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ALEXANDROFF SPACES

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ABSTRACT. In this paper we mean by an Alexandroff space a topological space such that every point has a minimal neighborhood. We do not assume that the space is T_0. There spaces were first introduced by P. Alexandroff in 1937 in [**1**] and have become relevant for the study of digital topology. We make a systematic study of them from several points of view, including quasi-uniform spaces.

1. INTRODUCTION

In this paper we mean by an Alexandroff space (or an space with the property of Alexandroff) a topological space such that every point has a minimal neighborhood, or equivalently, has unique minimal base. This is also equivalent to the fact that the intersection of every family of open sets is open. The minimal neighborhood is denoted by $V(x)$ and is the intersection of all open sets containing x. We do not assume that the space is T_0.

Although they were first introduced by P. Alexandroff in 1937 in [**1**] with the name of **Diskrete Räume**, these spaces have not between systematically studied, perhaps because there were no reasons to do it. In fact the only references that the author was able to find before the eighties were two ([**8**] and [**7**]) that are mainly concerned with finite spaces (the most important particular case, undoubtely) and make claims (easily proved) of the type **this property also hold for Alexandroff spaces**.

In the eighties, the interest in Alexandroff spaces was a consequence of the very important role of finite spaces in digital topology and the fact that these spaces have all the properties of finite spaces relevant for such theory (see [**6**], [**5**]). However nobody has intended (as far as I know) a systematic study of all topological properties of these spaces, independently the property has direct digital applications or not. That is the main purpose of this paper.

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2. Topological Properties of Alexandroff Spaces

We begin with the following characterization of a minimal base.

Theorem 2.1. *Let X be an Alexandroff space and $\mathcal{U}$ a family of open sets. Then $\mathcal{U}$ is the minimal base for the topology of X if and only if:*

1. *$\mathcal{U}$ covers X.*
2. *If $A, B \in \mathcal{U}$ there exists a subfamily $\{U_i : i \in I\}$ of $\mathcal{U}$ such that $A \cap B = \bigcup_{i \in I} U_i$.*
3. *If a subfamily $\{U_i : i \in I\}$ of $\mathcal{U}$ verifies $\bigcup_{i \in I} U_i \in \mathcal{U}$, then there exists $i_0 \in I$ such that $\bigcup_{i \in I} U_i = U_{i_0}$.*

We show now in the next result how minimal bases can be induced in a subspace or in a product, so the Alexandroff property is hereditary and finitely productive.

Theorem 2.2. *Let X and Y be Alexandroff spaces with minimal bases $\mathcal{U}$ and $\mathcal{V}$. Then*

1. *If X is a subspace of Y, then $\mathcal{U} = \{V \cap X : V \in \mathcal{V}\}$.*
2. *$X \times Y$ is an Alexandroff space with minimal base $\mathcal{U} \times \mathcal{V} = \{U \times V : U \in \mathcal{U},\ V \in \mathcal{V}\}$.*

These first two results are quoted without proof from [**8**]. Note that the property of Alexandroff is not countably productive, since discrete spaces are Alexandroff and any compact metric totally disconnected space is a subspace of a countable product of finite discrete spaces (see [**9**, 29.15]) and the Cantor set is not Alexandroff, for example.

The following result comes from [**1**].

Theorem 2.3. *Let X be an Alexandroff space. X is T_0 if and only if $V(x) = V(y)$ implies $x = y$.*

Note that if X is Alexandroff, X is T_1 if and only if $V(x) = \{x\}$, and in that case the space is discrete. The interest of this result (and the reason to impose the hypothesis of being T_0 in the rest of the paper) is that allows to consider a functional equivalence between the categories of T_0 Alexandroff spaces and partially ordered sets (posets in the following):

Given a poset P we construct the T_0-Alexandroff space $X(P)$ as the set P with the topology generated by $\{]\leftarrow, x] : x \in X\}$, which is a T_0-Alexandroff space with $V(x) =]\leftarrow, x]$. Conversely, given a T_0-Alexandroff space X, we construct a poset $P(X)$ as the set P with the order $x \leq y$ if and only if $x \in V(y)$. It is straightforward that $X(P(X)) = X$ and $P(X(P)) = P$ and that under the functors, continuous mappings become order preserving mappings and conversely (see Section 4 of [**7**]). Note that the orders can be defined in the reversed way.

