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A FUJITA–TYPE THEOREM FOR THE LAPLACE EQUATION WITH A DYNAMICAL BOUNDARY CONDITION

H. AMANN AND M. FILA

ABSTRACT. We find a critical exponent for global existence of positive solutions of the Laplace equation on a half-space with a dynamical boundary condition.

1. INTRODUCTION

Given a nonempty open subset X of $\mathbb{R}^m$ we denote by $BUC(X)$ the Banach space of all bounded and uniformly continuous functions on X, endowed with the supremum norm $\|\cdot\|_\infty$. We also put $BUC_+(X) := \{ u \in BUC(X) \ ; \ u(x) \geq 0$ for $x \in X \}$. Moreover, $\mathbb{H}^n := \mathbb{R}^{n-1} \times (0,\infty)$ is the open upper half-space in $\mathbb{R}^n$, and its boundary $\partial\mathbb{H}^n$ is identified with $\mathbb{R}^{n-1}$.

We fix $q \in (1,\infty)$ and consider the following system:

$$\begin{aligned} \Delta u &= 0 && \text{in} \quad \mathbb{H}^n \times (0,\infty) \ , \\ \partial_t u - \partial_n u &= u^q && \text{on } \partial\mathbb{H}^n \times (0,\infty) \ , \\ u(\cdot,0) &= \varphi && \text{on } \partial\mathbb{H}^n \ , \end{aligned} \tag{1.1}$$

where $\Delta = \partial_1^2 + \cdots + \partial_n^2$ is the Laplacian with respect to $x = (x', x_n) \in \mathbb{R}^{n-1} \times \mathbb{R}$. By a solution of (1.1) on $[0,T)$ we mean a function

$$u \in C\big([0,T), BUC_+(\mathbb{H}^n)\big) \cap C^1\big((0,T), BUC(\mathbb{H}^n)\big) \tag{1.2}$$

such that $u(t) \in C^2(\mathbb{H}^n) \cap C^1(\bar{\mathbb{H}}^n)$ for $t > 0$, and u satisfies (1.1) point-wise, where $u(x,t) := u(t)(x)$. Note that this requires φ to belong to $BUC_+(\mathbb{R}^{n-1})$. Of course, each solution, being harmonic, is analytic in $\mathbb{H}^n$ for $0 < t < T$.

A function $u \in C\big([0,T_\varphi), BUC_+(\mathbb{H}^n)\big)$ is a maximal solution of (1.1) if u is a solution on $[0,T_\varphi)$ and $[0,T_\varphi)$ is a maximal interval with this property. If $T_\varphi = \infty$ and u is a solution on $[0,\infty)$ then u is a global solution of (1.1).

The following theorem is the main result of this paper:

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Theorem. *Problem* (1.1) *possesses for each* $\varphi \in BUC_+(\mathbb{R}^{n-1})$ *a unique maximal solution* u_φ. *If* $T_\varphi < \infty$ *then* u_φ *'blows up', that is,*

$$\lim_{t \to T_\varphi} \|u_\varphi(t)\|_\infty = \infty \ .$$

If $q \le n/(n-1)$ *then every nonzero maximal solution blows up in finite time (that is,* $T_\varphi < \infty$*). If* $q > n/(n-1)$ *then there are solutions that exist globally as well as solutions that blow up in finite time.*

Observe, in particular, that each nonzero maximal solution blows up in finite time if $n = 1$.

Intuitively, the above result can be explained as follows: If $\varphi \in L_1 \cap BUC_+$ then the solution of the linear problem

$$\begin{aligned} \Delta u &= 0 && \text{in } \ \mathbb{H}^n \times (0,\infty) \ , \\ \partial_t u - \partial_n u &= 0 && \text{on } \partial\mathbb{H}^n \times (0,\infty) \ , \\ u(\cdot,0) &= \varphi && \text{on } \partial\mathbb{H}^n \ , \end{aligned} \tag{1.3}$$

is global and decays with rate t^{1-n}, as will be shown in Section 2. On the other hand, each solution of the ordinary differential equation $\dot{u} = u^q$ blows up in finite time with rate $(T-t)^{1/(1-q)}$, where $T := T\big(u(0), q\big)$. Therefore the solution q $(\, = n/(n-1))$ of equation $n - 1 = 1/(q-1)$ can be expected to be a critical exponent for global existence.

