname{cov}(\mathcal{I}) = c$. Now, pick any x_α in $A_\alpha \setminus \cup\mathcal{F}_\alpha$. If the induction is finished, set $E = \{x_\alpha : \alpha < c\}$.

To show that E is an (s^0) set, consider any P_α. By the construction, $Q_\alpha \cap E \subseteq \{x_\beta : \beta < \alpha\}$. So, by Lemma 0.1, there is a perfect subset of Q_α (thus of P_α) which misses E.

To show $E \notin \mathcal{I}$, suppose that $E \in \mathcal{I}$ and choose $\tilde{E} \in \tilde{\mathcal{I}}$ containing E. Then $X \setminus \tilde{E} = A_\alpha$ for some $\alpha < c$. We have $A_\alpha \subseteq X \setminus E$, which implies $x_\alpha \notin E$, a contradiction. $\square$

Since $\operatorname{cov}(\mathcal{I}) \le \operatorname{cof}(\mathcal{I})$, it is suffices to assume in (a) that $c \le \operatorname{cov}(\mathcal{I})$ and $\operatorname{cof}(\mathcal{I}) \le c$. Property (P) seems rather strong. Note that it is not necessary for $\mathcal{I}_0 \setminus \mathcal{I} \neq \emptyset$. Indeed, if $\mathcal{I}$ is an ideal with property (P) and $\mathcal{I}_0 \setminus \mathcal{I} \neq \emptyset$, then throwing out all perfect subsets of a fixed perfect set from $\mathcal{I}$, we get the ideal $\mathcal{I}^*$ for which

(P) fails to hold and $\mathcal{I}_0 \setminus \mathcal{I}^* \neq \emptyset$. In Section 2 we give examples of quite large and regular ideals without property (P) which do not contain all (s^0) sets.

Coming back to the sources of Criterion 1; i.e. to Example 3 from [**Br**], consider the case when $\mathcal{I}$ is the ideal $\mathcal{L}$ of the Lebesgue null sets in the real line $\mathbb{R}$. It is obvious that $\operatorname{cof}(\mathcal{L}) \leq c$, and that $\mathcal{L}$ has property (P). The statement $\operatorname{cov}(\mathcal{L}) = c$ is implied by CH (or by MA) but is not equivalent. By Criterion 1, we get $\mathcal{I}_0 \setminus \mathcal{L} \neq \emptyset$. This easily implies the existence of an (s^0) set which is nonmeasurable (cf. Corollary 3.2). That was obtained in [**W2**] within ZFC. The first step of the proof is Theorem 2.2 from [**W1**] (for a generalization, see Section 2 of our paper). The second step uses the Fubini theorem. The same technique repeats when the measure is replaced by category (for other cases, see [**Ba2**]. However, if we have no analogues of the Fubini theorem, it can be unclear how to continue the first step. Thus Criterion 1 may help.

1.2. Example. Consider an infinite $K \subseteq \omega = \{0, 1, 2, \dots\}$ and a set $E \subseteq 2^\omega$ where 2^ω is the Cantor space of all infinite sequences with terms from $\{0, 1\}$. Let $\Gamma(E, K)$ be the following game between two players I and II. They choose consecutive terms of a sequence $x = \langle x(0), x(1), \dots\rangle \in 2^\omega$. Player I picks $x(i)$ if $i \notin K$ and Player II — if $i \in K$. Player I wins if $x \in E$ and Player II — if $x \notin E$. Let $V_{II}(K)$ be the set of all $E \subseteq 2^\omega$ such that Player II has a winning strategy in $\Gamma(E, K)$. Now, consider a system $\{K_s : s \in 2^{<\omega}\}$ (where $2^{<\omega}$ denotes the set of all finite sequences with terms from $\{0, 1\}$) fulfilling the conditions

$$K_{s0} \cup K_{s1} \subseteq K_s \quad \text{and} \quad K_{s0} \cap K_{s1} = \emptyset$$

for all $s \in 2^{<\omega}$ where si ($i \in \{0, 1\}$) extends s by the (last) term i. The family

$$\mathcal{M} = \cap\{V_{II}(K_s) : s \in 2^{<\omega}\}$$

is an ideal defined by Mycielski in [**My**]. It is interesting that there exists a set E in $\mathcal{M}$ such that $2^\omega \setminus E$ is of the first category and of measure zero (in 2^ω we consider the usual product measure which is isomorphic to the Lebesgue measure on $[0, 1]$). The ideal $\mathcal{M}$ has a base consisting of G_δ sets. The above facts are observed in [**My**]. Thus we have $\operatorname{cof}(\mathcal{M}) \leq c$. It was proved in [**Ba1**] that $\mathcal{M}$ has property (P). Rosłanoski showed that $\operatorname{cov}(\mathcal{M}) = \omega_1$ (see [**R,** Th. 2.3(a)]). Hence, if we assume CH, Criterion 1 yields $\mathcal{I}_0 \setminus \mathcal{M} \neq \emptyset$. (Can it be proved within ZFC?)