There is another way to assign a topological space to a poset, via the geometric realization of a simplicial complex, as can be found in Section 9 of [**2**]. That is, given a poset P, the order complex $\Delta(P)$ of P is the simplicial complex whose k-faces are k-chains in P. Conversely, given a simplicial complex K the face poset $P(K)$ is K ordered by inclusion. There is also a standard way to topologize a simplicial complex called the geometric realization. We denote it by $|K|$.

In Section 2 of [**7**], the simplicial complex $\Delta(P(X))$ is called barycentric subdivision, so we are going to denote it by sd X; on the other hand it is customary (see again Section 9 of [**2**]) to call sd $K = \Delta(P(K))$ the barycentric subdivision of the simplicial complex K. There is no confusion between the two notations.

Now, to relate topological properties of the space $|\operatorname{sd}(X)|$ (what is called the geometric realization of X) with those of X, we quote with our notation Theorems 2 and 3 of [**7**].

Theorem 2.4. *There exists a weak homotopy equivalence $f_X\colon |\operatorname{sd}(X)| \to X$ defined as $f_X(u) = x_0$ where u is in the unique open simplex of $|\operatorname{sd}(X)|$ with vertices $\{x_0, \ldots, x_n\} \in \operatorname{sd}(X)$ and $x_0 < \cdots < x_n$ in $P(X)$.*

Each mapping $\phi\colon X \to Y$ between T_0-Alexandroff spaces induces a simplicial mapping $|\phi|\colon |\operatorname{sd}(X)| \to |\operatorname{sd}(Y)|$ such that $\phi \circ f_X = f_Y \circ |\phi|$.

Theorem 2.5. *There exists a weak homotopy equivalence $g_K\colon |K| \to X(P(K))$, defined as $g_K = f_{X(P(K))}$, since $\Delta(P(X(P(K)))) = \operatorname{sd} K$ and $|\operatorname{sd} K| = |K|$.*

Each simplicial mapping $\psi\colon K \to L$ between simplicial spaces induces a mapping $\psi^*\colon X(P(K)) \to X(P(L))$ such that $\psi^* \circ f_K$ is homotopic to $f_L \circ |\psi|$.

The following result shows the behaviours of these functors.

Theorem 2.6. *Let X and Y be T_0-Alexandroff spaces and let P and Q be posets.*

1. *$\Delta(P(\operatorname{sd} X)) = \operatorname{sd} \Delta(P(X))$ and $\Delta(P(\operatorname{sd}^n X)) = \operatorname{sd}^n \Delta(P(X))$.*
2. *$P(\oplus_{i\in I} X_i) = \oplus_{i\in I} P(X_i)$.*
3. *$P(X \times Y) = P(X) \times P(Y)$ ($\times$ is the direct product between the posets $P(X)$ and $P(Y)$).*

Proof. Straightforward. □

Note that $X(P \times Q)$ is $P \times Q$ topologized by $V(x, y) = V(x) \times Q \cup \{x\} \times V(y)$, so is not $X(P) \times X(Q)$.

Now we shall study the connectivity properties of these spaces.

Theorem 2.7. *Let X be a T_0-Alexandroff space. The following statements are equivalent.*

1. *X is path-connected.*
2. *X is connected.*
3. *X is chain-connected.*
4. *For every $a, b \in X$, there exist $a_0, \ldots, a_{n+1} \in X$ such that $a_0 = a$, $a_{n+1} = b$ and $V(a_i) \cap V(a_j) \neq \emptyset$ if $|i - j| \leq 1$.*

5. *For every $a, b \in X$, there exist $a_0, \dots, a_{m+1} \in X$ such that $a_0 = a$, $a_{m+1} = b$ and $\overline{V(a_i)} \cap \overline{V(a_j)} \neq \emptyset$ if $|i - j| \leq 1$.*
6. *For every $a, b \in X$, there exist $a_0, \dots, a_{k+1} \in X$ such that $a_0 = a$, $a_{k+1} = b$ and $\overline{\{a_i\}} \cap \overline{\{a_j\}} \neq \emptyset$ if $|i - j| \leq 1$.*

Proof. (1) $\Rightarrow$ (2) $\Rightarrow$ (3) are valid in any topological space, (4) is the form that (3) has in these spaces and (4) $\Rightarrow$ (5) $\Rightarrow$ (6) $\Rightarrow$ (1) is straightforward. Note that if we rewrite (4) by means of the poset $P(X)$ we obtain [**8**, 3.5]. $\square$

The following results are an account of point-set topological properties satisfied by T_0 (and non-T_0) Alexandroff spaces.