Let us mention here that problem (1.1) possesses positive stationary states iff $q \ge n/(n-2)$ (cf. [**H**], [**CSF**], [**CCFS**]).

In the case of bounded domains, problems analogous to (1.1) have been studied recently in [**E1–3**], [**K**], [**FQ**], for example. References to earlier work can be found in [**E1**].

Beginning with the classical paper by Fujita [**F**], blow-up results of the above type have been established for many classes of parabolic problems (see [**L**] for a survey, and [**DFL**], [**EL**], [**FL**], [**FLU**], [**GL1–2**], [**HY1–3**], [**KO**], [**LQ**], [**MS**], [**MY**], and [**S**] for some more recent results). 'Fujita-type theorems' are also known for nonlinear Schrödinger and wave equations (also cf. [**L**] and the references given therein). The interest in our (model) problem stems from the fact that system (1.1) is equivalent to an evolution equation of the form

$$\dot{u} + Au = u^q \ , \quad t > 0 \ , \qquad u(0) = \varphi \tag{1.4}$$

in $BUC(\mathbb{R}^{n-1})$, where A is a pseudodifferential operator of degree 1 (see Section 2). Thus the Theorem is a Fujita-type result for a new class of equations.

2. Preliminaries

The case $n=1$ is trivial. Therefore we assume henceforth that $n \geq 2$ and study (1.1) by means of a 'variation-of-constants formula', which we establish now.

We denote by $\widehat{u} = \mathcal{F}u$ the (partial) Fourier transform with respect to $x' \in \mathbb{R}^{n-1}$ (in the space $\mathcal{S}'(\mathbb{R}^{n-1})$ of temperate distributions). Then the first equation in (1.3) yields

$$\partial_n^2 \widehat{u}(\cdot, x_n, t) - |\xi'|^2\, \widehat{u}(\cdot, x_n, t) = 0 \ , \qquad x_n, t > 0 \ .$$

Hence

$$\widehat{u}(\cdot, x_n, t) = \widehat{u}(\cdot, 0, t) e^{-x_n\, |\xi'|} \ , \tag{2.1}$$

and we infer from the second equation in (1.3) that

$$\partial_t \widehat{u}(\cdot, 0, t) + |\xi'|\, \widehat{u}(\cdot, 0, t) = 0 \ , \qquad t > 0 \ .$$

Thus $\widehat{u}(\cdot, 0, t) = e^{-t\,|\xi'|}\widehat{\varphi}$ for $t \geq 0$, and it follows from (2.1) that

$$\widehat{u}(\cdot, x_n, t) = e^{-(x_n+t)\,|\xi'|}\widehat{\varphi} \ , \qquad x_n > 0 \ , \quad t > 0 \ . \tag{2.2}$$

Put $\Lambda(x') := \left(1 + |x'|^2\right)^{1/2}$ for $x' \in \mathbb{R}^{n-1}$ and $c_n := \|\Lambda^{-n}\|_1^{-1}$, where $\|\cdot\|_r$ is the norm in $L_r := L_r(\mathbb{R}^{n-1})$ for $1 \leq r \leq \infty$, and let

$$p_\tau(x') := \tau^{1-n} c_n \Lambda^{-n}(x'/\tau) \ , \qquad \tau > 0 \ , \quad x' \in \mathbb{R}^{n-1} \ . \tag{2.3}$$

Then $\{\, p_\tau \ ; \ \tau > 0 \,\}$ is the $(n-1)$-dimensional Poisson kernel, and we deduce from (2.2), by taking the inverse Fourier transform, that

$$u(\cdot, x_n, t) = p_{x_n+t} * \varphi =: P(x_n + t)\varphi \ , \qquad x_n \geq 0 \ , \quad t > 0 \ , \tag{2.4}$$