Now we give an example of an ideal for which Criterion 1 works in ZFC.

1.3. Example. Let $\mathcal{F}$ be a disjoint family of perfect sets with the union equal to $X = 2^\omega$, we shall define sets Q_α, $\alpha < c$. Let $\mathcal{R}$ be the family of all sets $Q \subseteq X$ such that $Q \cap P$ is countable for any $P \in \mathcal{F}$. If $\mathcal{R} = \emptyset$, put $Q_\alpha = \emptyset$ for all $\alpha < c$. If $\mathcal{R} \neq \emptyset$; pick any $Q_0 \in \mathcal{R}$. Assume that $\alpha < c$ and Q_γ for $\gamma < \alpha$ are defined. If there is a $Q \in \mathcal{R}$ such that $Q \cap Q_\gamma$ is countable for all $\gamma < \alpha$, put

$Q_\alpha = Q$, and let $Q_\alpha = \emptyset$ in the opposite case. Next, put $\mathcal{F}^+ = \mathcal{F} \cup \{Q_\alpha : \alpha < c\}$ and $\mathcal{I} = \{E \subseteq X : E \subseteq \cup\tilde{\mathcal{F}}$ for some finite $\tilde{\mathcal{F}} \subseteq \mathcal{F}^+\}$. Then $\operatorname{cof}(\mathcal{I}) \leq c$ since $|\{\tilde{\mathcal{F}} \subseteq \mathcal{F}^+ : \tilde{\mathcal{F}}$ is finite$\}| = c$. Now, observe that $\operatorname{cov}(\mathcal{I}) = c$. Indeed, if $\operatorname{cov}(\mathcal{I}) = \kappa < c$, there is an $\mathcal{F}_0 \subseteq \mathcal{F}^+$ such that $|\mathcal{F}_0| = \kappa$ and $\cup\mathcal{F}_0 = X$. Let $\mathcal{F}_0 = \{P_\alpha : \alpha < \kappa\}$. Consider a fixed $P \in \mathcal{F} \setminus \mathcal{F}_0$. $P \cap P_\alpha$ is countable for all $\alpha < \kappa$, therefore $c = |P| = |P \cap \cup F_0| = |\cup_{\alpha<\kappa} P \cap P_\alpha| \leq \kappa \cdot \omega = \kappa < c$, a contradiction. The ideal $\mathcal{I}$ has property (P) since, by the construction, each perfect set P either belongs to $\mathcal{F}^+$, thus is in $\mathcal{I}$, or $P \cap Q$ is uncountable for some $Q \in \mathcal{F}^+$, thus a perfect part of $P \cap Q$ belongs to $\mathcal{I}$.

2. The Second Criterion

For a family $\{D_\alpha : \alpha < c\} \subseteq \mathcal{P}(X)$, we denote

$$D_0^* = D_0 \quad \text{and} \quad D_\alpha^* = D_\alpha \setminus \cup_{\alpha<\gamma} D_\gamma \text{ if } 0 < \alpha < c\,.$$

The following proposition generalizes Theorem 2.2 from [**W1**].

2.1. Proposition. *Let $\mathcal{D} = \{D_\alpha : \alpha < c\}$ be a family of analytic subsets of X such that $|D_\alpha^*| = c$ for all $\alpha < c$. Then there exists a selector E of $\{D_\alpha^* : \alpha < c\}$ being an (s^0) set.*

Proof. Let $\mathcal{P}$ be the family of all perfect subsets of X. If there exists $P \in \mathcal{P}$ meeting each member of D_α at $< c$ points (consequently, in a countable set of points), then let $\{Q_\alpha : \alpha < c\}$ consists of all such sets P. In the opposite case, let $Q_\alpha = \emptyset$ for all $\alpha < c$. Pick $x_0 \in D_0$ and choose inductively

$$x_\alpha \in D_\alpha^* \setminus \cup_{\gamma<\alpha}(Q_\gamma \cup \{x_\gamma\}) \quad \text{for } 0 < \alpha < c\,.$$