Theorem 2.8. *Let X be a T_0-Alexandroff space.*

1. *X is locally path-connected.*
2. *X is first countable.*
3. *X is orthocompact.*
4. *X is paracompact if and only if every $V(x)$ meets only a finite number of $V(y)$, so if X is paracompact, them X is locally finite (but not conversely).*
5. *X is second countable if and only if it is countable.*
6. *X is separable if and only if $X = \bigcup_{n=1}^{\infty} \overline{\{x_n\}}$.*
7. *X is Lindelöf if and only if $X = \bigcup_{n=1}^{\infty} V(x_n)$.*
8. *There exists Lindelöf T_0-Alexandroff spaces that are not separable and separable T_0-Alexandroff spaces that are not Lindelöf.*
9. *If X is finite, then X is compact.*
10. *If X is locally finite, then it is locally compact.*
11. *X is countable if and only if X is locally countable and Lindelöf.*
12. *If X is locally finite, X is compact if and only if X is finite.*

Proof. 1. Apply the preceding theorem to the minimal neighborhood.

2. Obvious,

3. The minimal base is an open interior/preserving refinement of every open covering.

4. The minimal base is a refinement of every open cover, so the first assertion is clear from the definition of paracompactness.

To prove assertion, suppose not, that is, there exists $x \in X$ such that $V(x)$ is infinite. Then $V(y) \subset V(x)$ for every $y \in V(x)$, a contradiction. $X = \mathbb{N}$ with $V(n) = \{1, \dots, n\}$ is locally finite (and countable) but not paracompact, since $V(1)$ meets every $V(n)$.

5. Countable and first countable, in any topological space, implies second countable. Since second countable mean that the minimal base is countable, so is the space.

6. If D is a countable dense subset, $D = \{x_n : n \in \mathbb{N}\}$ and for every $x \in X$, $V(x) \cap D$ is nonempty, hence $x_n \in V(x)$ for some $n \in \mathbb{N}$, what means $x \in \overline{\{x_n\}}$, so $X = \bigcup_{n=1}^{\infty} \overline{\{x_n\}}$. The converse is in the same way.

7. Apply that X is Lindelöf to the covering $\{V(x) : x \in X\}$. For the converse, if $\mathcal{U}$ is an open covering, for each $U \in \mathcal{U}$ there exists $n \in \mathbb{N}$ such that $x_n \in U$. Rename that U as U_n and note that $V(x_n) \subset U_n$.

8. If $X = \mathbb{R}_0^+$ and $V(0) = X$, $V(x) = \{x\}$ for every $x > 0$, we have that $\overline{\{0\}} = \{0\}$ and $\overline{\{x\}} = \{0, x\}$ for every $x > 0$, so X is Lindelöf but not separable.

On the other hand, if $X = [0, \omega_1[$, the first uncountable ordinal and $V(x) = \{y \in X : y \leq x\}$, $X = \overline{\{0\}}$, so X is separable, but $\bigcup_{n=1}^{\infty} V(x_n) = [0, \operatorname{Sup}\{x_n : n \in \mathbb{N}\}]$ and $\operatorname{Sup}\{x_n : n \in \mathbb{N}\} = \alpha < \omega_1$ (the countable supremum of countable ordinals is countable).

9. Obvious.

10. Obvious.

11. From (7) X is a countable union of countable subsets.

12. Apply compactness to the covering $\{V(x) : x \in X\}$. □

Theorem 2.9. *Let X be an Alexandroff space.*

1. *X is regular if and only if $V(x)$ is closed for every $x \in X$ (hence X is 0-dimensional).*
2. *If X is regular and compact, then X is locally compact.*
3. *If X is regular and separable, then X is perfectly normal.*
4. *X is pseudo-metrizable if and only if $V(x)$ is closed and finite for every $x \in X$.*

Proof. 1. X is regular means X locally closed.