denoting by $\left\{\, P(\tau) \ ; \ \tau \geq 0 \,\right\}$ the Poisson convolution semigroup on $\mathbb{R}^{n-1}$. It is well-known that $\left\{\, P(\tau) \ ; \ \tau \geq 0 \,\right\}$ is a strongly continuous analytic semigroup of contractions on $BUC(\mathbb{R}^{n-1})$ whose negative infinitesimal generator equals the $BUC(\mathbb{R}^{n-1})$-realization of $A := \mathcal{F}^{-1}\, |\xi'|\, \mathcal{F}$. By invoking standard properties of this semigroup (e.g., [**A**]) it is not difficult to see that u, as given by (2.4), is a global solution of (1.3) and the only one (in our class of solutions satisfying (1.2)). Note that (2.3) and (2.4) also imply that if $\varphi \in L_1 \cap BUC_+$ then

$$\|u(t)\|_\infty = O(t^{1-n}) \ , \qquad t \to \infty \ ,$$

and it is easily seen that this decay rate is exact.

Let γ be the trace operator for $\partial\mathbb{H}^n$, that is, $\gamma w(x') := w(x', 0)$ for $w \in C(\bar{\mathbb{H}}^n)$. Then an obvious modification of the above considerations also shows that u is a solution of (1.1) on $[0,T)$ iff $v := \gamma u$ is a solution of (1.4) on $[0,T)$. In fact, if v is a solution of (1.4) on $[0,T)$ then

$$u(\cdot, x_n, t) := P(x_n)v(t) \ , \qquad x_n \geq 0 \ , \quad t \geq 0 \ , \tag{2.5}$$

is its unique harmonic extension over $\mathbb{H}^n \times [0,T)$ such that $u(\cdot,\cdot,t)$ is bounded on $\mathbb{H}^n$ for $0 < t < T$.

As usual, we associate with (1.4) the Volterra integral equation

$$v(t) = P(t)\varphi + \int_0^t P(t-\tau)v^q(t)\,d\tau \ , \qquad t \geq 0 \ , \tag{2.6}$$

in $BUC(\mathbb{R}^{n-1})$. Standard arguments guarantee that it possesses a unique maximal solution

$$v_\varphi \in C\big([0,T_\varphi), BUC_+(\mathbb{R}^{n-1})\big) \tag{2.7}$$

and that

$$\lim_{t\to T_\varphi} \|v_\varphi(t)\|_\infty = \infty \tag{2.8}$$

if $T_\varphi < \infty$. Thus v_φ is a mild solution of (1.4). Using Besov spaces and regularity properties of the Poisson semigroup in these spaces it can be shown that v_φ is a classical solution of (1.4), that is, $v_\varphi(t) \in \mathrm{dom}(A)$ for $t \in (0,T_\varphi)$ and Av_φ belongs to $C\big((0,T_\varphi), BUC_+(\mathbb{R}^{n-1})\big)$, and v_φ satisfies (1.4) on $[0,T)$ in the point-wise sense. In fact, it can be shown that

$$v_\varphi \in C^1\big((0,T_\varphi), BUC(\mathbb{R}^{n-1})\big) \cap C\big((0,T_\varphi), BUC^2(\mathbb{R}^{n-1})\big) \ , \tag{2.9}$$

where $u \in BUC^2$ iff $\partial^\alpha u \in BUC$ for $|\alpha| \leq 2$. Since we do not need these regularity results for the blow-up considerations we refrain from giving details and refer the interested reader to [**A**].

Note that (2.7) and (2.9) imply that u_φ, as defined by (2.5) with v replaced by v_φ, is the unique maximal solution of (1.1). Moreover, (2.8) shows that the supremum norm of $u_\varphi(t)$ over $\mathbb{H}^n$ blows up as $t \to T$, provided T_φ is finite. Also note that u_φ is characterized by being the unique maximal solution of the integral equation

$$u(\cdot, x_n, t) = P(x_n + t)\varphi + \int_0^t P(x_n + t - \tau)u^q(\cdot, 0, \tau)\,d\tau \ , \qquad x_n > 0 \ , \quad t \geq 0 \ ,$$

in $BUC(\mathbb{H}^n)$, as follows from the semigroup property of P.