This can be done since $|D_\alpha^*| = c$ and $|D_\alpha^* \cap (\cup_{\gamma<\alpha}(Q_\gamma \cup \{x_\gamma\})| < c$. Define $E = \{x_\alpha : \alpha < c\}$. Certainly, $E \cap D_\alpha^* = \{x_\alpha\}$ for each $\alpha < c$. Now, consider any perfect P. If $P = Q_\alpha$ for some $\alpha < c$, then $E \cap P \subseteq \{x_\beta : \beta \leq \alpha\}$. So, by Lemma 0.1, there is a perfect subset of P which misses E. If $P \neq Q_\alpha$ for all $\alpha < c$, then $|P \cap D_\alpha| = c$ for some $\alpha < c$. For this α, we have

$$P \cap D_\alpha \cap E \subseteq \{x_\beta : \beta \leq \alpha\}\,.$$

So, by Lemma 0.1, there is a perfect subset of $P \cap D_\alpha$ (consequently, of P) disjoint from E. Hence E is an (s^0) set. □

Let $\mathcal{D} \subseteq \mathcal{P}(X)$, $|\mathcal{D}| = c$ and $\cup\mathcal{D} = X$. We say that an ideal $\mathcal{I} \subseteq \mathcal{P}(X)$ is $(< c)$-generated by $\mathcal{D}$ if

$$\mathcal{I} = \{E \subseteq X : E \subseteq \cup\tilde{\mathcal{D}} \text{ for some } \tilde{\mathcal{D}} \subseteq \mathcal{D}, |\tilde{\mathcal{D}}| < c\}\,.$$

2.2. Corollary (Criterion 2). *If $\mathcal{D} = \{D_\alpha : \alpha < c\}$ is a family of analytic subsets of X such that $\cup\mathcal{D} = X$ and $|D_\alpha^*| = c$ for all $\alpha < c$, then $\mathcal{I}_0 \setminus \mathcal{I} \neq \emptyset$ where $\mathcal{I}$ is the ideal $(< c)$-generated by $\mathcal{D}$.*

Proof. Consider the set E from Proposition 2.1. Then $E \in \mathcal{I}_0$ and, since E is a selector of $\{D_\alpha^* : \alpha < c\}$, we get $E \notin \mathcal{I}$. □

We say that a family $\mathcal{F}$ of perfect subsets of X is almost disjoint if $P \cap Q$ is countable for any distinct $P, Q \in \mathcal{F}$. By Zorn's lemma, each almost disjoint family of perfect sets can be extended to a maximal one. From Criterion 2 we get

2.3. Corollary. *If $\mathcal{D}$ is an almost disjoint family of perfect subsets of X, such that $|\mathcal{D}| = c$ and $\cup\mathcal{D} = X$, then $\mathcal{I}_0 \setminus \mathcal{I} \neq \emptyset$ where $\mathcal{I}$ is the ideal $(< c)$-generated by $\mathcal{D}$.*

2.4. Examples. (a) Let $I = [0, 1]$ and $X = I^2$. Put

$$\mathcal{D} = \{I \times \{x\} : x \in I\} \cup \{\{x\} \times I : x \in I\}.$$

Then $\mathcal{D}$ is an almost disjoint family of perfect sets fulfilling the assumptions of 2.3. Note that $\mathcal{D}$ is not maximal since, for instance, the diagonal meets each set from $\mathcal{D}$ at exactly one point. By that reason, property (P) fails to hold for the ideal $\mathcal{I}$ $(< c)$-generate by $\mathcal{D}$ since the diagonal has no perfect subset in $\mathcal{I}$. So, Criterion 1 cannot be applied to $\mathcal{I}$.

(b) Let P be a perfect subset on $\mathbb{R}$ such that $|P \cap (P + x)| \leq 1$ for all $x \neq 0$ (here $P + x$ denotes the set of all sums $t + x$ for $t \in P$); see [**Ru-S**]. Then, for any perfect $Q \subseteq P$, the collection $\mathcal{D} = \{Q + x : x \in \mathbb{R}\}$ is an almost disjoint family of perfect sets fulfilling all the assumptions of 2.3. If there exists a perfect $S \subseteq P \setminus Q$, it is clear that $\mathcal{D}$ is not maximal since $\mathcal{D} \cup \{S\}$ extends $\mathcal{D}$. Hence again, the respective ideal $\mathcal{I}$ has not property (P). Note that $\mathcal{I}$ is translation invariant.