2. Obvious.

3. Note first that if X is regular, from (1) we have that $\overline{\{x_n\}} \subset V(x_n)$ and (6) and (7) of 2.8 and the separability of X give X is Lindelöf. Recall also that if X is regular and Lindelöf, it is also normal. As in (6) of 2.8, we can write any open set A as $A = \bigcup_{n=1}^{\infty} \overline{\{x_n\}}$ (the fact that $\overline{\{x_n\}} \subset A$ comes from regularity), so every open set is F_σ; this together with the normality gives perfect normality.

4. From the Nagata-Smirnov's pseudo-metrization theorem, X is semimetrizable if and only if X is regular and has a σ-locally finite base. Since a theorem of Michael says that a space is paracompact if and only if every open covering has a σ-locally finite refinement, with a reasoning similar to that of (3) of 2.8 we have that X has a σ-locally finite base if and only if X is paracompact. But under the additional condition of being $V(x)$ closed for every $x \in X$ we have that paracompact is equivalent to locally finite ($V(y) \cap V(X) \neq \emptyset$ means $y \in \overline{V(x)} = V(x)$, so the set of y's such that $V(y)$ meets $V(x)$ is the set of elements of $V(x)$). □

Note that the only T_0-Alexandroff pseudo-metrizable spaces are the discrete ones (which are metrizable in fact).

3. Alexandroff Spaces and Spaces of Functions. Homotopy

Let X be a topological space and let Y be a T_0-Alexandroff space. Denote by $C(X,Y)$ the space of continuous mappings from X to Y with the compact open topology. We partially order $C(X,Y)$ by $f \leq g$ if and only if $f(x) \leq g(x)$ for every $x \in X$. The following result can be proved as Proposition 9 is in [**8**].

Theorem 3.1. *Let X be a topological space and let Y be a T_0-Alexandroff space. The intersection of all open sets in $C(X,Y)$ containing the map f is $\{g \in C(X,Y) : g \leq f\}$. That is, $C(X,Y)$ is T_0-Alexandroff and the order in $P(C(X,Y))$ is just the order defined above.*

We also quote from [**4**] the following standard result:

Lemma 3.2. *Let X and Y be topological spaces and suppose that X is locally compact. Then:*

1. *If $\phi\colon X \times I \to Y$ is continuous, so is $\phi_1\colon I \to C(X,Y)$ defined as $\phi_1(t)\colon X \to Y$ for every $t \in I$, where $\phi_1(t)(x) = \phi(t,x)$ for every $x \in X$.*
2. *If $\psi\colon I \to C(X,Y)$ is continuous, so is $\psi_1\colon X \times I \to Y$ defined as $\psi_1(x,t) = \psi(t)(x)$.*

In particular, these conditions are satisfied if X is an Alexandroff and locally finite space.

Thus one has:

Corollary 3.3. *Let X and Y be Alexandroff spaces and suppose that X is locally finite. Then*

1. *The homotopy classes of maps from X to Y are in one-to-one correspondence with the connected components of $C(X,Y)$.*
2. *If $f,g\colon X \to Y$ and $f \leq g$, then f is homotopic to g by a homotopy which keeps pointwise fixed the set $\{x \in F : f(x) = g(x)\}$.*

Now we are going to obtain a homotopy-type classification in a way similar to that of Section 4 of [**8**] for finite spaces.

Definition 3.4. Let X be an Alexandroff space.

1. $x \in X$ is linear if when it exists $y, z > x$ then $z \geq y$.
2. $x \in X$ is colinear if when it exists $y, z < x$ then $z \leq y$.
3. X is said to be a core (with base point $p \in X$) if it is T_0 and there exists no linear or colinear points (except possibly p).
4. A core of the space X (with base point $p \in X$) is a subspace X_1 of X (with the same base point) such that X_1 $((X_1,p))$ is a core and such that X_1 is a strong deformation retract of X.