The above considerations prove the existence and uniqueness assertion of the Theorem and also the fact that u_φ blows up if T_φ is finite. It should be mentioned that uniqueness is lost if we drop the condition that $u_\varphi(t)$ be bounded on $\mathbb{H}^n$ for $0 \leq t < T_\varphi$. Indeed, if φ is a positive constant and g is any C^1-function of one variable satisfying $g(0) = \varphi$ and $\dot{g}(t) \geq g^q(t)$ for $0 \leq t < T$, then

$$u(x,t) := \big(\dot{g}(t) - g^q(t)\big)x_n + g(t) \ , \qquad (x,t) \in \mathbb{H}^n \times [0,T) \ ,$$

is a solution of (1.1).

3. Blow-Up and Global Existence

Thanks to the observations in Section 2 it suffices to study the blow-up behavior of the unique maximal solution of integral equation (2.6) in the Banach space $E := BUC(\mathbb{R}^{n-1})$. Note that E is ordered by the positive cone $E_+ := BUC_+(\mathbb{R}^{n-1})$ and that $\{\, P(t)\ ;\ t \geq 0\,\}$ is a positivity preserving semigroup on E.

For abbreviation we write (2.6) in the form

$$u(t) = P(t)\varphi + P * f(u)(t)\ , \qquad t \geq 0\ , \tag{3.1}$$

where $f(u) := u^q$. Then we prove the following comparison principle:

Lemma 1. *Let v be a solution of (3.1) on $[0,T]$ and suppose that $w \in C\big([0,T], E_+\big)$ satisfies*

$$w(t) \geq P(t)\psi + P * f(w)(t)\ , \qquad 0 \leq t \leq T\ , \tag{3.2}$$

for some $\psi \in E_+$. Then $\psi \geq \varphi$ implies $w \geq v$.

Proof. Suppose that $\delta \in (0,T]$ and $a_j \in C\big([0,\delta], E_+\big)$ for $j = 0,1$ with $a_0 \geq a_1$. Also suppose that $u_j \in C\big([0,\delta], E_+\big)$ satisfies $u_j = a_j + P * f(u_j)$ on $[0,\delta]$ and that $u_j = \lim_{k\to\infty} u_j^k$ in $C\big([0,\delta], E_+\big)$, where the sequence $(u_j^k)_{k\in\mathbb{N}}$ is obtained by the iteration scheme $u_j^0 := a_j$ and $u_j^{k+1} := a_j + P * f(u_j^k)$ for $k \in \mathbb{N}$ and $j = 0,1$. Then $a_0 \geq a_1$, the positivity of P, and the fact that f is increasing imply by induction that $u_0^k \geq u_1^k$ for $k \in \mathbb{N}$. Hence $u_0 \geq u_1$.

From (3.2) we infer the existence of $b \in C\big([0,T], E_+\big)$ such that

$$w = b + P(\cdot)\psi + P * f(w)$$

on $[0,T]$. Thus

$$a_0 := b + P(\cdot)\psi \geq P(\cdot)\varphi =: a_1$$

and $u_0 := w$ and $u_1 := v$ satisfy $u_j = a_j + P * f(u_j)$ on $[0,T]$. Since f is locally Lipschitz continuous it is well-known that there exists $\delta \in (0,T]$ such that u_j can be obtained on $[0,\delta]$ by the above iteration scheme. Hence $w\,|\,[0,\delta] \geq v\,|\,[0,\delta]$ by the first part of the proof. Since (3.1) is autonomous we can apply this argument once more with ψ and φ replaced by $w(\delta)$ and $v(\delta)$, respectively, to find that $w\,|\,[0,\delta_1] \geq v\,|\,[0,\delta_1]$ for some $\delta_1 \in (0,T]$. Then standard arguments guarantee that $w \geq v$ as long as both solutions exist, that is, on all of $[0,T]$. $\square$

Since the Poisson kernel is positive and satisfies $\int p_t(x')\,dx' = 1$ for $t > 0$, it follows from Jensen's inequality that

$$P(\tau)u^q = P(\tau)f(u) = p_\tau * f(u) \geq f(p_\tau * u) = (p_\tau * u)^q$$

for $\tau \geq 0$ and $u \in E_+$. Thus the proof of Theorem 5 in [**W1**] implies the existence of a constant $c := c(q)$ such that

$$t^{1/(q-1)} P(t)\varphi \leq c \ , \qquad 0 \leq t < T_\varphi \ , \quad \varphi \in E_+ \ . \tag{3.3}$$

Using this estimate we can derive the following blow-up result along the lines of the proof of Theorem 1 in [**W2**]. For the reader's convenience we include the details.