(c) Observe that $\mathcal{F}^+$ from Example 1.3 can form a maximal almost disjoint family of perfect sets. By 2.3, there is an (s^0) set outside the ideal $(< c)$-generated by $\mathcal{F}^+$. That ideal contains $\mathcal{I}$ considered in 1.3. Thus, now we get more that $\mathcal{I}_0 \setminus \mathcal{I} \neq \emptyset$.

(d) Let X be the set of all infinite subsets of ω. Then X can be embedded into the Cantor set 2^ω via the characteristic functions. Thus X inherits the product topology from 2^ω and forms a dense-in-itself space which is Polish since it is embedded into 2^ω as a G_δ set (apply the Alexandrov theorem, see [**Kr,** §33.VI]). Let $A \subseteq X$ be a family of c sets which meet pairwise on finite sets (see [**Kn,** Th. 1.2(b), p. 48]). Let $\mathcal{A} = \{A_\alpha : \alpha < c\}$ and $D_\alpha = \{K \in X : K \cap A_\alpha \in X\}$, $\alpha < c$. It is easy to verify that $\mathcal{D} = \{D_\alpha : \alpha < c\}$ consists of perfect sets and $\cup\mathcal{D} = X$. This is not an almost disjoint family since $|D_\alpha \cap D_\beta| = c$ for any distinct $\alpha, \beta < c$. Indeed, there exist c distinct subsets of ω meeting either of the sets A_α and A_β in infinite sets. On the other hand, Criterion 2 can be applied to $\mathcal{D}$ since $|D^*_\alpha| = c$ for $\alpha < c$. This follows from the fact that A_α has c infinite subsets and each of them is in D^*_α.

Finally, note that, for any ideal $\mathcal{I}$ fulfilling the conditions of Criterion 2, it is consistent with ZFC that $\mathrm{cf}\,(\mathcal{I}) > c$. Indeed, we have $\mathrm{cf}\,(\mathcal{I}) \geq 2^{\omega_1}$ hence $\mathrm{cf}\,(\mathcal{I}) > c$ holds in the model in which $\omega_1 < c$ and $2^{\omega_1} > c$ are true (see [**Kn,** Th. 6.18(c), p. 216]). So, in this case, Criterion 1 is not useful.

3. Further Remarks

In Sections 1 and 2 we have concentrated on the problem "When $\mathcal{I}_0 \setminus \mathcal{I} \neq \emptyset$?", while the results for Brown and Walsh which we try to generalize deal mainly with the question "When $\mathcal{I}_0 \setminus S_{\mathcal{I}} \neq \emptyset$?" where $S_{\mathcal{I}}$ is a respective σ-field associated with $\mathcal{I}$. Now, we shall show that, in some cases, these two problems are equivalent.

For a family $\mathcal{F} \subseteq \mathcal{P}(X)$ and an ideal $\mathcal{I} \subseteq \mathcal{P}(X)$, by $\mathcal{F}(\mathcal{I})$ we denote the collection of all sets $E \subseteq X$ expressible as the symmetric differences $B \triangle C$ where $B \in \mathcal{F}$ and $A \in \mathcal{I}$. In particular, one can consider as $\mathcal{F}$ the σ-field $\mathcal{B}$ of all Borel sets in X; then $\mathcal{B}(\mathcal{I})$ is the smallest σ-field containing $\mathcal{B} \cup \mathcal{I}$. We shall also consider projective pointclasses Σ^1_n and Π^1_n for $n \geq 1$ (see [**Mo**], for the definitions); here we restrict them to the space X. We say that a pointclass Λ fulfils Perfect Set Theorem (abbr. PST) if each uncountable set from Λ contains a perfect set. It is known that Σ^1_1 fulfils PST and, for $n \geq 2$, the statement "Σ^1_n fulfils PST", is not provable in ZFC; however, it can be treated as a strong axiom of set theory (cf. [**Mo**]).

For $\mathcal{F} \subseteq \mathcal{P}(X)$, we denote $\neg\mathcal{F} = \{X \setminus A : A \in \mathcal{F}\}$.