The proof of Theorem 2 of [**8**] cannot be extended to Alexandroff spaces, even if they are countable and locally finite, since the space $\mathbb{N}$ topologized by $V(n) = \{0, \ldots, n\}$ is an Alexandroff space with $P(\mathbb{N})$ the natural numbers with the usual order, have core (take $F = \{0\}$ and the obvious mapping verifies $f \leq$ indentity, so is a strong deformation retract) but that core cannot be constructed as in [**8**], since you may take $F_0 = \mathbb{N} - \{0\}$ as the first step of induction ($\{0\}$ is a linear point) and you will never obtain a core.

On the other hand, $X(\mathbb{Z}, \leq)$, where $\leq$ is the usual order in $\mathbb{Z}$, is a countable but not locally finite T_0-Alexandroff space without core. So it arise the question of a different proof of Theorem 2 of [**8**] for locally finite T_0-Alexandroff spaces (or a counterexample). Anyway, assuming there is a core, Theorems 3 and 4 can be easily generalized.

Theorem 3.5. *Let X (or (X,p)) be a locally finite core. Then any mapping $f \colon X \to X$ (preserving the base point) which is homotopic to the identity (relative to base points) is the identity.*

Proof. The same of Theorem 3 of [**8**]; local finiteness let us to make the same introduction as in that proof and also let us to apply 3.3. □

Theorem 3.6. *Let X and Y be two locally finite Alexandroff spaces (with base point $p \in X$ and $q \in Y$) and suppose they have cores X_1 and Y_1. Then X is homotopy equivalent to Y if and only if X_1 is homeomorphic to Y_1 (relative to base points).*

Proof. The same as in Theorem 4 of [**8**]. □

Since contractible means homotopy equivalent to a point, it follows:

Corollary 3.7. *Let X be a locally finite Alexandroff space. Then X is contractible if and only if some point of X is a strong deformation retract of X.*

Finally, the same proof given in [**8**] for Proposition 12 gives for Alexandroff spaces the following:

Proposition 3.8. *Let (X,p) be a core, $x \in X$. Then:*

1. *x is less than two distinct maximal points, or*
2. *x is maximal, or*
3. *x is linear under a maximal point; hence $x = p$.*

and

1. *x is greater than two distinct minimal points, or*
2. *x is minimal, or*
3. *x is colinear over a minimal point; hence $x = p$.*

4. Alexandroff Spaces and Quasi-Uniform Spaces

Another interesting way to handle Alexandroff spaces is the use of quasi-uniformities. As it is well-known, quasi-uniformities provide an useful tool to develop properties of any topological space. In the case of Alexandroff spaces we obtain a very simple characterization of these spaces on terms of bases of quasi-uniformities.

Theorem 4.1. *Let X be a topological space. X is an Alexandroff space if and only if there exists a subset A of $X \times X$ containing the diagonal Δ such that $\{A\}$ is a base for a quasi-uniformity compatible with the topology of X.*

Proof. According to [**3**], the whole topology is an interior preserving family (since the intersection of every family of open sets is open), so the finest quasi-uniformity associated to X is $\mathcal{FT} = \{U_{\mathcal{C}} : \mathcal{C} \subset \mathcal{T}\}$, where $U_{\mathcal{C}} = \{(x, y) \in X \times X : x \in X, y \in \bigcap_{x \in C \in \mathcal{C}} C\} = \bigcup_{x \in X} \{x\} \times W(x, \mathcal{C})$, where $W(x, \mathcal{C}) = \bigcap_{x \in C \in \mathcal{C}} C$. Since $V(x) \subset W(x, \mathcal{C})$ for every $\mathcal{C} \subset \mathcal{T}$, we have that $\{A\}$ is a base for $\mathcal{FT}$ if $A = \bigcup_{x \in X} \{x\} \times V(x)$, which clearly contains the diagonal.

Conversely, from Proposition 1.4 of [**3**], if $\{A\}$ is a base for a quasi-uniformity compatible with the topology of X, then $\{A(x)\}$ is a neighborhood base for x in X, so every point of X has a minimal neighborhood and then X is Alexandroff.

In fact, since $A(x) = \{y \in X : (x, y) \in A\}$ and clearly A can be written as $A = \bigcup_{x \in X} \{x\} \times A(x)$, we have that the finest quasi-uniformity associated to the topological space associated to $\{A\}$ is just the given quasi-uniformity. $\square$

Thus, the theorem essentially says that Alexandroff spaces are the only spaces whose topology is determined by only one set.