Lemma 2. *If $q \leq n/(n-1)$ then each nonzero solution blows up in finite time.*

Proof. Suppose that $\varphi \in E_+ \backslash \{0\}$ and $T_\varphi = \infty$. Thanks to Lemma 1 we can assume that $\varphi \in L_1$. Since $t^{n-1} p_t \to c_n$ point-wise as $t \to \infty$, it follows that

$$\lim_{t \to \infty} t^{n-1} P(t)\varphi = c_n \left\| \varphi \right\|_1 \tag{3.4}$$

point-wise. This contradicts (3.3) if $q < n/(n-1)$.

Suppose that $q = n/(n-1)$. From $1 + |x' - y'|^2 \leq 2(1 + |x'|^2)(1 + |y'|^2)$ we infer

$$P(1)\varphi = p_1 * \varphi \geq \alpha p_1 \ ,$$

where $\alpha := 2^{-n/2} \left\| \Lambda^{-n} \varphi \right\|_1$. Hence (3.1) implies

$$u_\varphi(t+1) \geq P(t+1)\varphi = P(t)P(1)\varphi \geq \alpha P(t) p_1 = \alpha p_{t+1} \ , \qquad t \geq 0 \ ,$$

and

$$\begin{aligned} \|u_\varphi(t+1)\|_1 \geq \left\| P * f(u_\varphi)(t+1) \right\|_1 &\geq \alpha^q \int_0^{t+1} \left\| P(t+1-\tau) f(p_{\tau+1}) \right\|_1 d\tau \\ &= \alpha^q \int_0^{t+1} \|f(p_{\tau+1})\|_1 \, d\tau \end{aligned} \tag{3.5}$$

for $t \geq 0$, since $\|P(t)v\|_1 = \|v\|_1$ for $v \in E_+$ by Fubini's theorem. Note that

$$\|f(p_{\tau+1})\|_1 = (\tau+1)^{-1} c_n^q \left\| \Lambda^{-nq} \right\|_1$$

since $q = n/(n-1)$. From this, together with (3.5) we deduce that

$$\lim_{t \to \infty} \|u_\varphi(t)\|_1 = \infty \ . \tag{3.6}$$

On the other hand, (3.3) (with $q = n/(n-1)$) and (3.4) guarantee the existence of a constant c such that $\|\varphi\|_1 \leq c$ for any $\varphi \in E_+$ with $T_\varphi = \infty$. Thus, in particular, $\|u_\varphi(t)\|_1 \leq c$ for $t \geq 0$, which contradicts (3.6). ☐

Lemma 3. *If $q > n/(n-1)$ then there are global solutions.*

Proof. Note that $\alpha := n - (1-1/q)^{-1} > 0$. Put $c := \alpha^{1/(q-1)}/c_n$ and $v(t) := ct^\alpha p_t$ for $t > 0$. Then $\dot{p}_t = -Ap_t$ for $t > 0$ and

$$\dot{v}(t) = \alpha c t^{\alpha-1} p_t + ct^\alpha \dot{p}_t = \alpha t^{-1} v(t) - Av(t) \ , \qquad t > 0 \ .$$

It is easily verified that $\alpha t^{-1} v(t) \geq f\big(v(t)\big)$ so that $\dot{v} + Av \geq f(v)$ on $(0,\infty)$. From this it follows that $w(t) := v(t+1)$ satisfies $w(t) \geq P(t)v(1) + P * f(w)(t)$ for $t \geq 0$. Now the assertion is a consequence of Lemma 1. □

Lastly, suppose that u_φ is a solution on $[0, T_\varphi)$ for some $\varphi \in E_+ \backslash \{0\}$. Then it follows from (3.3) that there exists $k_0 > 0$ such that $T_{k\varphi} < T_\varphi$ for $k \geq k_0$. Hence there exist solutions that blow up in finite time.

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H. Amann, Institute for Mathematics, University of Zürich, Winterthurerstr. 190, CH–8057 Zürich, Switzerland

M. Fila, Institute of Applied Mathematics, Comenius University, 842 15 Bratislava, Slovakia