3.1. Proposition. *Assume that $\mathcal{F} \subseteq \mathcal{P}(X)$ is closed under finite intersections and $\mathcal{I} \subseteq \mathcal{P}(X)$ is an ideal with a base $\tilde{\mathcal{I}} \subseteq \neg\mathcal{F}$ such that each set from $\mathcal{F} \setminus \mathcal{I}$ contains a perfect set. Let $E \subseteq X$ be totally imperfect. Then $E \notin \mathcal{I}$ and $E \notin \mathcal{F}(\mathcal{I})$ are equivalent.*

Proof. Obviously, $E \notin \mathcal{F}(\mathcal{I})$ implies $E \notin \mathcal{I}$. Now, assume that we have a totally imperfect $E \in \mathcal{I}$. Suppose that $E \in \mathcal{F}(\mathcal{I})$. Then $E = B \triangle A$ where $B \in \mathcal{F}$ and $A \in \mathcal{I}$. Of course, $B \notin \mathcal{I}$. Choose $\tilde{A} \in \tilde{\mathcal{I}}$ containing A. Then, for $\tilde{B} = B \setminus \tilde{A}$, we get $E = \tilde{B} \cup D$ where $D = (B \cap (\tilde{A} \setminus A)) \cup (A \setminus B) \in \mathcal{I}$. Observe that $\tilde{B}$ is in $\mathcal{F} \setminus \mathcal{I}$ and thus, by the assumption, it contains a perfect set. Hence E has a perfect subset, a contradiction. □

If an ideal $\mathcal{I} \subseteq \mathcal{P}(X)$ has a base $\tilde{\mathcal{I}}$ contained in a pointclass Λ, then $\tilde{\mathcal{I}}$ is called a Λ-base.

3.2. Corollary. *Let $\mathcal{I} \subseteq \mathcal{P}(X)$ be an ideal having a Π^1_1-base and containing all countable subsets of X. For any totally imperfect set E, the conditions $E \notin \mathcal{I}$ and $E \notin \Sigma^1_1(\mathcal{I})$ are equivalent.*

The same result holds when Π^1_1 and Σ^1_1 are replaced by $\mathcal{B}$.

3.3. Corollary. *For $n \geq 2$, assume that Σ^1_n fulfils PST. Let $\mathcal{I} \subseteq \mathcal{P}(X)$ be an ideal having a Π^1_n-base and containing all countable subsets of X. For any totally imperfect set, the conditions $E \notin \mathcal{I}$ and $E \notin \Sigma^1_n(\mathcal{I})$ are equivalent.*

Applying the results of this section together with Criterion 1 or 2 to the respective ideals $\mathcal{I}$ and pointclasses Λ, we get conditions guaranteeing $\mathcal{I}_0 \setminus \Lambda(\mathcal{I}) \neq \emptyset$.

References

[**Ba1**] Balcerzak M., *On σ-ideals having perfect members in all perfect sets*, Demonstratio Math. **22** (1989), 1159–1168.

[**Ba2**] ———, *Another nonmeasurable set with property* (s^0), preprint.

[**Br**] Brown J., *The Ramsey sets and related sigma algebras and ideals*, Fund. Math. **136** (1990), 179–183.

[**C**] Corazza P., *Ramsey sets, the Ramsey ideal and other classes over* $\mathbb{R}$, preprint.

[**Kn**] Kunen K., *Set Theory. An introduction to Independence Proofs*, North Holland, 1980.

[**Kr**] Kuratowski K., *Topology I*, Academic Press, 1966.

[**Mo**] Moschovakis Y., *Descriptive Set Theory*, North Holland, 1980.

[**My**] Mycielski J., *Some new ideals of sets on the real line*, Colloq. Math. **20** (1969), 71–76.

[**R**] Rosłanowaki, *On game ideals*, Colloq. Math. **59** (1990), 159–168.

[**Ru-S**] Ruziewicz and Sierpiński W., *Sur un ensemble parfait qui a avec toute sa translation au plus un point commun*, Fund. Math. **19** (1932), 17–21.

[**Sz**] Szpilrajn (Marczewski) E., *Sur une classe de fonctions de M. Sierpiński et la classe correspondante d'ensembles*, Fund. Math. **24** (1935), 17–34.

[**W1**] Walsh J. T., *Marczewski sets, measure and the Baire property*, Fund. Math. **129** (1988), 83–89.

[**W2**] ———, *Marczewski sets, measure and the Baire property II*, Proc. Amer. Math. Soc. **106** (1989), 1027–1030.

M. Balcerzak, Institute of Mathematics, Łódź University, ul. S. Banacha 22, 90-238 Łódź, Poland