From this theorem we can say that a quasi-uniform space is Alexandroff if it has a base of the quasi-uniformity with only one member.

Recall now that given an uniformity $\mathcal{U}$ we can construct two uniformities $\mathcal{U}^{-1}$ and $\mathcal{U}^*$. One can easily check that $\mathcal{U}^{-1}$ is the uniformity with $\{A^{-1}\}$ as a base and $\mathcal{U}^*$ is the uniformity with $\{A^*\}$ as a base, and A^* is just the diagonal when the space is T_0 (sketch: $(x, y) \in A^*$ if and only if $(x, y) \in A$ and $(y, x) \in A$, hence $V(x) = V(y)$, so $x = y$).

So any uniform property that relates properties of $\mathcal{U}, \mathcal{U}^{-1}$ and $\mathcal{U}^*$ becomes easy to handle, since the first two are generated by one set and the third is the discrete one. As an example we have the following.

Corollary 4.2. *Any Alexandroff uniform space is complete and bicomplete.*

Proof. $(X, \mathcal{U})$ is bicomplete if and only if $(X, \mathcal{U}^*)$ is complete, and the discrete uniformity always is.

About completeness, recall that in this case, a filter $\mathcal{F}$ is $\mathcal{U}$-Cauchy if there is $x \in X$: $A(x) \in \mathcal{F}$. Since $A(x) = V(x)$ and $V(x)$ is the minimal neighborhood of x, the filter $\mathcal{F}$ converges to x, hence $(X, \mathcal{U})$ is complete. $\square$

From 4.1, it is easy to associate a quasi-proximity to the topology of X.

Theorem 4.3. *Let X be an Alexandroff space. Then $A\delta B$ if and only if $A\cap\overline{B}\neq\emptyset$ is a quasi-proximity compatible with the topology (so $A\ll B$ if and only if $A\subset B^0$ is a strong inclusion compatible with the topology).*

Proof. The quasi-proximity associated to the finest quasi-uniformity compatible with X is $A\delta B$ if and only if $A\times B\cap U\neq\emptyset$ for every $\bigcup_{x\in X}\{x\}\times V(x)\subset U$, that is, $A\delta B$ if and only if there exists $(x,y)\in A\times B: y\in V(x)$, what is equivalent to $A\cap\overline{B}\neq\emptyset$. □

From the above results we have the functors $\mathcal{A}$ and $\mathcal{A}^{-1}$ between the full subcategory of Alexandroff topological spaces ($\mathcal{AT}$) and the full subcategory of Alexandroff quasi-uniform spaces ($\mathcal{AU}$) defined as $\mathcal{A}(X,T)=(X,\mathcal{U})$ and $\mathcal{A}(f)=f$ and $\mathcal{A}^{-1}(X,\mathcal{U})=(X,T)$ and $\mathcal{A}^{-1}(f)=f$, where T and $\mathcal{U}$ are defined according to 4.1.

It is clear that if f is quasi-uniformly continuous from one quasi-uniform Alexandroff space into another one, it is continuous between the induced topological spaces. Conversely, if $f\colon X\to Y$ is continuous, then $f^{-1}(W(f(x)))$ is a a neighborhood of x, so it contains $V(x)$, that is, $\{x\}\times V(x)\subset f^{-1}(f(x))\times f^{-1}(W(f(x)))$, hence $\bigcup_{x\in X}\{x\}\times V(x)\subset\bigcup_{y\in Y}\{f^{-1}(y)\}\times f^{-1}(W(y))=(f\times f)^{-1}(\bigcup_{y\in Y}\{y\}\times W(y))$, that is $A_X\subset(f\times f)^{-1}(A_Y)$ and the quasi-uniformities are defined as those subsets of the product containing A_X and A_Y respectively. So it only remains to apply the definition of quasi-uniformly continuous mapping.

So $\mathcal{A}$ and $\mathcal{A}^{-1}$ are both functors, each inverse of the other, and $\mathcal{AU}$ and $\mathcal{AT}$ are equivalent categories.

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